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METAL PLATE WORK

C. T. MILLIS.

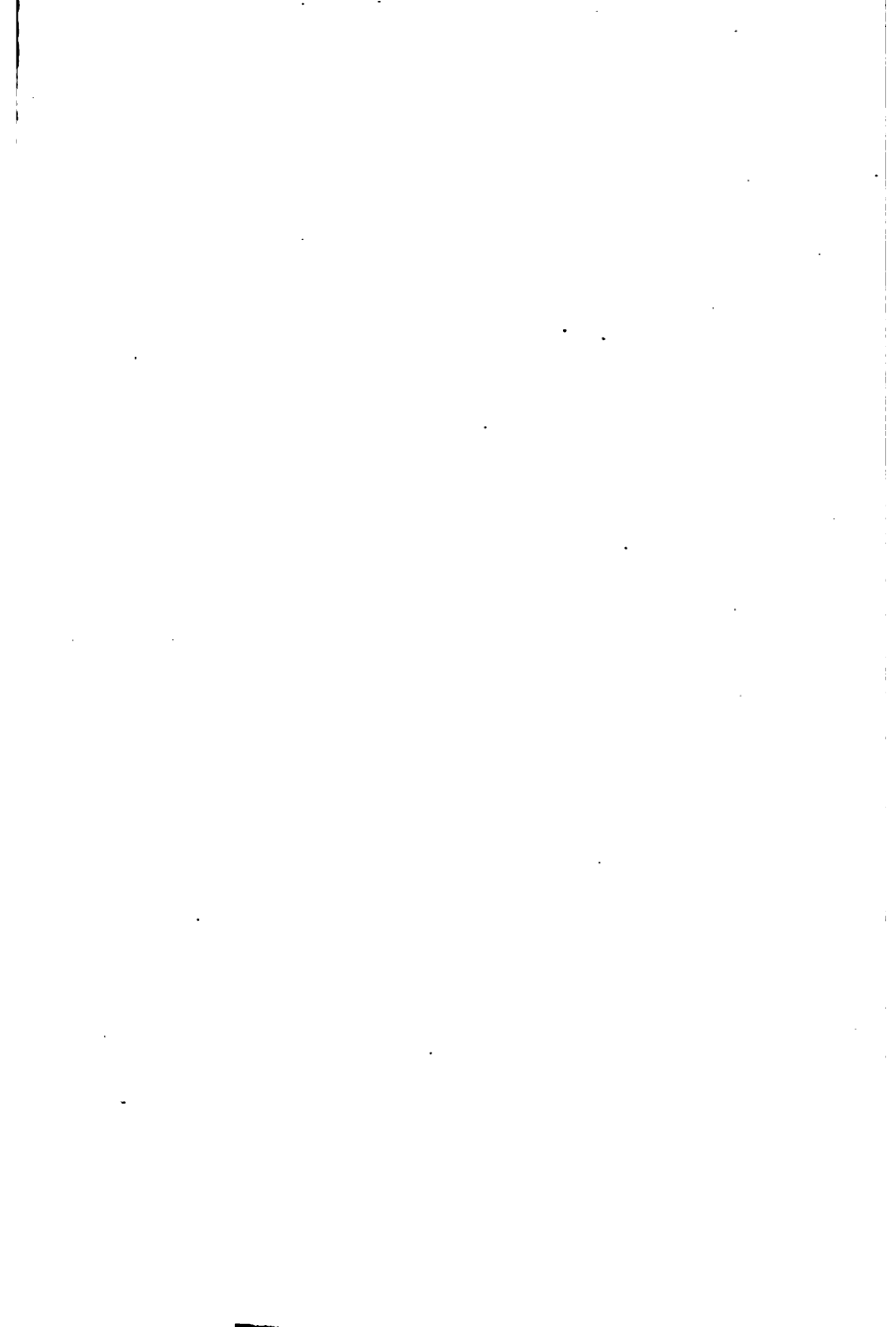
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METAL-PLATE WORK



Finsbury Technical Manuals

METAL-PLATE WORK

ITS PATTERNS AND THEIR GEOMETRY

ALSO

NOTES ON METALS, AND RULES IN MENSURATION

FOR THE USE OF

**TIN, IRON, AND ZINC PLATE WORKERS, COPPERSMITHS,
BOILER-MAKERS, PLUMBERS AND OTHERS**

BY

C. T. MILLIS, M.I. MECH. E.

**EXAMINER IN METAL PLATE WORK TO THE CITY AND GUILDS OF LONDON INSTITUTE;
PRINCIPAL, EDUCATIONAL DEPARTMENT, BOROUGH POLYTECHNIC INSTITUTE**

THIRD EDITION, CONSIDERABLY ENLARGED

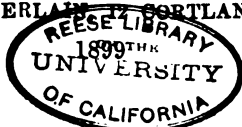


London

E. & F. N. SPON, LTD., 125 STRAND

New York

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PREFACE

TO

THE THIRD EDITION.

THE *Third Edition* of this work contains, with 50 new diagrams and illustrations, more than 80 pages of additional matter. Books I. and II. remain as before. Book III. contains several new and important problems in "penetrations." Book IV. is almost new. Towards Chapter I., which describes Pipe-Bending, help was furnished to the author by his former student and assistant, Mr. W. H. Bennett, now Instructor in *Metal-Plate Work* at the Regent Street Polytechnic. Chapter II. treats of the making up of Pipe Bends out of flat sheet, and deals with that subject as it has never before been dealt with—systematically and comprehensively. To Book V. has been added an article on "Tinning and Retinning."

The system of geometric construction for the patterns of sheet-metal articles set forth in this book, came of investigation made by the author about twenty-two years ago. In 1885, after several years of thorough testing, the system was illustrated in model and diagram at the South Kensington Inventions Exhibition, and in 1887 further made public in the First Edition. A workman, studying the book, learns to recognise easily the nature of any surface he meets with, whether flat or curved, and how to deal with it for his

purposes. Indirect evidence of the value of the book is afforded by the extent to which other writers—with no acknowledgement, however, of their indebtedness—have turned to it for “copy” and model; and direct evidence in the frequent communications received from actual sheet-metal workers, who, by the author's system, have readily solved problems, very difficult if not impossible of solution otherwise.

As with the Second Edition, so now, I have to acknowledge the very valuable assistance given me by my friend, Mr. H. W. Ley, in revising the new matter, and seeing the book through the press.

C. T. M.

BOROUGH POLYTECHNIC INSTITUTE, LONDON :

April, 1899.

PREFACE

TO

THE SECOND EDITION.

THIS *Second Edition*, long delayed through pressure of work, is the First Edition augmented by about 70 more pages of original matter, making the book still more useful to the sheet-metal worker who desires to know his trade, and become an all-round hand, instead of being cramped up in a narrow everyday groove. The specialisation in the shops brought Technical Education about ; the increasing tendency to it marks the need of a wide basis for such education.

The divisions of tin-plate, iron-plate, and zinc workers did not formerly exist ; they do not exist now among the older workmen, and should not among the younger. The shapes treated in this volume come before all sheet-metal workers ; the main differences of their work are not those of shape, but of size and consequent manipulation.

Blinn's 'Workshop Companion for Tin, Sheet Iron, and Copper Plate Workers,' is a book published in America and circulating here. More than one-quarter of that portion which deals with patterns in its 1891 edition is neither more nor less than a word for word appropriation, without acknowledgment, of original problems, diagrams and all, from my pages. The *Preface* to the book can well afford to make much of "the addition of new matter."

My thanks are due to Mr. H. W. Ley, Secretary of the Westminster Technical Institute during my Directorship of it, for many useful suggestions, and for very careful revision of its proof sheets.

C. T. M.

BOROUGH POLYTECHNIC INSTITUTE, LONDON,
March 1893.

PREFACE.

IN the pages that follow, the setting-out of patterns for metal-plate workers is *systematised* (for the first time, so far as I am aware), and by the system laid down, nearly all the patterns required by sheet-metal workers can be set out on one general geometrical principle. Thus treated, the subject becomes a comparatively easy one, and the workman learns how, on the given principle, to develop for himself the surface of any article he may have to make, to the saving, as against various methods practised hitherto, of both time and material.

Of the methods heretofore taught in the workshop or otherwise, some have no application beyond the particular article with respect to which they are described, some are absolutely impracticable. The methods that these pages explain are applicable always; and have been proved by abundant experience in my classes.

The commencement of the book was a series of articles that were written by me for *The Ironmonger*. These and my class and lecture notes have greatly aided me in my effort to make the book comprehensive, as well as at the same time a welcome workshop companion. Not only does it contain

patterns which are essentially those of numerous articles of every-day manufacture, but a *special feature* is made of large work ;—and how it may be dealt with under ordinary workshop conditions. Each of the problems is complete in itself; but although solved independently they follow each other in due order.

It is my hope that the book will be an aid to students in engineering and geometry, as well as to those for whom it is particularly written; that it will be a serviceable addition to the scanty and insufficient literature on the application of geometry to metal-plate work; and that it will in some degree assist the cause of technical education.

C. T. M.

CITY AND GUILDS OF LONDON TECHNICAL COLLEGE,
FINSBURY, LONDON,

June 1887.

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METAL-PLATE WORK.

BOOK I.

CHAPTER I.

CLASSIFICATION.

(1.) NOTWITHSTANDING the introduction of machinery and the division of labour in the various branches of metal-plate work, there is as great a demand for good metal-plate workers as ever, if not indeed a greater demand than formerly, while the opportunities for training such men are becoming fewer. An important part of the technical education of those connected with sheet-metal work is a knowledge of the setting-out of patterns. Such knowledge, requisite always by reason of the variety of shapes that are met with in articles made of sheet-metal, is nowadays especially needful; in that the number of articles made of sheet metal, through the revival of art metal-work, the general advance of science, and the introduction of new designs (which in many cases have been very successful), in articles of domestic use, has considerably increased. It is with the setting-out of patterns that this volume principally deals. To practical men, the advantages in saving of time and material, of having correct patterns to work from, are obvious. Whilst, however, the method of treatment here of the subject will be essentially practical, an

amount of theory sufficient for a thorough comprehension of the rules given will be introduced, a knowledge of rules without principles being mere 'rule of thumb,' and not true technical education.

(2.) Starting in the following pages with some introductory problems and other matter, we shall proceed from these to the articles for which patterns are required by sheet-metal workers and which may be thus conveniently classed and subdivided:

CLASS I.— <i>Patterns for Articles of equal taper or inclination</i> (pails, oval teapots, gravy strainers, &c.).	Sub- divisions.	a. Of round surfaces.
		b. Of plane or flat surfaces.
		c. Of curved and plane surface combined.

CLASS II.— <i>Patterns for Articles of unequal taper or inclination</i> (baths, hoppers, canister-tops, &c.).	Sub- divisions.	a. Of round surfaces.
		b. Of plane or flat surfaces.
		c. Of curved and plane surface combined.

CLASS III.—*Patterns for Miscellaneous Articles* (elbows, and articles of compound bent surface, as vases, aquarium stands, mouldings, &c.).

All these articles will be found dealt with in their several places.

We shall conclude with a few technical details in respect of the metals that metal-plate workers mostly make use of.

(3.) The setting out of patterns in sheet-metal work belongs to that department of solid geometry known as "Development of Surfaces," which may be said to be the spreading or laying out without rupture the surfaces of solids in the plane or flat, the plane now being sheet metal.

CHAPTER II

INTRODUCTORY PROBLEMS ; WITH APPLICATIONS.

DEFINITIONS.

Straight Line.—A straight line is the shortest distance between two points.

NOTE.—If not otherwise stated, lines are always supposed to be straight.

Angle.—An angle is the inclination of two lines, which meet, one to another. The lines AB , CB in Fig. 1 which are inclined to each other, and meet in B , are said to form an *angle* with one another. To express an angle, the letters which denote the two lines forming the angle are employed, the letter at the *angular point* being placed in the middle; thus, in Fig. 1, we speak of the angle ABC .

FIG. 1.

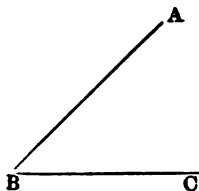


FIG. 2.

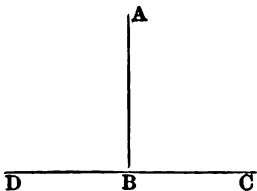
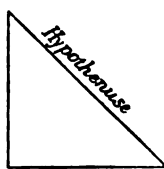


FIG. 3.



Perpendicular. Right Angles. — If a straight line, AB (Fig. 2), meets or stands on another straight line, CD , so that the adjacent angles (or angles on either side of AB) ABD , ABC , are equal, then the line AB is said to be *perpendicular* to, or *at right angles* with ('square with') DC , and each of the angles is a right angle.

Parallel Lines.—Parallel lines are lines which, if produced ever so far both ways, do not meet.

Triangle.—A figure bounded by three lines is called a triangle.

A triangle of which one of the angles is a right angle is called a *right-angled* triangle (Fig. 3); and the side which joins the two sides containing the right angle is called the *hypotenuse* (or hypotenuse). If all the sides of a triangle are equal, the triangle is *Equilateral*. If it has two sides equal, the triangle is *Isosceles*. If the sides are all unequal, the triangle is *Scalene*.

Polygon.—A figure having more than four sides is called a *polygon*. Polygons are of two classes, *regular* and *irregular*.

Irregular Polygons have their sides and angles unequal.

Regular Polygons have all their sides and angles equal, and possess the property (an important one for us) that they can always be inscribed in circles; in other words, a circle can always be drawn through the angular points of a regular polygon (Figs. 12 and 13).

Special names are given to regular polygons, according to the number of sides they possess; thus, a polygon of five sides is a *pentagon*; of six sides, a *hexagon*; of seven, a *heptagon*; of eight, an *octagon*; and so on.

Quadrilaterals.—All figures bounded by four lines are

FIG. 4a.

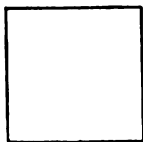
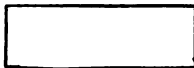


FIG. 4b.



called quadrilaterals. The most important of these are the *square* and *oblong* or *rectangle*. In a square (Fig. 4a) the sides are all equal and the angles all right angles, and consequently equal. An oblong or rectangle has all its angles right angles, but only its opposite sides are equal. (Fig. 4b.)

Circle.—A circle is a figure bounded by a curved line such that all points in the line are at an equal distance from a certain point within the figure, which point is called the *centre*.

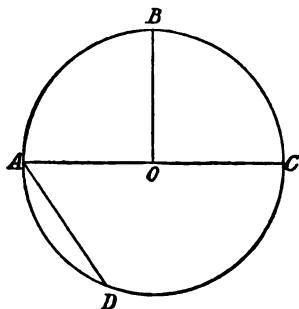
The bounding line of a circle is called its *circumference*. A part only of the circumference, no matter how large or small, is called an *arc*. An arc containing a quarter of the circumference is a *quadrant*. An arc containing half the circumference is a *semicircle*. A line drawn from the centre to any point in the circumference is a *radius* (plural, *radii*). The line joining the extremities of any arc is a *chord*. A chord that passes through the centre is a *diameter*.

A line drawn from the centre of, and perpendicular to, any chord that is not a diameter of a circle, will pass through its centre.

In practice a circle, or arc, is 'described' from a chosen, or given, centre, and with a chosen, or given, radius.

If two circles have a common centre, their circumferences are always the same distance apart.

FIG. 5.



In Fig. 5.

O is the centre.

A D (the curve) is an arc.

A B (or B C) is a quadrant.

A C B is a semicircle.

O A (or O B, or B C) is a radius.

A D (the straight line) is a chord.

A C is a diameter.

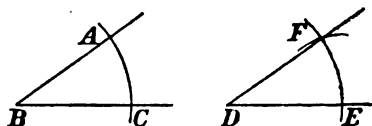
PROBLEM I.

To draw an angle equal to a given angle.

CASE I.—Where the 'given' angle is given by a drawing.

This problem, though simple, is often very useful in practice, especially for elbows, where the angle (technically called 'rake' or 'bevil') is marked on paper, and has to be copied.

FIG. 6.



Let ABC (Fig. 6) be the given angle. With B as centre and radius of any convenient length, describe an arc cutting BA , BC (which may be of any length, *see* Def.) in points A and C . Draw any line DE , and with D as centre and same radius as before, describe an arc cutting DE in E . With E as centre and the straight line distance from A to C as radius, describe an arc intersecting in F the arc just drawn. From D draw a line through F ; then the angle FDE will be equal to the given angle ABC .

CASE II.—Where the given angle is an angle in already existing fixed work.

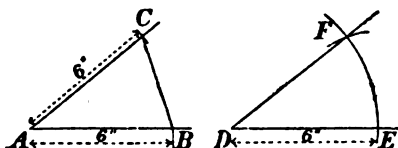
The angle to which an equal angle has to be drawn, may be an angle existing in already fixed work, fixed piping for instance; or in brickwork, when, suppose, a cistern may have to be made to fit in an angle between two walls. In such cases a method often used in practice is to open a two-fold rule in the angle which is to be copied. The rule is then laid down on the working surface, whatever it may be (paper, board, &c.), on which the work of drawing an angle equal to the existing angle has to be carried out, and lines are drawn on that surface, along either the outer or inner edges of the

rule. The rule being then removed, the lines are produced; meeting, they give the angle required.

CASE III.—Where the given angle is that of fixed work, and the method of CASE II. is inapplicable.

With existing fixed work, the method of CASE II. is not always practicable. A corner may be so filled that a rule cannot be applied. The method to be now employed is as follows. Draw lines on the fixed work, say piping, each way from the angle; and on each line, from the angle, set off any the same distance, say 6 in., and measure the distance between the free ends of the 6-in. lengths. That is, if A C, A B (Fig. 7)

FIG. 7.



represent the lines drawn on the piping, measure the distance between B and C. Now on the working surface on which the drawing is to be made, draw any line DE, 6 in. long; and with D as centre and radius DE, describe an arc. Next, with E as centre, and the distance just measured between B and C as radius, describe an arc cutting the former arc in F. Join FD; then the angle FDE will be equal to the angle of the piping.

NOTE.—When points are 'joined,' it is always by *straight* lines.

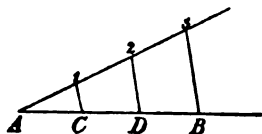
PROBLEM II.

To divide a line into any number of equal parts.

Let AB (Fig. 8) be the given line. From one of its extremities, say A, draw a line A3 at any angle to AB, and on it, from the angular point, mark off as many parts,—of any con-

venient length, but all equal to each other,—as AB is to be divided into. Say that AB is to be divided into three equal parts, and that the equal lengths marked off on AB are A to

FIG. 8.



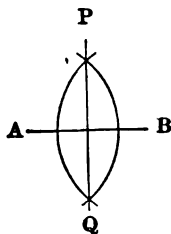
1, 1 to 2, and 2 to 3. Then join point 3 to the B extremity of AB , and through the other points of division, here 1 and 2, draw lines parallel to $3B$, cutting AB in C and D . Then AB is divided as required.

PROBLEM III.

To bisect (divide a line into two equal parts) a given line.

Let AB (Fig. 9) be the given line. With A as centre, and any radius greater than half its length, describe an indefinite arc; and with B as centre and same radius, describe an arc intersecting the former arc in points P and Q . Draw a line through P and Q ; this will bisect AB .

FIG. 9.



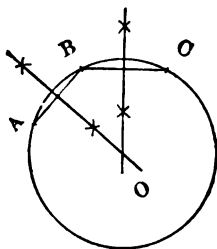
NOTE.—It is quite as easy to bisect AB by Problem II.; but the method shown gives, in PQ , not only a line bisecting AB , but a line perpendicular to AB . This must be particularly remembered.

PROBLEM IV.

To find the centre of a given circle.

Let $A B C$ (Fig. 10) be the given circle. Take any three points A, B, C , in its circumference. Join $A B, B C$; then $A B, B C$, are chords (*see* Def.) of the circle $A B C$. Bisect $A B, B C$; the point of intersection, O , of the bisecting lines is the centre required.

FIG. 10.



PROBLEM V.

To describe a circle which shall pass through any three given points that are not in the same straight line.

Let A, B, C (Fig. 10) be the three given points. Join $A B, B C$. Now the circle to be described will not be a circle through A, B, C , unless $A B, B C$, are chords of it. Let us therefore assume them such, and so treating them, find (by Problem IV.) O the centre of that circle. With O as centre, and the distance from O to A as radius, describe a circle; it will pass also through B and C , as required.

PROBLEM VI.

Given an arc of a circle, to complete the circle of which it is a portion.

Let $A C$ (Fig. 10) be the given arc; take any three points in it as A, B, C ; join $A B, B C$. Bisect $A B, B C$ by lines

intersecting in O. With O as centre, and O to A or to any point in the arc, as radius the circle can be completed.

PROBLEM VII.

To find whether a given curve is an arc of a circle.

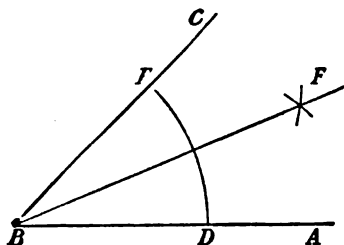
Choose any three points on the given curve, and by Problem V describe a circle passing through them. If the circle coincides with the given curve, the curve is an arc.

PROBLEM VIII.

To bisect a given angle.

Let ABC (Fig. 11) be the given angle. With B as centre and any convenient radius describe an arc cutting AB, BC in D and E. With D and E as centres and any convenient distance, greater than half the length of the arc

FIG. 11.



DE as radius describe arcs intersecting in F. Join F to B; then FB bisects the given angle.

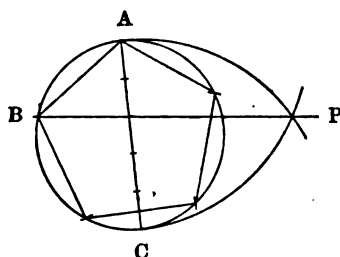
PROBLEM IX.

In a given circle, to inscribe a regular polygon of any given number of sides.

Divide (PROBLEM II.) the diameter AC of the given circle (Fig. 12) into as many equal parts as the figure is to have

sides, here say five. With A and C as centres, and C A as radius, describe arcs intersecting in P. Through P and the second point of division of the diameter draw a line P B

FIG. 12.



cutting the circumference in B; join B A, then B A will be one side of the required figure. Mark off the length B A from A round the circumference until a marking off reaches B. Then, beginning at point A, join each point in the circumference to the next following; this will complete the polygon.

NOTE.—By this problem a circumference, and therefore also one-half of it (semicircle), one-third of it, one-fourth of it (quadrant), and so on, can be divided into any number of equal parts.

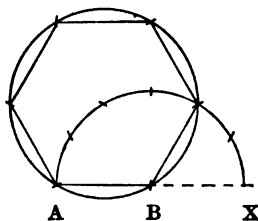
PROBLEM X.

To describe any regular polygon, the length of one side being given.

Let A B (Fig. 13) be the given side of, say, a hexagon. With either end, here B, as centre and the length of the given side as radius, describe an arc. Produce A B to cut the arc in X. Divide the semicircle thus formed into as many equal parts (PROBLEM IX., NOTE) as the figure is to have sides (six), and join B to the second division point of the semicircle counting from X. This line will be another side of the required polygon. Having now three points, A, B,

and the second division point from X, draw a circle through them (Problem V.), and, as a regular polygon can always be inscribed in a circle (*see* Def.), mark off the length B A round

FIG. 13.



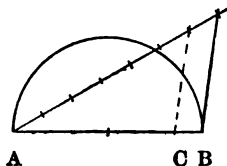
the circumference from A until at the last marking-off, the free extremity of the second side (the side found) of the polygon is reached, then, beginning at A, join each point in the circumference to the next following; this will complete the polygon (hexagon).

PROBLEM XI.

To find the length of the circumference of a circle, the diameter being given.

Divide the given diameter A B (Fig. 14) into seven equal parts (Problem II.). Then three times A B, with C B, one of the seven parts of A B, added, that is with one-seventh of

FIG. 14.

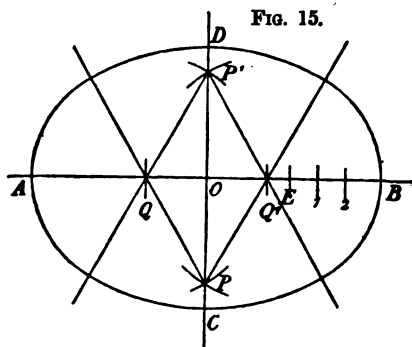


A B added, will be the required length of the circumference. The semicircle of the figure is superfluous, but may help to make the problem more clearly understood.

PROBLEM XII.

To draw an oval, its length and width being given.

Draw two lines AB , CD (the axes of the oval), perpendicular to one another (Fig. 15), and intersecting in O . Make



OA and OB each equal to half the length, and OC and OD each equal to half the width of the oval. From A mark off AE equal to CD the width of the oval, and divide EB into three equal parts. With O as centre and radius equal to two of the parts, as $E2$, describe arcs cutting AB in points Q and Q' . With Q and Q' as centres and QQ' as radius describe arcs intersecting CD in points P and P' . Join PQ , $P'Q$, $P'Q'$ and PQ' ; in these lines produced the end and side curves must meet. With Q and Q' as centres and QA as radius, describe the end curves, and with P and P' as centres and radius PD , describe the side curves; this will complete the oval.

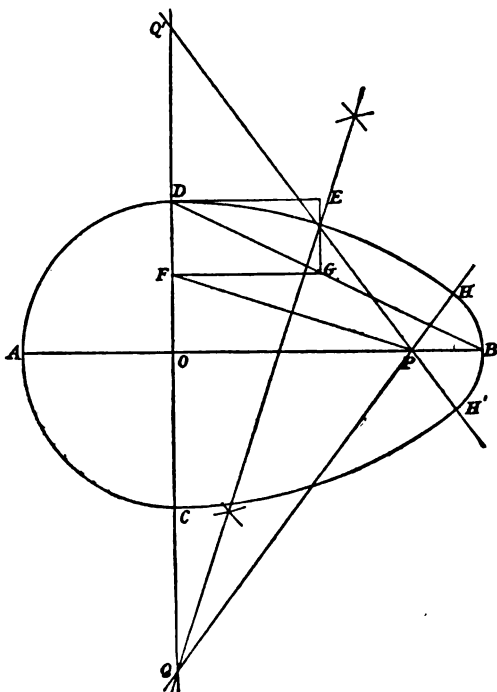
NOTE.—Unless care is taken, it may be found that the end and side curves will not meet accurately, and even with care this may sometimes occur. It is best if great accuracy be required in the length, to draw the end curves first, and then draw side curves to meet them; or, if the width is most important, to draw the side curves first. The centres (P and P') for the side curves come inside or outside the curves, according as the oval is broad or narrow. This figure is sometimes erroneously called an ellipse. It is, however, a good approximation to one, and for most purposes where an elliptical article has to be made, is very convenient.

PROBLEM XIII.

To draw an egg-shaped oval, having the length and width given.

Make $A B$ (Fig. 16) equal to the length of the oval, and from A set off $A O$ equal to half its width. Through O draw

FIG. 16.



an indefinite line $Q Q'$ perpendicular to $A B$, and with O as centre and $O A$ as radius describe the semicircle $C A D$. Join $D B$; and from D draw $D E$ perpendicular to $Q Q'$ and equal to $O D$. Also from E draw $E G$ parallel to $Q Q'$ and

intersecting DB in G, and from G draw GF parallel to DE and intersecting QQ' in F. From B set off BP equal to DF, and join PF. Bisect FP and through the point of bisection draw a line cutting QQ' in Q. Join QP and produce it indefinitely, and with Q as centre and QD as radius describe an arc meeting QP produced in H. Make OQ' equal to OQ, and join Q'P and produce it indefinitely. With Q' as centre, and Q'C (equal to QD) as radius, describe an arc meeting Q'P produced in H'. And with P as centre and PB as radius describe an arc to meet the arcs DH and CH' in H and H'; and to complete the egg-shaped oval.

PROBLEM XIV.

To describe an ellipse.

Before working this as a problem in geometry, let us draw an ellipse non-geometrically and get at some sort of a definition. This done, we will solve the problem geometrically, and follow that with a second mechanical method of describing the curve.

METHOD I.—MECHANICAL

A. Irrespective of dimensions.—On a piece of cardboard or smooth-faced wood, mark off any two points F, F' (Fig. 17) and fix pins securely in those points. Then take a piece of thin string or silk, and tie the ends together so as to form a loop; of such size as will pass quite easily over the pins. Next, place the point of a pencil in the string, and take up the slack so that the string, pushed close against the wood, shall form a triangle, as say, FDF', the pencil point being at D. Then, keeping the pencil upright, and always in the string, and the string taut, move the pencil along from left to right say, so that it shall make a continuous mark. Let us trace the course of the mark. Starting from D, the pencil, constrained always by the string, moves from D to P,

then on to B, P' , C, P^2 , P^3 , A, P^4 , and D again, describing a curve which returns into itself; this curve is an ellipse.

Having drawn the ellipse, let us remove the string and pins, draw a line from F to F' , and produce it both ways to terminate in the curve, as at B and A. Then AB is the *major axis* of the ellipse, and F, F' are its *foci*. The mid-point of AB is the *centre* of the ellipse. Any line through the centre and terminating both ways in the ellipse is a *diameter*. The major axis is the longest diameter, and is commonly called the length of the ellipse. The diameter through the centre at right angles to the major axis is the shortest diameter, or *minor axis*, or width of the ellipse.

Referring to the Fig. :—

ADPBC is an ellipse.

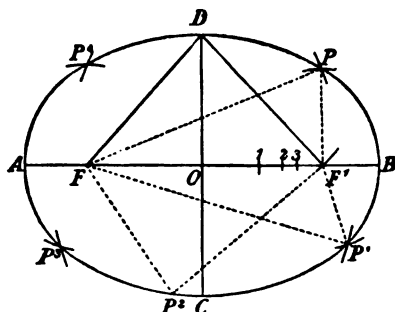
F, F' are its foci (singular, *focus*).

AB is the major axis.

CD is the minor axis.

O is the centre.

FIG. 17.



We notice with the string and pencil that when the pencil point reaches P, the triangle formed by the string is FPF' ; when it reaches P' , the triangle is $F'P'F$; when it reaches P^2 the triangle is FP^2F' ; and when P^3 is reached, it is FP^3F' . Looking at these triangles, it is obvious that

FF' is one side of each of them; from which it follows, seeing that the loop of string is always of one length, that the sum of the other two sides of any of the triangles is equal to the sum of the other two sides of any other of them; that is to say, FD added to DF' is equal to FP added to $P'F'$, is equal to FP' added to $P'F$, and so on.

Which leads us to the following definition.

DEFINITION.

Ellipse.—The ellipse is a closed curve (that is, a curve returning into itself), such that the sum of the distances of any point in the curve from certain two points (foci), inside the curve is always the same.

B. Length and width given.—Knowing now what an ellipse is, we can work to dimensions. Those usually given are the length (major axis), and width (minor axis). Draw AB, CD (Fig. 17), the given axes, and with either extremity, C or D , of the minor axis as centre, and half AB , the major axis as radius, describe an arc cutting AB in F and F' . Fix pins securely in F, F' and D (or C). Then, having tied a piece of thin string or silk firmly round the three pins, remove the pin at D (or C); put, in place of it, a pencil point in the string; and proceed to mark out the ellipse as above explained.

METHOD II.—GEOMETRICAL.—THE SOLUTION OF THE PROBLEM. LENGTH AND WIDTH GIVEN.

Draw AB, CD (Fig. 17), the major and minor axes. With C or D as centre, and half the major axis, OB say, as radius, describe arcs cutting AB in F and F' . On AB , and between O and F' , mark points—any number and anywhere, except that it is advisable to mark the points closer to each other as they approach F' . Let the points here be 1, 2, and 3. With F and F' as centres and $A2, B2$ as radii respectively, describe arcs intersecting in P ; with same centres and $A3, B3$ as radii respectively, describe arcs intersecting in P' . With F' and F as centres and $A3, B3$ as radii respectively,

describe arcs intersecting in P^3 . With same centres and $A 2, B 2$ as radii respectively, describe arcs intersecting in P^4 . Similarly obtain P^2 . We have thus nine points, $D, P, B, P', C, P^2, P^3, A$ and P^4 , through which an even curve may be drawn which will be the ellipse required. A greater number of points through which to draw the ellipse may of course be obtained by taking more points between O and F' , and proceeding as explained.

**METHOD III.—MECHANICAL.—LENGTH AND WIDTH
GIVEN.**

As it is not always possible to proceed as described at end of *Method I.*, for pins cannot always be fixed in the material to be drawn upon, we now give a second mechanical method. Having drawn (Fig. 18b) AB, CD , the

FIG. 18a.

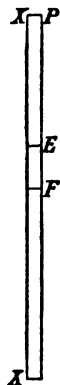
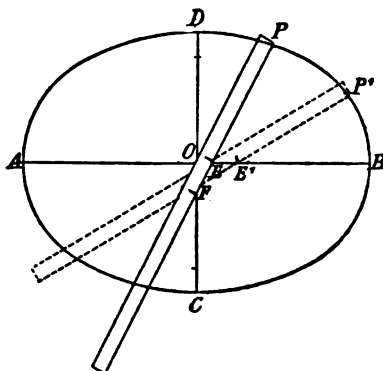


FIG. 18b.



given axes, then, on a strip of card or stiff paper XX (Fig. 18a), mark off from one end P , a distance PF equal to half the major axis (length), and a distance PE equal to half the minor axis (width). Place the strip on the axes in such a position that the point E is on the major axis, and the

point F on the minor, and mark a point against the point P. Now shift XX to a position in which E is closer to B, and F closer to C, and again mark a point against P. Proceed similarly to mark other points, and finally draw an even curve through all the points that have been obtained.

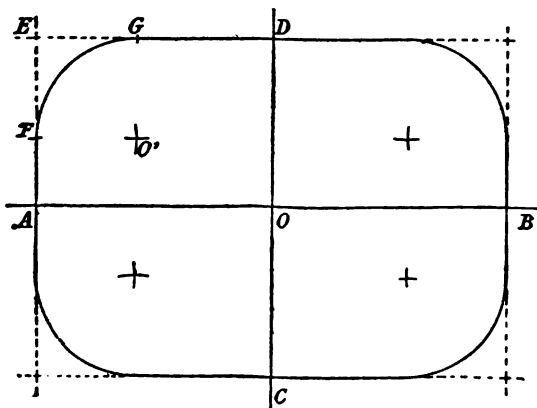
The following problems deal with shapes often required by the metal-plate worker, and will give him an idea of how to adapt to his requirements the problems that precede. The explanation of the measurement of angles that concludes the chapter will further assist him in his work.

PROBLEM XV.

To draw an oblong with round corners.

Draw two indefinite lines AB, CD (Fig. 19) perpendicular to one another and intersecting in O. Make OA

FIG. 19.



and OB each equal to half the given length; and OC and OD each equal to half the given width. Through O and D

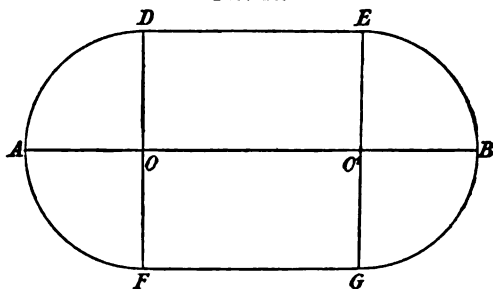
draw lines parallel to AB , and through A and B draw lines parallel to CD . We now have a rectangle or oblong, and require to round the corners, which are quadrants. Mark off from E along ED and EA equal distances EG and EF according to the size of corner required. With F and G as centres and EF or EG as radius, describe arcs intersecting in O' . With O' as centre and same radius describe the corner FG . The remaining corners can be drawn in similar manner.

PROBLEM XVI.

To draw a figure having straight sides and semicircular ends (oblong with semicircular ends).

Draw a line AB (Fig. 20) equal to the given length; make AO and BO' each equal to half the given width.

FIG. 20.



Through O and O' draw indefinite lines perpendicular to AB ; with O and O' as centres and OA as radius describe arcs cutting the perpendiculars through O and O' in D and F and G and E . Join DE , GF ; this will complete the figure required.

ANGLES AND THEIR MEASUREMENT.

The right angle BOC (Fig. 5) *subtends* the quadrant BC . If we divide that quadrant into 90 parts and call the parts

degrees, then a right angle subtends or contains 90 degrees (written 90°), or as usually expressed, is *an angle of 90 degrees*, the degree being the unit of measurement. If each division point of the quadrant is joined to O, the right angle is divided into 90 angles, each of which subtends or is an angle of 1 degree. That is to say, an angle is measured by the number of degrees that it contains. Suppose the quadrant BA is divided as was BC, then BOA also is an angle of 90 degrees. If the division is continued round the semicircle ADC, this will contain 180 degrees, and the whole circumference has been divided into 360 degrees. As an angle of 90, which is a fourth part of 360 degrees, subtends a quadrant or fourth part of the circumference of the circle, so an angle of 60, which is a sixth part of 360 degrees, subtends a sixth part of the circumference, and similarly an angle of 30 degrees subtends a twelfth part, an angle of 45 an eighth part, and so on. And this angular measurement is quite independent of the dimensions of the circle; the quadrant always subtends a right angle; the 60 degrees angle always subtends an arc of one sixth of the circumference; and the like with other angles. From our definition p. 5 we have it that a chord is the line joining the extremities of any arc. The chord of a sixth part of the circumference of any circle, we have now to add, is equal to the radius of that circle. This being the case, and as an angle of 60 degrees subtends the sixth part of the circumference of a circle, it follows that an angle of 60° subtends a chord equal to the radius.

SCALE OF CHORDS.

Construction.—We have now the knowledge requisite for setting out a scale of chords, by which angles may be drawn and measured.

On any line OB (Fig. 21) describe a semicircle OAB, and from its centre C draw CA perpendicular to OB. Divide OA into nine equal parts. Then, as OA, being a quadrant,

contains 90° , each of the nine divisions will contain 10° . The points of division, from O, of the quadrant, are marked 10, 20, 30, &c., up to 90 at A. With O as centre, describe arcs from each of these division points, cutting the line O B. Note that the arc from point 60 cuts O B in C, the centre of the semi-circle; the chord from O to 60 (not drawn in the Fig.), that

FIG. 21.

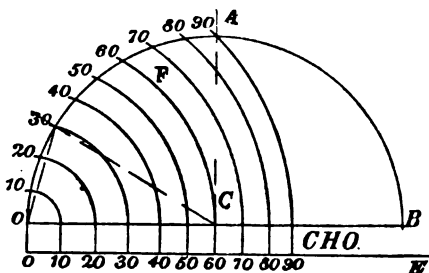
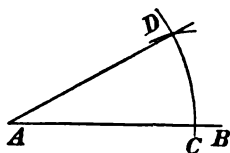


FIG. 22.



is, the chord of one-sixth of the circumference of the circle whose centre is C, being equal to the radius of that circle. Draw a line O E parallel to O B, and from O let fall O O perpendicular to O B. Also from each of the points where the arcs cut O B let fall perpendiculars to O B and number these consecutively to correspond with the numbers on the quadrant O A. The scale is now complete.

How to use. It is used in this way. Suppose from a point A in any line A B (Fig. 22) we have to draw a line at an angle of 30° with it. Then with A as centre and the distance from O to 60 on O E (Fig. 21) as radius, describe an arc C D cutting A B in C. And with C as centre and the distance from O to 30 on O E (Fig. 21) (the angle to be drawn is to be of 30°) as radius describe an arc intersecting arc O D in D. Join D A, then D A C will be the required angle of 30° . Similarly with angles of other dimensions.

In taking the lengths of arcs, we really take the length of their chords, and it is these lengths that (Fig. 21) we have

set off along O E. The angle (Fig. 21) O C F (the point F is the point 60) being an angle of 60° subtends a chord equal to the radius; therefore in O to 60 we have the radius C O. In the example (Fig. 22), the distance C D (O to 30) is the chord of 30° ; and it is clear that we must set this off on an arc C D of a circle of the same size as that employed in the construction of the scale, and this we do by making A C equal O to 60 on the same scale.

When a scale of chords has been constructed as explained, the semicircle may be cut away, and we thus get a scale convenient for shop use in the form of a rule.

CHAPTER III.

PATTERNS FOR ARTICLES OF EQUAL TAPER OR INCLINATION.

(CLASS I.)

(4.) It is necessary here at once to remark that ordinary workshop parlance speaks of 'slant,'—not as meaning an *angle*, but a *length*; not as referring to the angle of inclination of a tapering body, but to the length of its slanting portion. It is in this sense that we shall use the word, and shall employ the word 'taper' or the term 'inclination of slant' when meaning an angle.

(5.) In order that the rules for the setting out of patterns for articles of equal taper or inclination may be better understood and remembered, it is advisable to consider the principles on which the rules are based, as a knowledge of principles will often enable a workman himself to find rules for the setting out of patterns for odd work. The basis of the whole of the articles in this Class is the right cone. It is necessary, therefore, to define the right cone and explain some of its properties.

DEFINITION.

(6.) *Right Cone*.—A right cone is a solid figure generated or formed by the revolution of a right-angled triangle about one of the sides containing the right angle. The side about which the triangle revolves is the *axis* of the cone; the other side containing the right angle being its *radius*. The point of the cone is its *apex*; the circular end its *base*. The hypotenuse of the triangle is the slant of the cone. From the method of formation of the right cone, it follows that the axis is perpendicular to the base. The height of the cone is the length of its axis.

(7.) Referring to Fig. 1a, O B E represents a cone gene-

rated or formed by the revolution of the right-angled triangle OAB (Fig. 1b) about one of its sides containing the right angle, here the side OA . Similarly the cone ODF ,

FIG. 1a.

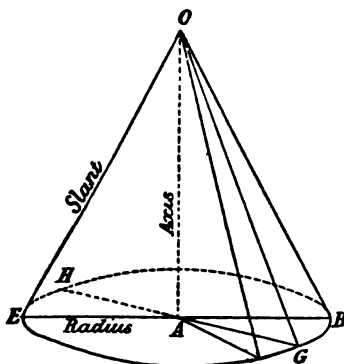


FIG. 1b.

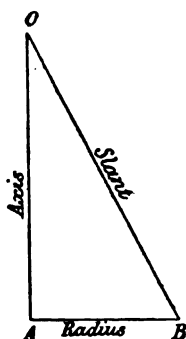


Fig. 2a, is formed by the revolution of OCD (Fig. 2b) about its side OC . As will be seen from the figs., OA , OC are respectively the axes of the cones OBE , ODF , as also their heights. Their bases are respectively $BGEH$, $DKFL$,

FIG. 2a.

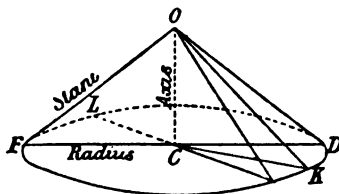
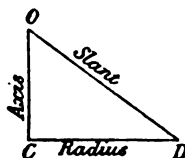


FIG. 2b.



and the radii of the bases are AB and CD . The slants of the cones are OB and OD , the apex in either being the point O . Other lines will be seen in figs., namely, those representing the revolving triangle in its motion of generating

the cone. The sides of these triangles that start from the apex and terminate in the base are all equal, it must be borne in mind ; and each of them is the slant of the cone. Likewise their sides that terminate in A are all equal, and each shows a radius of the base of the cone. How these particulars of the relations to one another of the several parts of the *right cone* apply in the setting-out of patterns will be seen in the problems that follow.

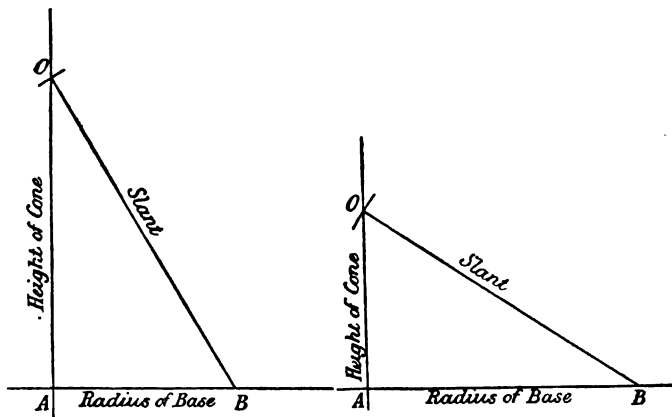
PROBLEM I.

To find the height of a cone, the slant and diameter of the base being given.

Draw any two lines O A, B A (Figs. 3 and 4) perpendicular to each other and intersecting in A. On either line

FIG. 3.

FIG. 4.



mark off from A half the diameter of the base, in other words, the radius of the base, as A B. With B as centre, and radius equal to the slant, describe an arc cutting A O in O. Then O A is the height of the cone.

PROBLEM II.

To find the slant of a cone, the height and diameter of the base being given.

Draw any two lines OA , BA (Figs. 3 and 4) perpendicular to each other and intersecting in A . On either line mark off from A half the diameter of the base (radius of the base), as AB , and make AO on the other line equal to the height of the cone; join OB . Then OB is the required slant

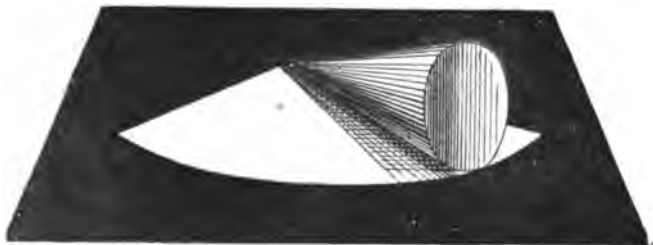
CHAPTER IV.

PATTERNS FOR ROUND ARTICLES OF EQUAL TAPER OR INCLINATION OF SLANT.

(CLASS I. *Subdivision a.*)

(8.) If a cone has its inclined or slanting surface painted, say, white, and be rolled while wet on a plane so that every portion of the surface in succession touches the plane, then the figure formed on the plane by the wet paint (see Fig. 5)

FIG. 5.



will be the pattern for the cone. As the cone rolls (the figure represents the cone as rolling), the portion of it touching the plane at any instant is a slant of the cone (see § 7).

(9.) Examining the figure formed by the wet paint, we find it to be a *sector* of a circle, that is, the figure contained between two radii of a circle and the arc they cut off. The length of the arc here is clearly equal to the length of the circumference of the base of the cone, and the radius of the

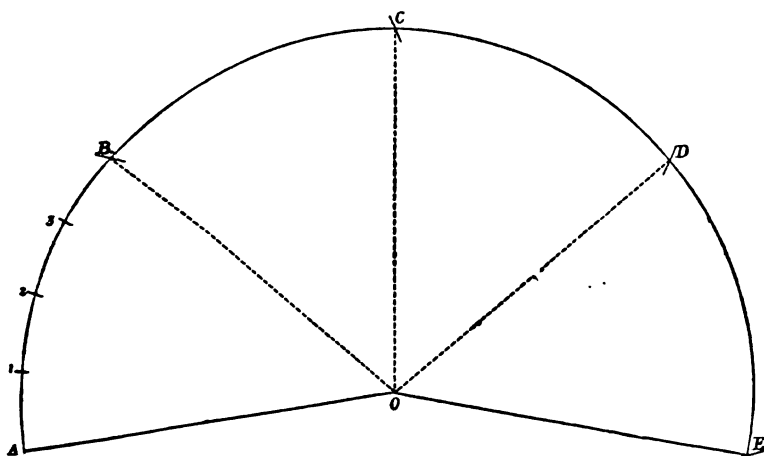
are evidently equal to the slant of the cone. From this it is obvious that to draw the pattern for a cone, we require to know the slant of the cone (which will be the radius for the pattern), and the circumference of the base of the cone.

PROBLEM III.

To draw the pattern for a cone, in one piece or in several pieces, the slant and diameter of the base being given.

PATTERN IN ONE PIECE.—With $O A$ (Fig. 6b) equal to the slant as radius, describe a long arc $A C E$. What has now

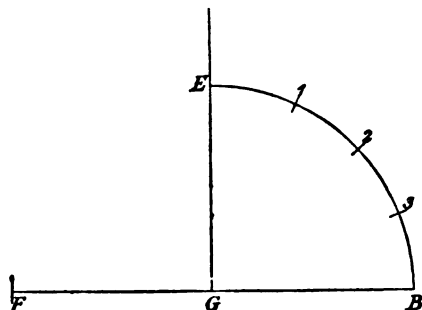
FIG. 6b.



to be done is to mark off a length of this arc equal to the circumference of the base of the cone. The best and quickest way for this is as follows. Draw a line $F B$ (Fig. 6a) equal to the given diameter of the base, and bisect it in G ; then $G B$ is a radius of the base. From G draw $G E$ perpendicular

to FB; and with G as centre and radius GB describe from B an arc meeting GE in E. The arc BE is a quadrant (quarter) of the circumference of the base of the cone. Divide this quadrant into a number of equal parts, not too

FIG. 6a.



many, say four, by points 1, 2, 3. From A (Fig. 6b) mark off along arc ACE four parts, each equal to one of the divisions of the quadrant, as from A to B. Take this length AB equal to the four parts, that is, equal to the quadrant, and from B set it off three times along the arc towards E as from B to C, C to D, D to E. Join E to O; then OACEO will be the pattern required.

NOTE.—It must be noted that when this pattern is bent round to form the cone, the edges OA and OE will simply butt up against each other, for no allowance has been made for *lap* or *seam*. Let us call the junction of OA and OE the line of butting. Nor, further, has any allowance been made for *wiring* of the edge ACE. These most essential matters will be referred to immediately.

PATTERN IN MORE THAN ONE PIECE.—If B be joined to O, then the sector OAB will be the pattern for *one-quarter* of the cone. If C be joined to O, then the sector OAC is the pattern for *one-half* of it. Similarly OAD will give

three-quarters of the cone. A cone pattern can thus be made in one, two, three, or four pieces. If the cone is required to be made in three pieces, then instead of dividing, as above, a quadrant of the circumference of the base, divide one-third of it into parts, say five; set off five of the parts along A C E from A, and join the last division point to the centre; the sector so obtained will be the pattern for *one-third* of the cone. If required to be made in five pieces, divide a fifth of the circumference of the base into equal parts, and proceed as before. Similarly for any number of pieces that the pattern may be required in.

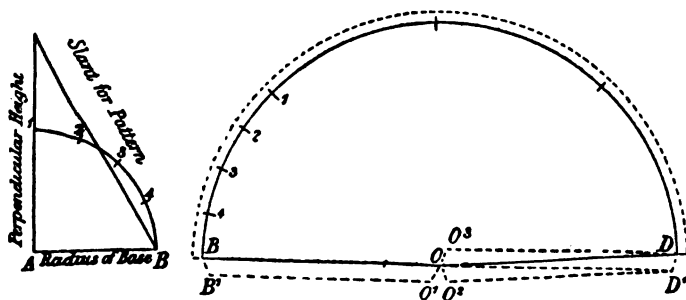
PROBLEM IV.

To draw the pattern for a cone, the height and the diameter of the base being given.

First find the slant O B (Fig. 7a) by PROBLEM II. Then with A as centre and radius A B, describe B C a quadrant of

FIG. 7a.

FIG. 7b.



the circumference of the base, and proceed, as in PROBLEM III., to draw the pattern Fig. 7b (the plain lines).

ALLOWANCE FOR LAP, SEAM, WIRING, &c.

(10.) It has already been stated that the geometric pattern Fig. 6b has no allowance for seam, wiring, or edging. (For the present it is assumed that these terms are understood; we shall come back to them later on.) In the pattern Fig. 7b the dotted line $O'B'$ parallel to the edge OB shows 'lap' for soldered seam. For a 'grooved' seam not only must there be this allowance, but there must be a similar allowance along the edge OD . These allowances, it must be distinctly remembered, are always *extras* to the geometric pattern; that is to say, the junction line of OD and OB , or line of butting (*see* NOTE, Problem III.) is not interfered with. And here a word of warning is necessary. Suppose instead of marking off a parallel slip or lap for soldered seam, a slip $DO'D'$ going off to nothing at the centre O , is marked off, and that then, for soldering up, there is actually used not this triangular slip, but a parallel one as $DD'O O^3$, the result brought about will be that the work will solder up untrue; there will be, in fact, a 'rise' at the base of the work. We can understand the result in this way. If the parallel slip $DD'O O^3$ used for soldering were cut off, there would remain a pattern which is not the geometric pattern, but a nondescript approximation, having a line of butting other than the true line. And it being thus to an untrue pattern that the parallel slip for seam is added, the article made up from the untrue pattern must of course itself necessarily be untrue. In the fig. the dotted line parallel to the curve of the pattern shows an allowance for wiring. For a grooved seam there must be on the edge OD an addition $OD'D' O^3$ similar to the addition on the edge OB , as above stated.

(11.) In working from shop patterns for funnels, oil-bottle tops, and similar articles, workmen often find that if they take a good lap at the bottom, and almost nothing at the top of the seam, the pattern is true. And so it is, for these patterns have the triangular slip $DO'D'$ added. Whereas, if a parallel piece $DD'O O^3$ is used for lap, the pattern is

untrue. Which again is the case, because, now, in addition to DOD' , an extra triangular piece DO^3O is used, and this extra *is taken off the geometric pattern*. Consequently, the line of butting is interfered with; that is to say, the two lines OB and OD , instead of meeting, overlap; OB forming a junction with O^3D instead of with OD ; with which OB must always form a junction, for the pattern to be true. In setting out patterns, to prevent error, the best rule to follow and adopt is, to first mark them out independent of any allowance for seams, or wiring, or edging, and to afterwards add on whatever allowances are intended or requisite. In future diagrams, allowances, where shown, will be mostly shown by dotted lines.

DEFINITION.

(12.) FRUSTUM.—If a right cone is cut by a plane parallel to

FIG. 8a.

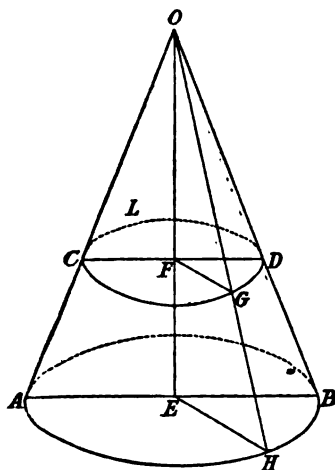
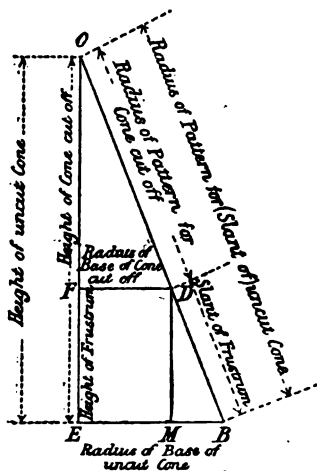


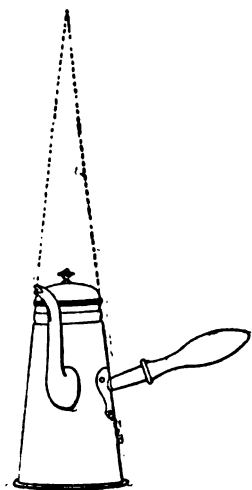
FIG. 8b.



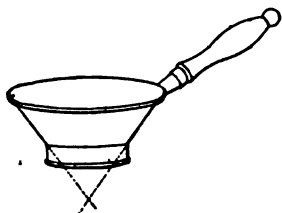
its base, the part containing the apex is a complete cone, as OCL (Fig. 8a), and the part $CABD$ containing the

base AHBK is a *frustum* of the cone. In other words a *frustum* of a right cone is a solid having circular ends, and of equal taper or inclination of slant everywhere between the ends. Conversely a round equally tapering body having top and base parallel is a frustum of a right cone.

(13.) Comparing such a solid with round articles of equal taper or inclination of slant, as pails, coffee-pots, gravy



FIGS. 9.



strainers, and so on (Fig. 9), it will be seen that they are portions (frusta) of right cones.

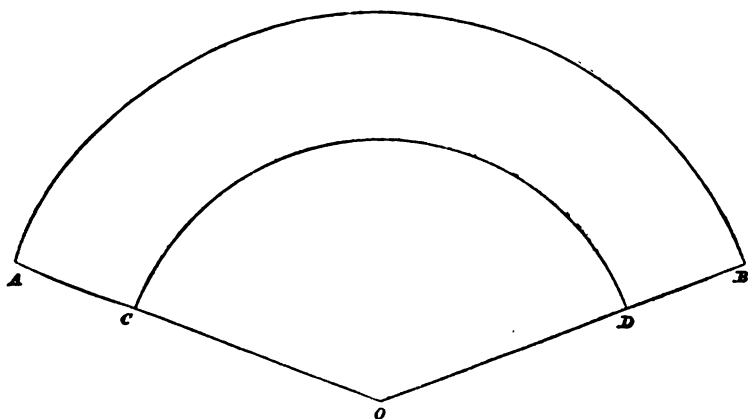
(14.) In speaking here of metal-plate articles as portions of cones, it must be remembered that all our patterns are of *surfaces*, seeing that we are dealing with metals

in *sheet*; and that these patterns when formed up are not solids, but merely simulate solids. It is, however, a convenience, and leads to no confusion to entirely disregard the distinction; the method of expression referred to is therefore adopted throughout these pages.

(15.) By Fig. 8b is shown the relations of the cone OAB of Fig. 8a with its portions OCD (complete cone cut off), and CABD (frustum). The portion OCD is a complete cone, as it is the solid that would be formed by the revolution of the right-angled triangle OFD (both figs.) around OF. The triangles OFG and OFC (Fig. 8a) represent the

triangle OFD in progress of revolution. The triangle OEB (both figs.) is the triangle of revolution of the uncut cone OAB (Fig. 8a) and OEH, OEA represent OEB in progress of revolution. The height of the cone OAB being OE (both figs.), the height of the cone OCD is OF (both figs.). The radius for the construction of pattern of the uncut cone OAB will be OB (both figs.), for the pattern of OCD , the cone cut off, the radius will be OD (both figs.). In FE , or DM , we have the height of the frustum. Just as (§8) the portion of the rolling cone touching the plane at any instant is a slant of the cone, so the slant of a frustum is that portion of it, which, if it were set rolling on a plane, would at any instant touch the plane. DB is a slant of the frustum $CABD$. The extremities of a slant of a frustum are 'corresponding points.' Other details of cone and frustum are shown in Fig. 8b.

FIG. 10.



(16.) It is obvious that, if the patterns for the cones OAB , OCD (Fig. 8a) be drawn (Fig. 10) from a common centre O , the figure $ACDB$ will be the pattern for the frustum.

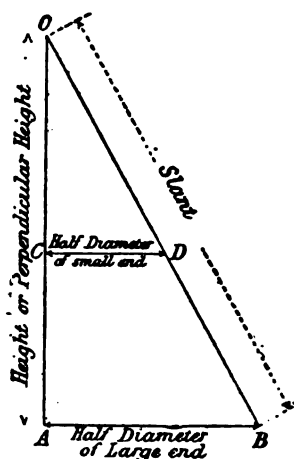
A C D B (Fig. 8a). From which we see that in order to draw the pattern for the frustum of a cone, we must know the slant of the cone of which the frustum is a portion, that is, we must know the radius for the construction of the pattern of that cone, and also the slant (radius for pattern) of the cone cut off.

PROBLEM V.

Given the dimensions of the ends of a round equal-tapering body (frustum of right cone), and its upright height. To find the slant, or the height, of the cone of which it is a portion.

Draw any two lines O A, A B (Fig. 11) at right angles to each other and intersecting in A. From A on either line,

FIG. 11.



say on B A, mark off A B equal to half the diameter of the larger of the given ends, and from A on the other line make A C equal to the given upright height. Draw a line C D

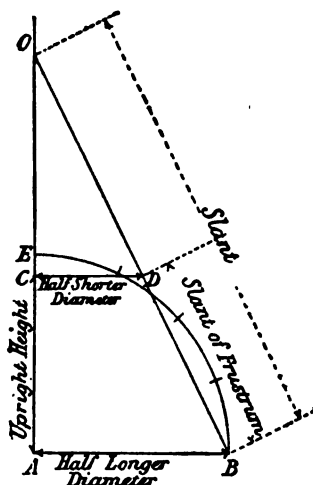
parallel to $A B$, or, which is the same thing, at right angles to $A O$, and make $C D$ equal to half the diameter of the smaller end. Join $B D$, and produce it, meeting $A O$ in O . Then $O A$ is the height of the cone of which the tapering body is a portion, and $O B$ the slant.

PROBLEM VI.

To draw the pattern for a frustum of a cone, the diameters of the ends of the frustum and its upright height being given.

The Frustum.—Draw any two lines $O A$, $B A$ (Fig. 12a) perpendicular to each other and meeting in A ; on one of the

FIG. 12a.

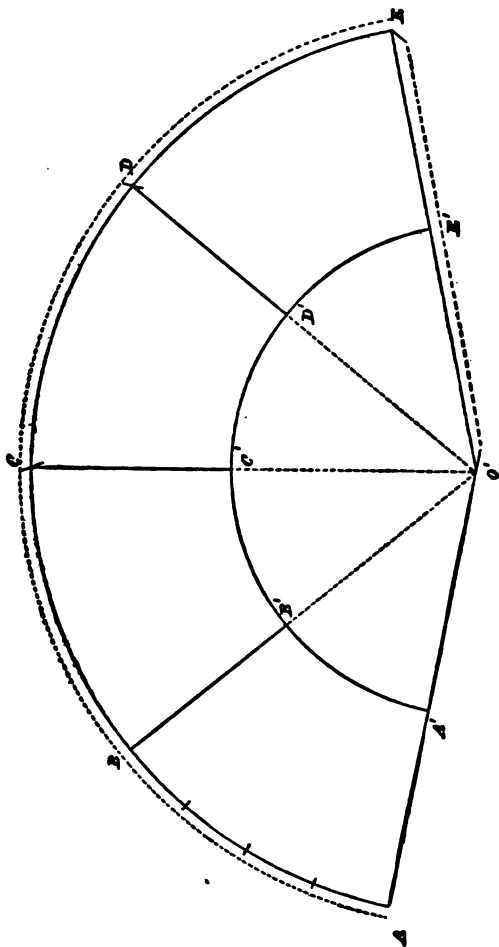


perpendiculars, say $B A$, make $A B$ equal to half the longer diameter (radius), and on the other make $A C$ equal to the given upright height. Draw a line $C D$ perpendicular to $A O$ and make $C D$ equal to half the shorter diameter. Join $B D$, and produce it, meeting $A O$ produced in O . With A

as centre, and radius AB , describe quadrant BE , which divide into any convenient number of equal parts, here four.

To draw the pattern (Fig. 12b) take any point O' as

FIG. 12b.



centre, and with radius OB (Fig. 12a) describe an arc ACE ; also with same centre and radius, OD (Fig. 12a), describe an arc $A'O'E'$. From any point in the outside curve, as A , draw a line through O' , and cutting the inner arc in A' . From A mark off successively parts equal to those into which the quadrant BE (Fig. 12a) is divided, and the same number of them, four, to B . And from B , along the outer curve, set off BC , CD , DE , each equal to AB . Join EO' , cutting the inner curve in E' . Then $AA'E'E$ is the pattern required.

Just as OB (Fig. 12a) is the slant of the cone that would be generated by the revolution of right-angled triangle OAB around OA , so DB is the slant of the frustum of which $AA'E'E$ (Fig. 12b) is the pattern. In the pattern the slant DB appears as AA' , BB' , CC' , &c.

Parts of the Frustum—If B be joined to O' , the figure $AA'B'B$ will be one-quarter of the pattern of the frustum; and if C be joined to O' , the figure $AA'C'C$ will be pattern for one-half of it, and so on. The paragraph "Pattern in more than one Piece" in PROBLEM III. should be re-read in connection with this "Parts of a Frustum."

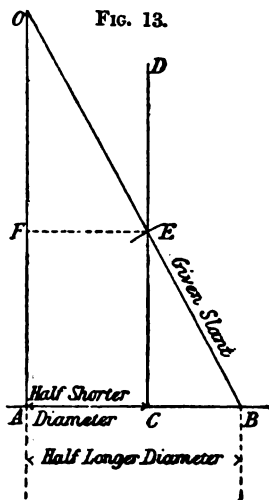
(17.) The problem next following is important, in that, in actual practice, the slant of a round equal-tapering body is very often given instead of its height, especially in cases where the taper or inclination of the slant is great; as for instance in ceiling-shades. The only difference in the working out of the problem from that of PROBLEM VI. is that the radii required for the pattern of the body are found from other data. Let us take the problem.

PROBLEM VII.

To draw the pattern for a round equal-tapering body (frustum of right cone), the diameter of the ends and the slant being given.

To find the required radii, draw any two lines OA , BA (Fig. 13) perpendicular to one another, and meeting in A .

On either line, as A B, make A B equal to half the longer of the given diameters and A C equal to half the shorter. From C draw C D perpendicular to A B. With B as centre and



the given slant as radius, describe an arc cutting C D in E. Join B E and produce it to meet A O in O. Then O B and O E are the required radii. By E F being drawn parallel to A B, comparison may be made between this Fig. and Fig. 12a, and the difference between PROBLEMS VI. and VII. clearly apprehended. To draw the pattern, proceed as in PROBLEM VI.

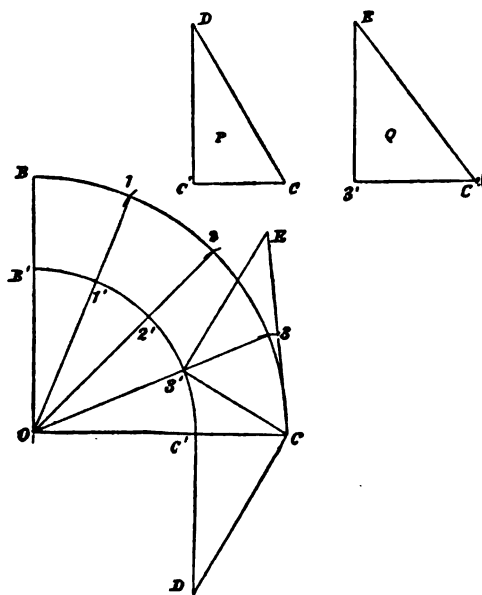
(18.) For large work and for round equal-tapering bodies which approximate to round bodies without any taper at all, the method of PROBLEM VI. is often not available, for want of space to use the long radii that are necessary for the curves of the patterns. The next problem shows how to deal with such cases; by it a working-centre and long radii can be dispensed with. The method gives fairly good results.

PROBLEM VIII.

To draw, without long radii, the pattern for a round equal-tapering body (frustum of right cone), the diameters of the ends and the upright height being given.

First draw one-quarter of the plan. (To do this, we fore-stall for convenience what is taught in the following chapter.)

FIG. 14.



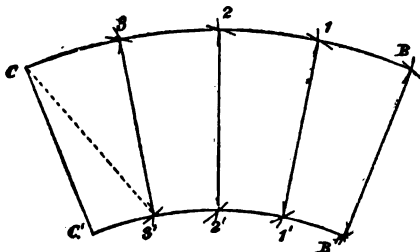
Draw any two lines BO , CO (Fig. 14) perpendicular to each other and meeting in O . With O as centre and radius equal to half the longer diameter, describe an arc meeting the lines BO , CO in B and C . With O as centre and radius equal to

half the shorter diameter describe an arc $B'C'$. This completes the one-quarter plan.

Now divide BC , the largest arc, into any number of equal parts, say four; and join the points of division to O by lines cutting $B'C'$ in $1', 2', 3'$. Join $3'C$, and through $3'$ draw $3'E$ perpendicular to $3'C$, and equal to the given upright height. Join CE ; then CE may be taken as the true length of $C3'$. Through C' draw $C'D$ perpendicular to CO and equal to the upright height. Join CD ; then CD is the true length of CC' . If it is inconvenient to find these true lengths on the plan, it may be done apart from it, as by the triangles P and Q .

To set out the pattern. Draw (Fig. 15) any line CC' equal to CD (Fig. 14). With C' and C as centres and radii respectively CE and $C3$ (Fig. 14) describe arcs intersecting in 3 (Fig. 15). With C and C' as centres and radii respectively CE and $C'3'$ (Fig. 14) describe arcs intersecting in $3'$ (Fig. 15). Then C and 3 are two points in the outer

FIG. 15.



curve of the pattern, and $C'3'$ two points in the inner curve. To find points 2 and $2'$, proceed as just explained, and with the same radii, but $3'$ and 3 as centres instead of C' and C . Similarly, to find points $1'$ and 1 , and B' and B . A curved line drawn from C through $3, 2$, and 1 to B will be the outer curve of one-quarter of the required pattern, and a curved

line from C' through $3'$, $2'$, and $1'$ to B' its inner curve; that is $C'C'B'B$ is one-quarter of the pattern. Four times the quarter is of course the required pattern complete.

NOTE.—In cases where this method will be most useful, the pattern is generally required so that the article can be made in two, three, four, or more pieces. If the pattern is required in three pieces, one-third of the plan must be drawn (see end of Problem III., p. 31) instead of a quarter, as in Fig. 14; the remainder of the construction will then be as described above.

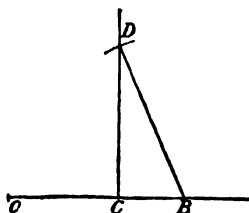
(19.) It is often desirable in the case of large work to know what the slant or height, whichever is not given, of a round equal-tapering body (frustum of right cone) will be, before starting or making the article. Here the following problems will be of service.

PROBLEM IX.

To find the slant of a round equal-tapering body (frustum of right cone), the diameters of the ends and the height being given.

Mark off (Fig. 16) from a point O in any line OB the lengths of half the shorter and longer diameters, as OC , OB .

FIG. 16.



From C draw CD perpendicular to OB . Make CD equal to the given height, and join BD . Then BD is the slant required.

PROBLEM X.

To find the height of a round equal-tapering body (frustum of right cone), the diameters of the ends and the slant being given.

Mark off (Fig. 17) from a point O in any line OB the lengths of half the shorter and longer diameters, as in PROBLEM IX., and from O draw CD perpendicular to OB. With B as centre and radius equal to the given slant, describe an arc cutting CD in E. Then CE is the height required.

Essentially this problem has already been given, in the working of PROBLEM VII.

FIG. 17.

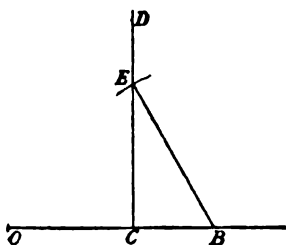
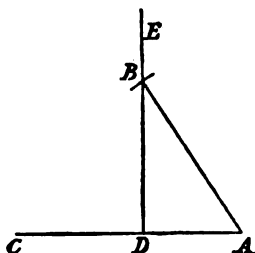


FIG. 18.



PROBLEM XI.

Given the slant and the inclination of the slant of a round equal-tapering body; to find its height.

Let AB (Fig. 18) be the slant, and the angle that AB makes with CA the inclination of the slant. From B let fall BD perpendicular to AC. Then BD is the height required.

(20.) In the workshop, the inclination of the slant of a tapering body is sometimes spoken of as the body being so many inches "out of true." This will be explained in the following chapter. If the inclination of the slant

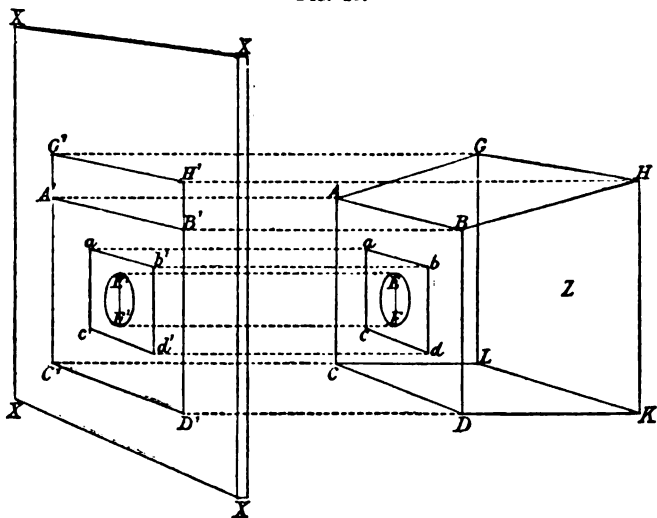
is given in these terms the problem is worked thus. From any point *D* in any line *CA* (Fig. 18) make *DA* equal in length to the number of inches the body is "out of flue," and draw *DE* perpendicular to *CA*. With *A* as centre and radius equal to the given slant, describe an arc intersecting *DE* in *B*. Then *BD* will be the height required.

CHAPTER V.

EQUAL-TAPERING BODIES OF WHICH TOP AND BASE ARE
PARALLEL, AND THEIR PLANS.

(21.) First let us understand what a plan is. Fig. 19 represents an object Z, made of tin, say, having six faces,

FIG. 19.



of which the ABCD and GHKL faces are parallel, as also the BDKH and CALG. The ABCD and CDKL faces are square. The ABCD face has, soldered flat on it centrally, a smaller square of tin *abcd* with a central

circular hole in it. Now suppose wires (represented in the fig. by dotted lines), soldered perpendicularly to the A B C D face, at A, B, C, D, a , b , c , d , E, and F (the points E and F are points at the extremities of a diameter of the circular hole). Also suppose wires soldered at G and H parallel to the other wires, and that the free ends of all the wires are cut to such length that they will, each of them, butt up against a flat surface (plane), of glass say, X X X X, parallel to the A B C D face. Lastly suppose that all the points where the wires touch the glass are joined by lines corresponding to edges of Z (see the straight lines in the

FIG. 29.

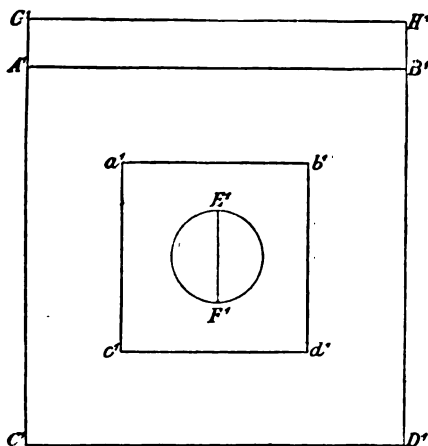


figure on the plane); also that E' and F' are joined, that the line joining them is bisected, and a circle described passing through E' and F' . Then the complete representation obtained is a *projection* of Z. Instead of actually *projecting* the points by wires, we may make the doing of it another supposition may, find, as if by wires, the required points, and draw the projection. The A B C D face being,

say 2 inches square, the flat piece, say 1 inch square, and the hole $\frac{1}{2}$ inch diameter, and the back face G H K L, say $2\frac{1}{2}$ in. by 2 in., then the projection that is upon the glass would be as shown in Fig. 20. The plane X X X X is here supposed vertical, and the projection G' C' D' H' is therefore an *elevation*; if the plane were horizontal, the projection would be a *plan*, and we might regard A B C D as the top of the body, and G H K L as its base, or *vice versâ*. We may define a plan then as the representation of a body obtained by projecting it on to a horizontal plane, by lines perpendicular to the plane.

(22.) The plane X X X X was supposed parallel to the A B C D face of Z; the plan A' B' C' D' of it is therefore of the same shape as A B C D, and in fact A B C D may be said to be its own plan. Similarly the G' H' D' C' is the plan of the back-face G H K L and is of the same shape as that face. But the plan of the face A G H B to which the plane is not parallel is by no means the same shape as that face, for the long edges B H and A G of the face A G H B are, in plan, the short lines B' H' and A' G'. We need not, however, go farther into this, because in the case of the bodies that now concern us, the horizontal plane on which any plan is drawn is always supposed to be parallel to the principal faces of the body, so that the plans of those faces are always of the same shape as the faces. In this paragraph the plane X X X X is supposed to be horizontal.

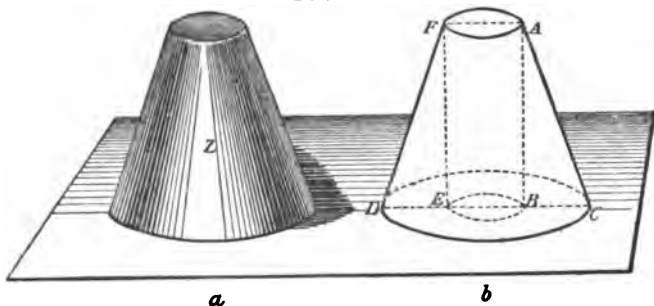
(22a.) We are now in a position to explain the getting at the true length of C C' in the fig. of PROBLEM VIII., p. 42; or, putting the matter generally, to explain the finding the true lengths of lines from their apparent lengths in their plans and elevations. Horizontal lines being excepted, there is, manifestly, for any line, however positioned in space, a vertical plane in which its elevation will appear as (if not a point) a vertical line. Let B E (Fig. 17, p. 44) be any line in the plane of the paper, and let C D be the vertical plane seen edgewise on which the elevation E C of B E is a vertical line. Then if O B be a horizontal plane seen edgewise

passing through C, the line joining the B extremity of BE to the C extremity of its elevation will be the plan of BE. We get thus the figure ECB, a figure in one plane, the plane of the paper, a right-angled triangle in fact, of which the EC side is the elevation of BE, the CB side its plan, and the hypotenuse the line itself; a figure, which, as combining a line, its plan, and its elevation, we have under no other conditions than when the elevation in question is a vertical line. In the plane passing through EC and BE, that is, in the plane in which these lines wholly lie, we have in the line that we get by joining C with B the plan, full length, of BE. In respect of this plan of BE, we are concerned with no other measurement, because, in a right-angled triangle representing a line and its plan and elevation, no other measurement of the plan line can come in. Not so, however, with the elevation line of BE. Here other measurement of it than its length can and does come in, because that length varies according to the position of the vertical plane with regard to it; the plan length is always the same. But to have in the three sides of ECB, the representation of BE, and its plan and elevation, it is evident that the plane which contains BE and its plan CB must also wholly contain the elevation EC; that is, the plane must be perpendicular to the plane of the triangle. Now, no matter on what vertical plane the line BE is projected, although the length of the projection will vary, the vertical distance between its extremities, that is, its height, never varies. Hence, if, in any right-angled triangle, we have in the hypotenuse the representation of a line, in one of its sides the plan of the line, and in the other side, not necessarily the elevation that comes out vertical, but the *height* of any elevation of the line, it comes to the same thing as if in the latter side we had the actual elevation that is vertical. And hence, further, if we have given the plan-length of an unknown line, and the vertical distance between its extremities, we can, by drawing a line, say CB, equal to the given plan-length, then drawing from one of its extremities and at

right angles to it, a line, say CE , equal to the given vertical distance, and finally joining the free extremities, as by BE , of these two lines, construct a right-angled triangle, the hypotenuse, BE , of which must be the true length of the unknown line; for there is no other line than BE of which CB and CE can be, at one and the same time, plan and elevation. We have explained this true-length matter fully, because we have to make use of it abundantly in problems to come.

(23.) Proceeding to the bodies we have to consider, we

FIG. 2L

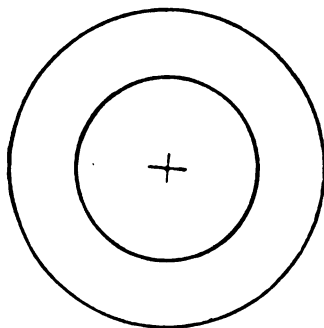


take first a frustum of a cone, Fig. 21a. To draw its plan, let us suppose the extremities of a diameter of its smaller face top (namely points A and F of the skeleton drawing Fig. 21b) (neither drawing is to dimensions), to be projected, in the way just explained, on to a plane parallel to the face, then, also as there explained, we can draw the circle which is a projection of that face. Suppose the smaller circle of Fig. 22 to be that circle, and to be to dimensions. Projecting now, similarly, the extremities of a diameter of the larger face (base), namely the points C and D of the skeleton drawing, on to the same plane, we can get the projection of the larger face. Let the larger circle of Fig. 22 be that projection. The two circular projections will be *concentric* (having the

same centre) because the body Z is of equal taper, and they will, together, be the plan of Z, that is Fig. 22 is that plan.

A C and F D each show the slant, and B A and E F the height. B C and D E each show the distances between the plans of corresponding points.

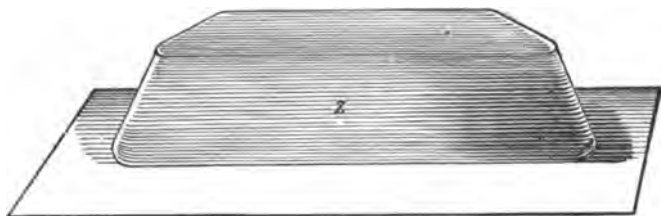
FIG. 22.



(24.) Turn to the skeleton drawing of Z. Here A C shows a slant of the frustum (§ 15), A B its height (see D M, Fig. 8b), and A and C are 'corresponding points' (§ 15). Looking at C D E B as at the plan of the frustum, we have, in the point B, the plan of the point A. Joining B C, we get a right-angled triangle A B C; the slant A C is its hypotenuse, the height A B is one of the sides containing the right angle, and the other side containing the right angle, B C, is the distance between the plans of the corresponding points A and C, as also between plans of corresponding points of Z anywhere. This distance is that of how much the body is 'out of flue' (a workshop expression that was referred to at the end of the previous chapter), in other words, how much A C is out of parallel with A B. What points, in the plan of a frustum, are the plans of corresponding points is shown

by the fig., as the line joining the plans of corresponding points (the line joining B and C or that joining D and E, for instance) will always, if produced, pass through the centre of the circles that constitute the plan of the frustum; the centre of the circles being the plan of the apex of the cone of which the frustum is a part. Which leads us to this; that the distance, actually, between the plans of corresponding points in the plan of a frustum is equal to half the difference of the diameters of its two circles; for, the difference between EB and DC is the sum of DE and BC, and DE and BC are equal; in other words, either DE or BC is half the difference between EB and DC.

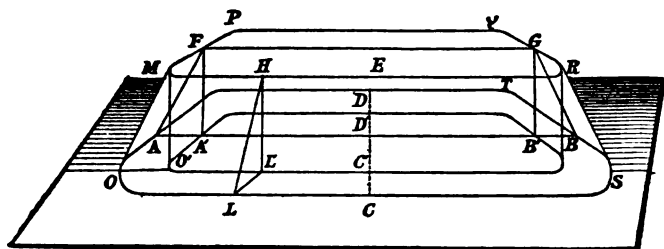
FIG. 23a.



(25.) Let us now consider another equal-tapering body which has top and base parallel, and we will suppose it to have flat parallel sides, flat ends, and round (quadrant) corners. Such a body is represented, except as to dimensions, in Z, Figs. 23a and b; Fig. 23b being a skeleton drawing of the body represented in Fig. 23a. Extending our definition of 'slant' to apply to such a body, a 'slant' becomes the shortest line that can be drawn anywhere on the slanting surface; and 'corresponding points' become, in accordance, the extreme points of such line. Either of the lines FA, GB, EC, HL, or MO represent a slant of the body, and F and A are corresponding points; as also are G and B, E and C, H and L, and M and O. The height

of the body is represented by either of the lines $F A'$, $G B'$, $E C'$, $H L'$, or $M O'$. The plane for the plan being parallel to the $M P Q R$ face (here the top) of Z , the plan of that face is of the same shape as the face. The round-cornered rectangle $A' F' G' D' B' C'$ of Fig. 29 is the plan to dimensions. For the same reason the plan of the $O A T S$ face (here the base) is of the same shape as that face. The round-cornered rectangle $A F G D B C$ of Fig. 29 is the plan to dimensions. How actually to draw these plans we shall deal with presently as a problem. The two circles constituting the plan of the frustum were concentric, that is, symmetrically disposed with respect to one another, because the frustum

Fig. 23b.



was an equal-tapering body; and the plans of top and base of the body we are now dealing with are symmetrical to each other for the same reason. The two plans (Fig. 29) together are the plan of the body Z .

(26.) Looking at $A B C D A' B' C' D'$ of the skeleton drawing (Fig. 23b) as at the plan of Z , we have, just as with the cone frustum, in the point A' the plan of F , in the point B' the plan of G , in the point C' the plan of E , in the point L' the plan of H , and in the point O' the plan of M . Further as in the case of the frustum, if we join any point in the plan of the base, as A , to the plan of its corresponding point A' , then we have a right-angled triangle, $F A A'$, of which the

hypotenuse FA represents the slant of the body, FA' , one of the sides containing the right angle, its height, and $A'A$, the other side containing the right angle, the distance between the plans of the corresponding points F and A , which is also the distance between B and B' , C and C' , L and L' , O and O' , and between plans of corresponding points of the body anywhere, the body being of equal taper. As with Fig. 21*b* what points, in the plan, are the plans of corresponding points is clear from the fig. Where the plan of the body consists of straight lines, the plans of corresponding points are always the extremities of lines joining these straight lines perpendicularly; the extremities of AA' , BB' , CC' , and LL' , for instance. Where the plan of the body consists of arcs, the plans of corresponding points (compare with cone frustum) are the extremities of lines joining the arcs, and which, produced, will pass through the centre from which the arcs are described; the line OO' for instance. To make all this quite plain, reference should again be made to Fig. 29; also to Fig. 28, which is the plan of an equal-tapering body with top and base parallel, and having flat sides, and semicircular ends. In Fig. 29, AA' , BB' , CC' , DD' , are lines joining the plan lines of the flat sides and ends perpendicularly, and the extremities of each of these lines are plans of corresponding points, that is to say, A and A' are plans of corresponding points, as are also B and B' , C and C' , and D and D' . Also F and F' are plans of corresponding points, being the extremities of the line FF' which is a line joining the ends of the arcs which are the plans of one of the quadrant corners of the body. Similarly G and G' are plans of corresponding points. In Fig. 28, FF' , GG' , DD' , EE' , are lines joining perpendicularly the plan lines of the flat sides of the body at their extremities where the semicircular ends begin; and F and F' , G and G' , D and D' , E and E' are plans of corresponding points. Also A and A' are plans of corresponding points, and B and B' , seeing that the lines joining these points, produced, pass respectively through O and O' , the centres from which the semicircular ends are described.

In the cone frustum, the actual distance between the plans of corresponding points was, we saw, equal to half the difference of the diameters of the two circles constituting its plan. Similarly with the body Z of Fig. 23a, and indeed with any equal-tapering body of which the top and base are parallel, if we have the lengths of the top and base given, or their widths, the distance between the plans of corresponding points (number of inches 'out of flue') is always equal to half the difference between the given lengths or widths. Thus, the distance between the plans of corresponding points of Z is equal to half the difference between A B and A' B' (Fig. 29) or between C D and C' D'; and the distance between the plans of corresponding points of the body of which Fig. 28 is the plan, is equal to half the difference between A B and A' B' of that fig., or between F D and F' D'.

Summarising we have

a. In the plans of equal-tapering bodies which have their tops and bases parallel, there is, all round, an equal distance between the plans of corresponding points of the tops and bases.

b. *Conversely*.—If, in the plan of a tapering body with top and base parallel, there is an equal distance all round between the plans of corresponding points of the top and base, then the tapering body is an equal-tapering body, that is, has an equal inclination of slant all round.

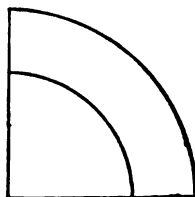
c. The plan of a round equal-tapering body having top and base parallel, consists of two concentric circles. The plan of a portion of a round equal-tapering body having top and base parallel, consists of two arcs having the same centre.

The corners of the body Z (Fig. 23a) are portions (quarters) of a round equal-tapering body; their plans are arcs (quadrants) of circles having the same centre.

d. *Conversely*.—If the plan of a tapering body having top and base parallel, consists of two concentric circles, then the body is a frustum of a right cone. Also if the plan of a tapering body having top and base parallel, consists of two

arcs having the same centre, then the body is a portion of a frustum of a right cone.

FIG. 24.



The plan of each end of the tapering body represented in plan in Fig. 28 consists of two arcs (semicircles) having the same centre; the ends are portions (halves) of a frustum of a right cone. The plan of each corner of the tapering body Z (Fig. 23a) consists of two arcs (quadrants, Fig. 29) having the same centre; the corners are portions (quarters) of a frustum of a right cone. The fig. annexed represents a quadrant corner in plan separately.

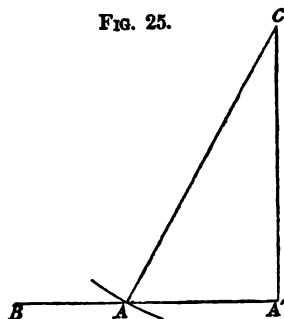
We conclude the chapter with some problems.

PROBLEM XII.

Given the height and slant of an equal-tapering body with top and base parallel; to find the distance between the plans of corresponding points of the top and base (number of inches 'out of flue').

Let CA' (Fig. 25) be the given height. Draw $A'B$ perpendicular to $A'C$; with C as centre and the given slant as

FIG. 25.



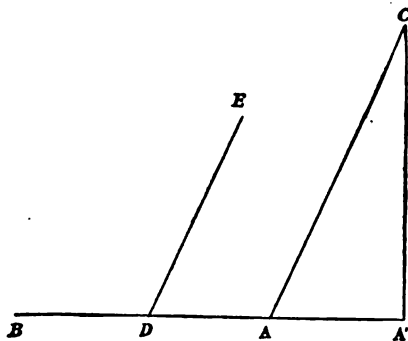
radius, describe an arc cutting BA' in A . Then AA' is the distance required.

PROBLEM XIII

Given the height of an equal-tapering body with top and base parallel, and the inclination of slant (number of inches 'out of flue'); to find the distance between the plans of corresponding points of the top and base.

Let CA' (Fig. 26) be the given height. Through A' draw a line $A'B$ perpendicular to CA' ; from any point, D , in $A'B$ draw a line DE making with $A'B$ an angle equal to that of the given inclination. From C draw CE parallel to ED and cutting $A'B$ in A ; then AA' is the distance required.

FIG. 26.



PROBLEM XIV.

To draw the plan of a round equal-tapering body with top and base parallel (frustum of right cone), the diameter of either end being given and the height and slant.

CASE I.—Given the height and slant and the diameter of the smaller end.

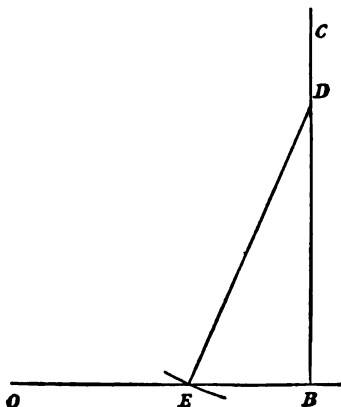
On any line OB (Fig. 17) set off OC equal to half the given diameter, and from C draw CD perpendicular to OB .

Mark off CE equal to the given height, and with E as centre and radius equal to the given slant, describe an arc intersecting OB in B ; then CB will be the distance in plan between corresponding points anywhere in the frustum; that is to say (by c, p. 55) OC will be the radius for the plan of the smaller end of the frustum, and OB the radius for the plan of the larger end.

CASE II.—Given the height and slant and the diameter of the larger end.

On any line OB (Fig. 27), set off OB equal to half the given diameter, and now work from B towards O instead of from O towards B ; thus. From B draw BC perpendicular to OB . Mark off BD equal to the given height, and with D as centre and radius equal to the given slant, describe an arc intersecting OB in E ; then BE will be the distance in

Fig. 27.



plan of corresponding points anywhere in the frustum; that is to say (by c, p. 55) OB will be the radius for the plan of the larger end of the frustum, and OE the radius for the plan of the smaller end.



PROBLEM XV.

To draw the plan of a round equal-tapering body with top and base parallel (frustum of right cone), the diameter of either end being given, and the number of inches 'out of flue' (distance between plans of corresponding points).

CASE I.—Given the number of inches 'out of flue,' and the diameter of the smaller end.

The radius for the smaller circle of the plan will be half the given diameter; the radius for the larger circle of the plan will be this half diameter with the addition of the number of inches 'out of flue.'

CASE II.—Given the number of inches 'out of flue,' and the diameter of the larger end.

The radius for the larger circle of the plan will be half the given diameter; the radius for the smaller circle of the plan will be the half diameter less the number of inches 'out of flue.'

(27.) It should be noted that with the dimensions given in this problem, we can draw plan only, we could not draw a pattern. To do that we must also have height given, for a plan of small height and considerable inclination of slant is also the plan of an infinite number of other frusta (*plural of frustum*) of all sorts of heights and inclinations of slant.

PROBLEM XVI.

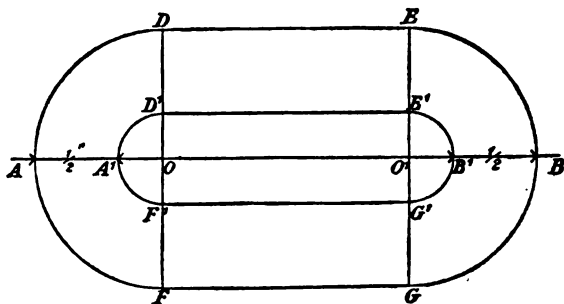
To draw the plan of an oblong equal-tapering body with top and base parallel, and having flat (plane) sides and semicircular ends.

CASE I.—Where the length and width of the top are given, and the length of the bottom.

Commencing with the plan of the top, we know from § 25 that it will be of the same shape as the top; we have there-

fore to draw that shape. On any line AB (Fig. 28) mark off AB equal to the given length of the top. From A set off AO , and from B set off BO' each equal to half the given width of the top. Through O and O' draw lines perpendicular to AB ; and with O and O' as centres and OA or $O'B$ as radius describe arcs meeting the perpendiculars in DF and EG . As DF and EG pass through the centres O and O' respectively they are diameters, and the arcs are semicircles; these diameters, moreover, are each equal to the given width. Join DE , FG , and the plan of the top is complete.

FIG. 28.



The plan of the base will be of the same shape as the top, and we will suppose it smaller than the top. What we have then to do is to draw a figure of the same shape as the base, and to so place it in position with the plan of the top that we shall have a complete plan of the body we are dealing with. By *a*, p. 55, we know that the distances between the plans of corresponding points of the top and base all round the full plan will be equal. We have therefore first to ascertain the distance between the plans of any two corresponding points. This by § 26 will in the present instance be equal to half the difference between the given

lengths of the top and base. Set off this half-difference, as the base is smaller than the top, from A to A'. Then with O and O' as centres, and O A' as radius, describe the semicircles D' A' F', E' B' G'. Join D' E', F' G', and we have the required plan of the body.

CASE II.—Where the length and width of the top are given, and the height and slant, or the height and the inclination of the slant (number of inches 'out of flue').

First draw the plan of the top as in Case I. Then if the height and slant are given, find by Problem XII. the distance between the plans of corresponding points. If the height and inclination of slant are given, find the distance by Problem XIII. If the inclination of the slant is given in the form of 'out of flue,' the number of inches 'out of flue' is the required distance. Set off this distance from A to A' in the fig. of Case I., and complete the plan as in Case I.

CASE III.—Where the length and width of the base (bottom) are given, and the height and slant, or the height and the inclination of the slant.

On any line A B (Fig. 28) mark off A' B' equal to the given length of the bottom. From A' set off A' O and from B' set off B' O' each equal to half the given width of the bottom. Through O and O' draw indefinite lines D F, E G perpendicular to A B; and with O and O' as centres, and O A' as radius describe the semicircles F' A' D', G' B' E', join D' E', F' G', and we have the plan of the bottom. Now by Problem XII. or Problem XIII., as may be required, find the distance between the plans of corresponding points, or take the number of inches 'out of flue,' if this is what is given. Set off this distance from A' to A. With O and O' as centres and O A as radius describe semicircles meeting the perpendiculars through O and O' in D and F and in E and G. Join D E, F G, and the plan of the body is completed.

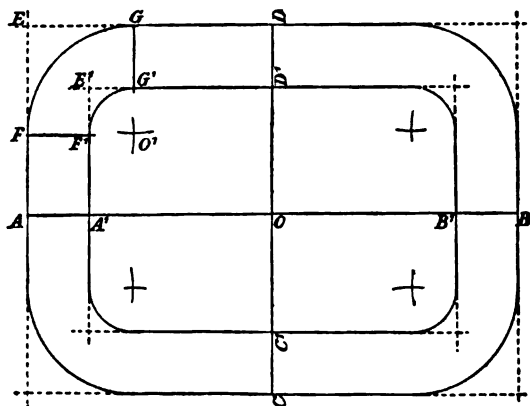
PROBLEM XVII.

To draw the plan of an oblong equal-tapering body with top and base parallel, and having flat sides, flat ends, and round (quadrant) corners.

CASE I.—Where the length and width of top and bottom (base) are given.

Draw any two lines AB , CD (Fig. 29) perpendicular to each other and intersecting in O . Make OA and OB each equal to half the length of the top, which we will suppose

FIG. 29.



larger than the bottom, and OA' and OB' each equal to half the length of the bottom. Also make OC and OD each equal to half the width of the top, and OC' and OD' each equal to half the width of the bottom. Through C , D , C' , and D' draw lines parallel to AB , and through A , B , A' , and B' draw lines parallel to CD and intersecting the lines parallel to AB . We have now two rectangles or oblongs, and we require to draw the round corners, which are quarters of circles.

From the intersecting point E along the sides of the rectangle mark equal distances E F and E G, according to the size of quadrant corners required. With F and G as centres and E F or E G as radius, describe arcs intersecting in O'; and with O' as centre and same radius, describe the arc F G, which will be a quadrant because if the points F and G be joined to O' the angle F O' G will be a right angle (p. 21). Draw F F' parallel to A B and G G' parallel to C D, and with O' as centre and radius O' F' describe the arc F' G', which also will be a quadrant. We have now the plan of one of the quadrant corners; the other corners can be drawn in like manner.

(27a.) It is important to notice that the larger corner determines the smaller one. In practice it is therefore often best to draw the smaller corner first, otherwise it may sometimes be found, after having drawn the larger corner, that it is not possible to draw the smaller curve sufficiently large, if at all. To draw the smaller corner first, mark off from the intersecting point E' equal lengths E' F', E' G', according to the size determined on for the corner. With F' and G' as centres and E' F' or E' G' as radius describe arcs intersecting in O'. Then O' will be the centre for the smaller corner. It will also be the centre for the larger corner, which may be described in similar manner to the smaller corner in the preceding paragraph.

CASE II.—Where the dimensions of the top are given and the height and slant, or the height and the inclination of the slant.

Draw the plan of the top, A D B C. Find the distance between the plans of corresponding points of the top and base by Problem XII., or Problem XIII., according to what is given; and set off this distance, as the base is smaller than the top, from A and B inwards towards O on the line A B, and from D and C inwards towards O on the line D C. Complete the plan by the aid of what has already been explained.

CASE III.—Where the dimensions of the bottom are given, and the length and slant, or the height and the inclination of the slant.

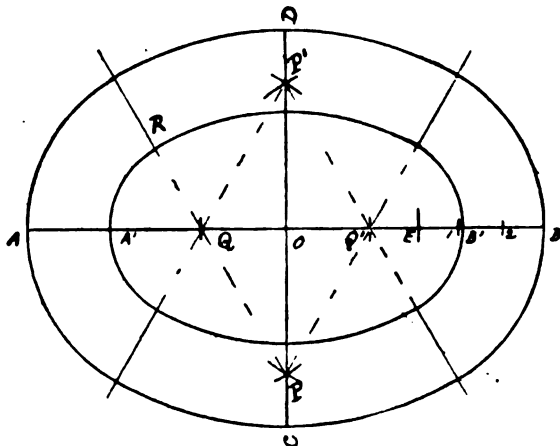
Draw the plan of the bottom, $A' D' B' C'$, find the distance between the plans of corresponding points of top and bottom, set this off outwards from A' , D' , B' , and C' and complete the plan by aid of what has already been stated.

PROBLEM XVIII.

To draw the plan of an oval equal-tapering body with top and base parallel, the length and width of the top and bottom being given.

Draw (Fig. 30) any two lines $A B$, $C D$ intersecting each other at right angles in O . Make $O A$ and $O B$ each equal

FIG. 30.



to half the given length of the larger oval (top or bottom, as may be), and $O C$ and $O D$ each equal to half its given width. $A B$ and $C D$ will be the *axes* of the oval. From A , on $A B$,

mark off A E equal to C D the width of the oval, and divide E B into three equal parts. With O as centre and radius equal to two of these parts, as from E to 2, describe arcs cutting A B in Q and Q'. With Q and Q' as centres and Q Q' as radius describe arcs intersecting in P and P'; and from P and P' draw lines of indefinite length through Q and Q'. With P and P' as centres and radius P D describe arcs (the side arcs), their extremities terminating in the lines drawn through Q and Q'; and with Q and Q' as centres, and radius Q A, describe arcs (the end arcs) to meet the extremities of the side arcs. This completes the plan of the larger oval.

To draw the plan of the smaller oval. Make O A' and O B' each equal to half the length of the smaller oval, and with Q and Q' as centres and Q A' as radius describe the end curves, their extremities terminating, as do the outer end-curves, in the lines drawn through Q and Q'; the point R is an extremity of one of the smaller curves. With P and P' as centres and radius P R, describe the side curves. The plan of the oval equal-tapering body is then complete; of which either the larger or smaller ovals are plan of top and bottom according to the purpose the article may be required for.

(28.) The plans of corresponding points in the plan of an oval equal-tapering body will be the extremities of any line joining the inner and outer curves anywhere, and that, produced, will pass through the centre from which the curves where joined by the line are described.

To draw the plan of an oval equal-tapering body with top and base parallel, other dimensions than the above may be given. For instance the top or bottom may be given, and either the height and slant, or the height and the inclination of the slant (number of inches out of flue). It will be a useful practice for the student to work out these cases for himself by the aid of the instruction that has been given.

CHAPTER VI.

PATTERNS FOR ARTICLES OF EQUAL TAPER OR INCLINATION
OF SLANT, AND HAVING FLAT (PLANE) SURFACES.(CLASS I. *Subdivision b.*)

DEFINITION.

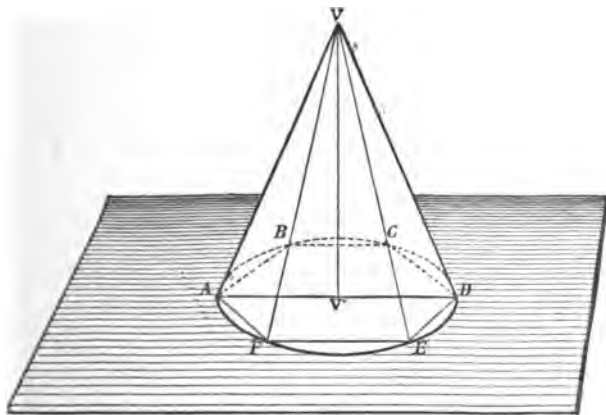
(29.) Pyramid.—A pyramid is a solid having a base of three or more sides and triangular faces meeting in a point above that base, each side of the figure forming the base being the base of one of the triangular faces, and the point in which they all meet being the *apex*. The shape of the base of a pyramid determines its name; thus a pyramid with a triangular base is called a *triangular* pyramid; with a square base, a *square* pyramid; with a hexagon base, an *hexagonal* pyramid (Fig. 31); and so on. The centre of the base of a pyramid is the point in which perpendicular lines bisecting all its sides will intersect. If the apex of a pyramid is perpendicularly above the centre of its base, the pyramid is a *right* pyramid (Fig. 31 represents a right pyramid), in which case the base is a regular polygon and the triangular faces are all equal and all equally inclined. In a pyramid, the line joining the apex to the centre of the base is called the *axis* (the line V V', Fig. 31) of the pyramid.

(30.) An important property that a right pyramid possesses is that it can be *inscribed in a right cone*.

(31.) A pyramid is said to be inscribed in a cone when both the pyramid and the cone have a common apex, and the base of the pyramid is inscribed in the base of a cone; in other words, when the angular points of the base of the pyramid are on the circumference of the base of the cone and the apex of cone and pyramid coincide.

(32.) Fig. 31 shows a right pyramid inscribed in a right cone. The apex V is common to both pyramid and cone, and the $A, B, C, \&c.$, of the base of the pyramid are on the circumference of the base of the cone. Also the axis VV' is common to both cone and pyramid. Further, the edges $VA, VB, VC, \&c.$, of the pyramid are lines on the surface of the cone, such lines or edges being each a slant of the cone, or in other words a radius of the pattern of the cone in

FIG. 31.



which the pyramid is inscribed. It hence follows, that if the pattern of the cone in which a right pyramid is inscribed be set out with the lines of contact of cone and pyramid, as $VA, VB, \&c.$, on it, and the extremities of these lines be joined, we shall have the pattern for the pyramid. Thus, the drawing a pattern for a right pyramid resolves itself into first determining the cone which circumscribes the pyramid, and next drawing the pattern of that cone with the lines of contact of pyramid and cone upon it.

PROBLEM XIX.

To draw the pattern for an hexagonal right pyramid, its height and base being given.

Draw (Fig. 32a) the plan A B C D E F of the base of the pyramid, which will be of the same shape as the base (see Chap. V.); the base in fact will be its own plan. Next draw any two lines O A, B A (Fig. 32b), perpendicular to each other and meeting in A; make A B equal to the radius of the circumscribed circle (Fig. 32a), and A O equal to the given height of the pyramid. Join B O; then B O is a slant of the

FIG. 32a.

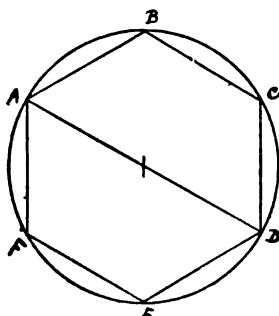
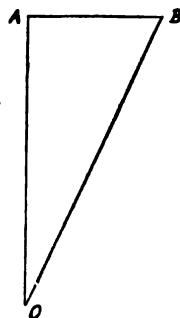


FIG. 32b.



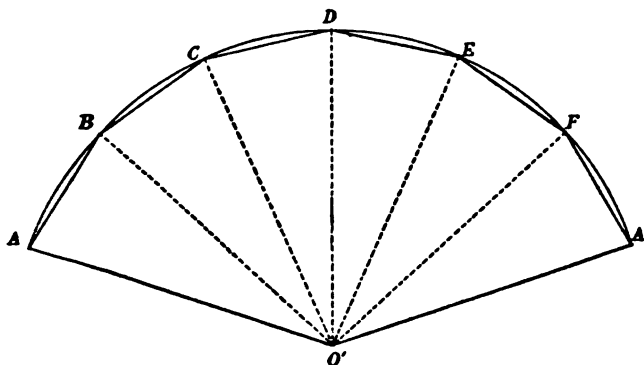
cone in which the pyramid can be inscribed, that is to say, is a radius of the pattern of that cone. The line B O is also a line of contact of the cone, in which the pyramid can be inscribed, that is, is one of the edges of the pyramid.

To draw the pattern (Fig. 32c). With any point O' as centre and B O (Fig. 32b) as radius, describe an arc A D A, and in it take any point A. Join A O', and from A mark off A B, B C, C D, D E, E F, and F A, corresponding to A B, B C, C D, D E, E F, and F A of the hexagon of Fig. 32a, and join the points B, C, D, E, F and A to O'. Join A B, B C, C D, D E, E F, and F A, by straight lines; and the figure bounded

by $O'A$, the straight lines from A to A , and $A O'$, will be the pattern required. The lines BO' , CO' , &c., correspond to the edges of the pyramid, and show the lines on which to 'bend up' to get the faces of the pyramid, the lines $O'A$ and $O'A$ then butting together to form one edge.

Similarly the pattern for a right pyramid of any number of faces can be drawn, the first step always being to draw the plan of the base of the pyramid; the circle passing through the angular points of which will be the plan of the base of the cone in which the pyramid can be inscribed.

FIG. 32c.



Suppose, instead of the dimensions from which to draw the plan of the base of the pyramid, the actual plan be given. The centre from which to strike the circumscribing circle can then be found by the Definition § 29.

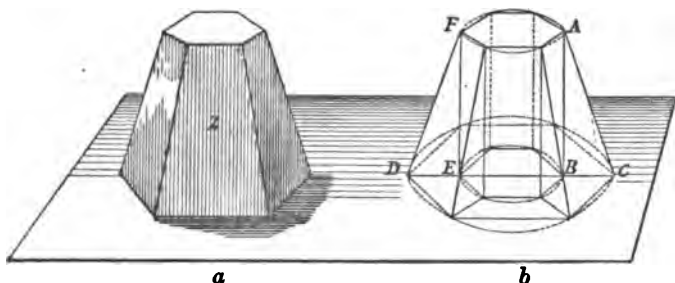
DEFINITION.

(33.) Truncated pyramid. Frustum of pyramid.—If a pyramid be cut by a plane parallel to its base, the part containing the apex will be a complete pyramid, and the other part will be a tapering body, the top and base of which are of the same shape but unequal. This tapering body is

called a *truncated pyramid*, or a *frustum of a pyramid*. The faces of a truncated pyramid which is a frustum of a right pyramid are all equally inclined. In Fig. 33 is shown such a frustum standing on a horizontal plane.

(34.) Comparison should here be made between this defini-

FIG. 33.



tion and that of a frustum of a cone (*see* § 12), which it closely follows; also between Fig. 21 and Fig. 33.

(35.) Articles of equal taper or inclination of slant and having flat (plane) surfaces and top and base parallel (hexagonal coffee-pots; hoods; &c.), are portions of right pyramids (truncated pyramids), or portions of truncated pyramids.

(36.) Exactly as a pyramid can be inscribed in a cone, so a truncated pyramid can be inscribed in a frustum of a cone, and the edges of the truncated pyramid are lines on the surface of that frustum. The skeleton drawing, Fig. 33b, shows a right truncated pyramid inscribed in a cone frustum. It also represents the plan of the cone frustum, and that of the pyramid frustum, with the lines of projection (*see* Chapter V.), of the smaller end of the latter on to the horizontal plane. This inscribing in a cone gives an easy construction for setting out the pattern of a truncated pyramid; which construction is, to first draw the pattern for the pyramid of

METAL-PLATE WORK.

which the truncated pyramid is a portion; and then mark off on this pattern the pattern for the pyramid that is cut off. Here again comparison should be made with what has been stated about the pattern of a frustum of a cone (*see* § 16), and the resemblance noted.

PROBLEM XX.

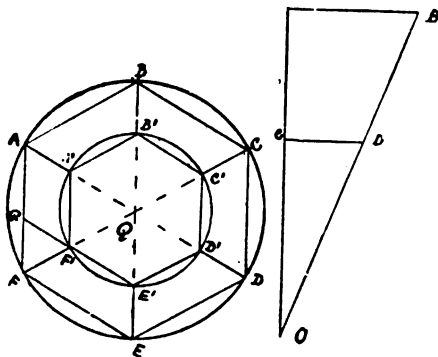
To draw the pattern for an equal-tapering body made up of flat surfaces (truncated right pyramid), the height, and top and bottom being given.

Suppose the equal-tapering body to be hexagonal.

To draw the required plan of the frustum. The plans of the top and bottom are respectively of the same shape as the top and bottom (§ 25). Draw (Fig. 34a) A B C D E F the larger hexagon (Problem X., Chap. II.) and its diagonals

FIG. 34a.

FIG. 34b.

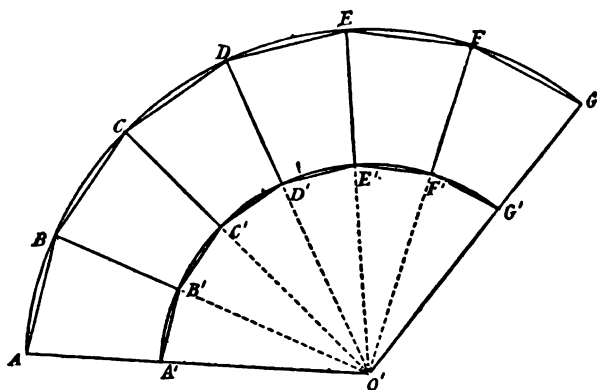


A D, B E, C F, intersecting in Q. On any one of the sides of this hexagon mark off the length of a side of the smaller hexagon, as A G on A F, and through G draw G F' parallel to the diagonal A D, and cutting the diagonal F C in F'. With Q as centre and Q F' as radius describe a circle. The

points in which this cuts the diagonals of the larger hexagon will be the angular points of the smaller hexagon. Join each of these angular points, beginning at F' , to the one next following, as $F'E'$, $E'D'$, &c. Then $F'E'D'C'B'A'$ is the plan of the smaller hexagon, and, so far as needed for our pattern, the plan of the 'equal-tapering body made up of flat surfaces' is complete. The lines AA' , BB' , CC' , will be the plans (see and compare lines DE and BC of Fig. 33b) of the slanting edges of the frustum.

Next draw (Fig. 34b) two lines OA , BA , perpendicular to each other, and meeting in A ; make AB equal to the radius of the circle circumscribing the larger hexagon of plan, and

FIG. 34a.



AC equal to the given height of the body. Through C draw CD perpendicular to AO , and make CD equal to the radius of the circle which passes through the angular points of the smaller hexagon (Fig. 34a). Join BD , and produce it to meet AO in O .

To draw the required pattern (Fig. 34c). Draw any line $O'A$, and with O' as centre and OB , OD (Fig. 34b) as radii, describe arcs ADG , $A'D'G'$ (Fig. 34c). Then take the

straight line length AB (Fig. 34a), and set it off as a chord from A (Fig. 34c) on the arc ADG . Do the same successively, from point B , with the straight line lengths (Fig. 34a) BC , CD , DE , EF , FG , the terminating point of each chord as set off, being the starting point for the next, the chord FG (Fig. 34c) corresponding to the straight line length FA (Fig. 34a). Join (Fig. 34c) the points B , C , D , E , F and G to O' by lines cutting the arc $A'D'G'$ in B' , C' , D' , E' , F' , and G' . Join AB , BC , CD , &c., and $A'B'$, $B'C'$, $C'D'$, &c., by straight lines; then $ADGG'D'A'$ is the pattern required.

The frustum of pyramid is here hexagonal, but by this method the pattern for any regular pyramid cut parallel to its base can be drawn. The next problem will show methods for larger work.

(37.) If $O'ADG$ (Fig. 34c), the pattern for the cone in which the frustum of pyramid is inscribed, be cut out of zinc or other metal, and small holes be punched at the points A , B , C , &c., and A' , B' , C' , &c.; and if the cone be then made up with the lines $O'A$, $O'B'$, &c., marked on it inside, and wires be soldered from hole to hole successively to form the top and bottom of the truncated pyramid, then (1) the whole pyramid of which the truncated pyramid is a portion, (2) the pyramid that is cut off, as well as (3) the truncated pyramid, will be clearly seen inscribed in the cone. The making such a model will amply repay any one who desires to be thoroughly conversant with the construction of articles of the kind now under consideration.

PROBLEM XXI.

To draw, without using long radii, the pattern for an equal-tapering body made up of flat surfaces, the height and top and bottom being given.

Again suppose the body to be hexagonal.

CASE I.—For ordinarily large work.

Draw (Fig 35a) the plan as by last problem. Next join AB' , and draw $B'G$ perpendicular to AB' and equal to the given height of the body. Also draw $B'H$ perpendicular to

BB' ; and equal to the given height. Join AG and BH ; then AG and BH are the true lengths of AB' and BB' respectively.

FIG. 35a.

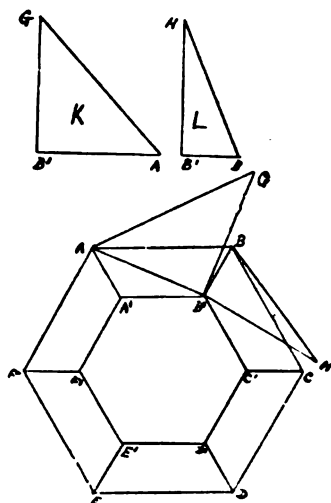
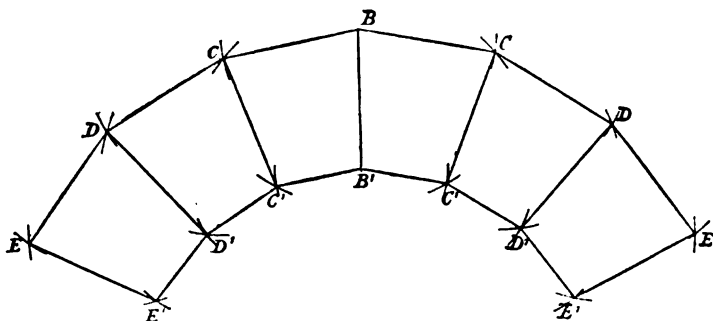


FIG. 35b.



To draw the pattern. Draw (Fig. 35b) BB' equal to BH (Fig. 35a). With B as centre and radius equal to BA

(Fig. 35a), and with B' as centre and radius equal to GA (Fig. 35a) describe arcs intersecting in C , right and left of BB' . With B as centre and the same GA as radius, and with B' as centre and radius $B'A'$, describe arcs intersecting in C' , right and left of $B'B$. Join $BC, CC', C'B'$ (Fig. 35b); then $BB'C'C$ is the pattern of the face $BB'C'C$ in plan (Fig. 35a). The other faces $CD C'D', BCB'C', \&c.$ (Fig. 35b) of the frustum are described in exactly similar manner, GA being the distance between diagonally opposite points of any face as well as of the face $BB'C'C$. The triangles $A'B'G$, $BB'H$, can be drawn apart from the plan, as shown at K and L .

(38.) It should be observed that if the pattern is truly drawn, the top and bottom lines of each face will be parallel, as $BC, B'C'$, of face $BCB'C'$ (Fig. 35 b); and that this gives an easy method of testing whether the pattern has been accurately drawn.

CASE II.—For very large work; where it is inconvenient to draw the whole of the plan.

Draw AB (Fig. 36a) equal in length to the end-line of one of the faces of the frustum at its larger end, and produce it both ways. With B as centre and radius BA describe a semicircle, which divide into as many equal parts as the frustum has faces (Problem IX.). Here it is hexagonal, and the points of division working from point 1, are 2, C , 4, &c. Through the second division point, here C , draw a line to B , then ABC is the angle made in plan by two faces of the frustum one with another, and AB, BC are two adjacent end-lines of the plan of its larger ends. Bisect the angle ABC (Problem VIII.) by BE ; and draw a line CC' from C making the angle $C'CB$ equal to the angle CBE (Problem I.). On BC set off BD equal to the end-line of one of the faces of the frustum at its smaller end; and draw DC' parallel to BE , cutting CC' in C' . Through C' draw $C'B'$ parallel to CB and meeting BE in B' ; and draw $B'A'$ parallel to BA and equal to BD or $C'B'$. Join $A'A$, and we

arcs, the radius of the arc B' G (Fig. 36b) increasing in length as the point G approaches nearer to A. If the pattern be truly drawn, B A will be perpendicular to B' G.

It must not be forgotten that these methods are, both of them, quite independent of the number of the sides of the pyramid. Also it should be noted that B E does not of necessity pass through a division point, nor of necessity is C C' parallel to A B. These are coincidences arising from the frustum being here hexagonal.

PROBLEM XXII.

To draw the pattern for an oblong or square equal-tapering body with top and base parallel, and having flat sides and ends. (The bottom is here taken as part of the body, and the whole pattern is in one piece.)

NOTE.—This problem will be solved in the problem next following. We adopt this course because the article there treated of is so important an example of the oblong equal-tapering bodies in question, that it is desirable to make that, the special problem, the primary one, and this, the general problem, secondary to it. Its solution will be found at the end of Case L

PROBLEM XXIII.

To draw the pattern for a baking-pan.

A baking-pan has not only to be water-tight, but also to stand heat; hence when made in one piece the corners are seamless.

CASE L.—Where the length and width of the bottom, the width of the top, and the slant are given.

Draw two lines X X, Y Y, intersecting at right angles in O (Fig. 37); make O A' and O B' each equal to half the length of the bottom, and O C' and O D' each equal to half its width. Through C' and D' draw lines parallel to Y Y; also through A' and B' draw lines parallel to X X and inter-

FIG. 37.

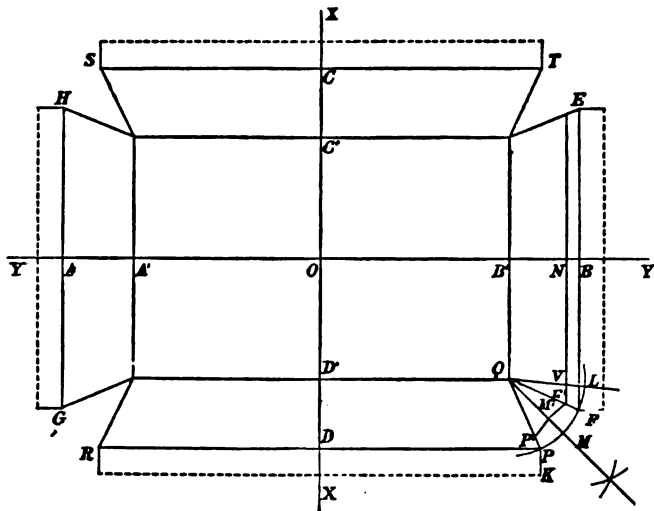
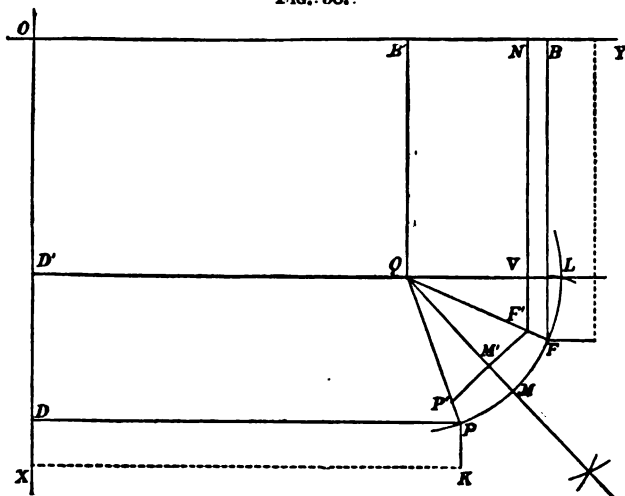


FIG. 38.



(QUARTER OF FIG. 37, ENLARGED.)

secting the lines drawn through C' and D' ; we get by this a rectangular figure, which is the shape of the bottom. Make $A'A$, $B'B$, $C'C$, and $D'D$ each equal to the given slant; through B and A draw EF and GH parallel to XX ; through C and D draw ST and RP parallel to YY ; and make BE , BF , AH , and AG each equal to half the given width of the top. Join QF and with Q , which is one of the angular points of the bottom, as centre and radius QF describe an arc cutting PR in P . Join QP (the working can here be best followed in Fig. 38); bisect the angle FQP by QM (Problem VIII., Chap. II.); and draw a line QL making with FQ an angle equal to the angle FQM . The readiest way of doing this is by continuing the arc PF to L , then setting off FL equal to FM , and joining LQ . Now on $B'B'$ set off BN equal to the thickness of the wire to be used for wiring, and through N draw NF' parallel to EF (Fig. 37) and cutting QL and QF in V and F' ; make QP' equal to QF' , and QM' equal to QV ; and join $P'M'$ and $M'F'$. Repeat this construction for the other three corners and the pattern will be completed. This is not done in Fig. 37, for a reason that will appear presently. We shall quite understand the corners if we follow the letters of the Q corner in Fig. 38. These are $BFF'M'P'P$. The dotted lines drawn outside the pattern (Fig. 37) parallel to EF , PR , GH , and ST show the allowance for fold for wiring. It is important that the ends of this allowance for fold shall be drawn, as, for instance, PK (both figs.), perpendicular to their respective edges.

Now as to bending up to form the pan. When the end adjacent and the side adjacent to the corner Q come to be bent up on $B'Q$ and $D'Q$ so as to bring FQ and PQ into junction, it is evident that as FQ and PQ are equal we shall obtain a true corner. To bring FQ and PQ into junction it is likewise manifest that the pattern will have also to be bent on the lines $F'Q$, $P'Q$, and $M'Q$. This fold on each other of $P'QM'$ and $F'QM'$ is generally still further bent round against QV .

(39.) The truth of the pan when completed, and the ease with which its wiring can be carried out, depend entirely on the accuracy of the pattern at the corners. This must never be forgotten. In marking out a pattern, only one corner need be drawn, as the like to it can be cut out separately and used to mark the remaining corners by. The points R, S, and T, the fixing of which will aid in this marking out, can be readily found, thus:—For the R corner to come up true it is clear that DR will have to be equal to DP, from which we learn that DP is half the length of the top. Having then determined the point P we have simply to set off DR, CS, and CT each equal to DP.

If the pattern Fig. 37 were completed with the corners as at R, S, and T, that would be the solution of Problem XXII., and the ends and sides being bent up, we should get an oblong equal-tapering body with top and bottom parallel and having flat sides and ends.

CASE II.—Where the length and width of the bottom, the length of the top, and the slant are given.

The only difference between this case and the preceding is that DP (Fig. 37), half the length of the top, is known instead of BF the half-width of top. To find the half-width of top; with Q as centre and QP as radius describe an arc cutting BF in F. Then BF will be half the width of the top, just as (§ 39, previous case) we saw that DP was half the length of the top. The remainder of the construction is now as in Case I.

(40.) It will be evident that, in this problem, choice of dimensions is not altogether arbitrary. The lines QF and QP the meeting of which forms the corner, must always be of the same length. This limits the choice; for with the dimensions of bottom given, and the slant, and the width of the top, the length of the top cannot be fixed at pleasure, but must be such as will bring QF and QP into junction; and *vice versa* if the length of the top is given. It is unnecessary to follow the limit with other data.

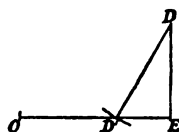
CASE III.—Where the length and width of the top, and the slant and the height are given.

The data in this case and the next are the usual data when a pan has to be made to order. The difference between this case and the preceding is that the size of the bottom is not given, but has to be determined from the data. Excepting as to finding the dimensions of this, the case is the same as Cases I. and II. All that we have now to do is therefore to find the dimensions of the bottom, thus:—

FIG. 39.



FIG. 40.



Draw OC (Fig. 39) equal to half the given length of the top, and through C draw CB perpendicular to OC and equal to the given height. With B as centre and radius equal to the given slant describe an arc cutting OC in B'. Then OB' is the required half-length of the bottom.

Next draw OE (Fig. 40) equal to half the given width of the top, and through E draw ED perpendicular to OE and equal to the given height. With D as centre and radius equal to the given slant describe an arc cutting OE in D'. Then OD' is the required half-width of the bottom.

CASE IV.—Where the length and width of the top and the length and inclination of the slant are given.

This is a modification of Case III.

Let BB' (Fig. 39) be the length of the given slant, and the angle that BB' make with B'C be the inclination of the slant. Through B draw BC perpendicular to B'C. Then half the given length of the top less B'C will be the required half-length of the bottom; and similarly half the given width of the top less B'C will be the required half-width of the bottom.

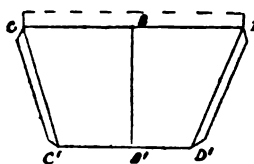
PROBLEM XXIV.

To draw the pattern for an equal-tapering body with top and base parallel, and having flat sides and ends (same as Problem XXII.), but with bottom, sides, and ends in separate pieces; the length and width of the bottom, the width of the top, and the slant being given.

To draw the pattern for the end.

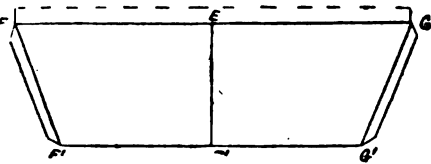
Draw $B B'$ (Fig. 41) equal to the given slant, and through B and B' draw lines $C D$ and $C' D'$ perpendicular to $B B'$. Make $B C$ and $B D$ each equal to half the given width of the top; also make $B' C'$ and $B' D'$ each equal to half the given width of the bottom. Join $C C'$ and $D D'$; then $C' C D D'$ will be the end pattern required.

FIG. 41.



END PATTERN.

FIG. 42.



SIDE PATTERN.

To draw the pattern for the side.

Draw $E E'$ (Fig. 42) equal to the given slant, and through E and E' draw $F G$ and $F' G'$ perpendicular to $E E'$. Make $E' F'$ and $E' G'$ each equal to half the given length of the bottom, and with F' and G' as centres and radius $D' D$ (Fig. 41) describe arcs cutting $F' G'$ in points F' and G' . Join $F' F$ and $G' G$; then $F' F G G'$ will be the side pattern required.

It should be noted that, as in the preceding problem, the width of the top determines the length of the top and *vice versa*, also that lines such as $D D'$, $G G'$ must be equal.

(41.) If the ends of the body are seamed ('knocked up') on to the sides, as is usual, twice the allowance for lap shown in Fig. 41, must be added to the figure $CC'D'D$. Similarly if the sides are seamed on to the ends, a like double allowance for lap must be added to $FF'G'G$, instead of to the end pattern.

CHAPTER VII.

PATTERNS FOR EQUAL-TAPERING ARTICLES OF FLAT AND CURVED SURFACES COMBINED.

(CLASS I. *Subdivision c.*)

From what has been stated about the plans of equal-tapering bodies, and from *d*, p. 55, it will be evident that the curved surfaces of the articles now to be dealt with, are portions of frusta of cones.

PROBLEM XXV.

To draw the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and equal semicircular ends (an 'equal-end' pan, for instance), the dimensions of the top and bottom of the body and its height being given.

Four cases will be treated of; three in this problem and one in the problem following.

CASE I.—Patterns when the body is to be made up of four pieces.

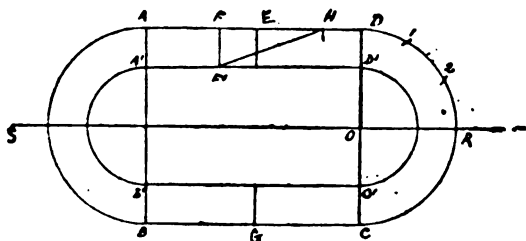
We may suppose the article to be a pan. Having drawn (Fig. 43) the plan *A R C S A' D' C' B'* by the method of Problem XVI. (as well as the plan, the lines of its construction are shown in the fig.), let us suppose the seams are to be at *A, B, C*, and *D*, where indeed they are usually placed. Then we shall require one pattern for the flat sides and another for the curved ends.

To draw the pattern for the sides.

anywhere in *AD* (Fig. 43) and perpendicular to it draw

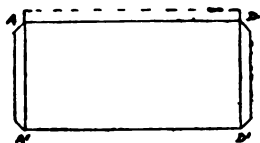
FF' , and make FH equal to the given height; join $F'H$; then $F'H$ is the length of the slant of the article, and therefore the width of the pattern for the sides.

FIG. 43.



Draw AA' (Fig. 44) equal to $F'H$ (Fig. 43). Through A and A' draw lines perpendicular to AA' . Make AD (Fig. 44) equal to AD (Fig. 43), and draw DD' parallel to

FIG. 44.



SIDE PATTERN.

AA' . The rectangle $ADD'A'$ will be the side pattern required. The fig. also shows extras for lap.

To draw the pattern for the ends.

Draw (Fig. 45a) two lines DA , OA , perpendicular to one another and meeting in A ; make DA equal to OD (Fig. 43), the larger of the radii of the semicircular ends; and on AO set off AG equal to the given height. Draw GD' perpendicular to AG and equal to OD' (Fig. 43), the smaller of the radii of the semicircular ends; join DD' and produce it, meeting AO in O . Then with any point O' (Fig. 45b), as

centre, and OD (Fig. 45a) as radius, describe an arc DC , and with the same centre and radius OD' (Fig. 45a) describe an arc $D'C'$. Draw any line DO' , cutting the arcs in points D and D' . Divide DR (Fig. 43) into any number of equal parts, say three. From D (Fig. 45b) mark off these three

FIG. 45a.

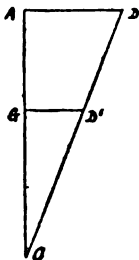
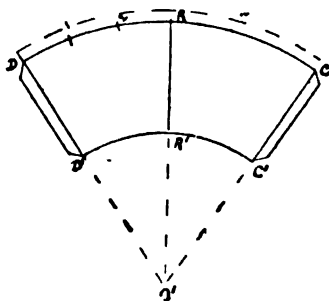


FIG. 45b.



dimensions to R , make RC equal to DR , and join CO' ; then $DC'C'D'$ will be the end pattern required.

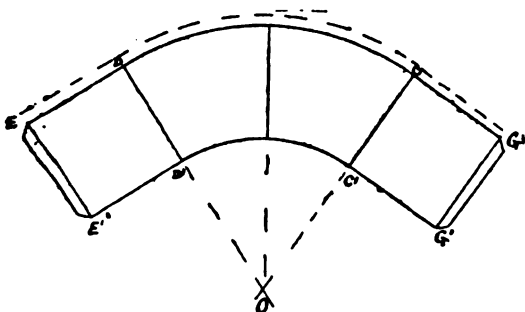
If R be joined to O' , then $D'D'R'R$ will be half the pattern. The centre line RR' is very useful, because, in making up the article, the point R' must meet the line SR (axis) of Fig. 43, otherwise the body will be twisted in consequence of the bottom not being true with the ends.

CASE II.—Pattern when the body is to be made up of two pieces.

Secondly, suppose the article can be made in two pieces (halves), with the seams at E and G (Fig. 43); the line EG (part only of it shown) being the bisecting line of the plan. It will be seen by inspection of the fig. that we require one pattern only, namely, a pattern that takes in one entire end of the article with two half-sides attached.

Draw the end pattern $DD'C'C$ (Fig. 46) in precisely the same manner that it is drawn in Fig. 45b. Through C and C' draw indefinite lines CG $C'G'$, perpendicular to CC' , and through D and D' draw indefinite lines DE , $D'E'$ perpendicular to DD' . Make DE , $D'E'$ each equal to DE (Fig. 43)

FIG. 46.



and join EE' ; also make CG , $C'G'$, each equal to CG (Fig. 43), and join GG' . Then $EE'G'G$ will be the pattern required.

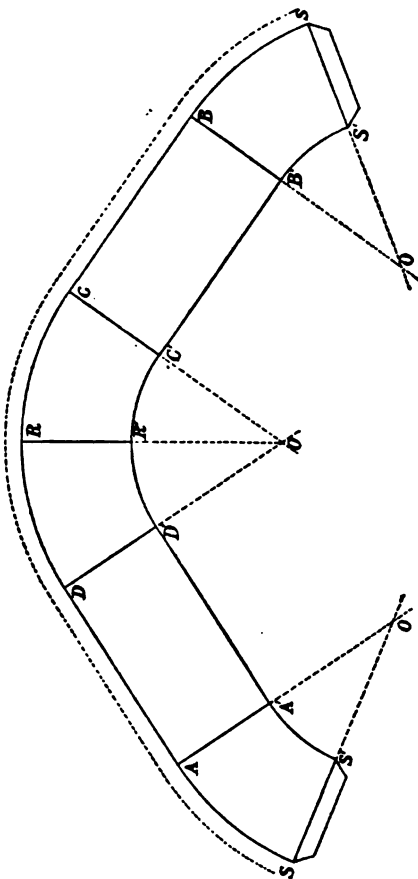
CASE III.—Pattern when the body can be made of one piece.

Thirdly, suppose the article can be made in one piece, with the seam at S (Fig. 43). Then evidently the pattern will be made up of the pattern of one entire end, side patterns attached to this, and half an end pattern attached to each side pattern.

Draw $DD'C'C$ (Fig. 47) the end pattern. Through C and C' draw lines perpendicular to CC' , and each equal to CB (Fig. 43); and join BB' . Produce BB' , and with B as centre and OD (Fig. 45a), the larger of the radii for the end pattern, as radius, describe an arc cutting the produced line

in O. With O as centre and OB and OB' as radii respectively, draw arcs BS, B'S'. Make BS equal to DR (Fig. 47)

FIG. 47.



and join SO. Repeating this construction for DD'A'A the other side pattern, and AA'S'S the remaining half-end pattern, will complete the pattern required.

in point 1 (Fig. 48b). With D' and D as centres and radii respectively D' 1' and E 1' (Fig. 48a) describe arcs intersecting in point 1' (Fig. 48b). By using points 1 and 1' (Fig. 48b) as centres, instead of D and D', and repeating the construction, the points 2 and 2' can be found. Next, using points 2 and 2' as centres and repeating the construction, find points 3 and 3', and so on for the remaining points necessary to complete the end pattern, which is completed by joining the various points, as 3 to 3' by a straight line; D, 1, 2, and 3, by a line of regular curve; and D', 1', 2' and 3', also by a line of regular curve. Only half the end pattern is shown in Fig. 48b. The side pattern can be drawn as shown in Fig. 44.

PROBLEM XXVII.

To draw the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and ends, and round corners (an oblong pan with round corners, for instance); the height and the dimensions of the top and bottom being given.

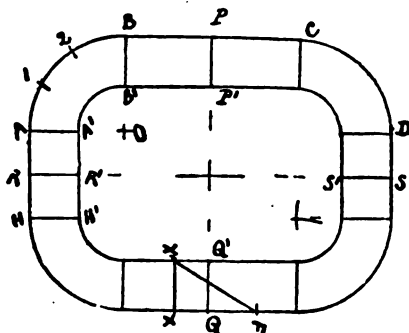
Again four cases will be treated of; three in this problem, and one in the problem following.

CASE I.—Pattern when the body is to be made up of four pieces.

The plan of the article, with lines of construction, drawn by the method of Problem XVII., is shown in Fig. 49. We will suppose the seams are to be at P, S, Q, and R, that is, at the middle of the sides and ends. The pattern required is therefore one containing a round corner, with a half-end and a half-side pattern attached to it. The best course to take is to draw the pattern for the round corner first, which, as will be seen from the plan, is a quarter of a frustum of a cone.

Draw two lines (Fig. 50a) OA , CA , meeting perpendicularly to one another in A ; make AC equal to OA (Fig. 49) the radius of the larger arc of the plan of one of the corners,

FIG. 49.



and make AD equal to the given height of the body. Draw DE perpendicular to AD and equal to the radius of the smaller arc of the plan of a corner, and join CE and produce

FIG. 50a.

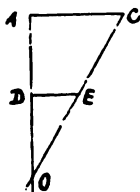
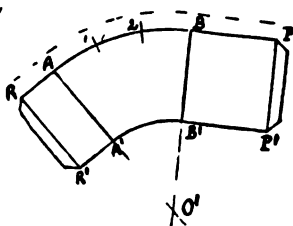


FIG. 50b.



it to meet AO in O . With any point O' (Fig. 50b) as centre and radius equal to OC (Fig. 50a), describe an arc AB , and with the same centre and radius equal to OE (Fig. 50a) describe an arc $A'B'$. Draw any line $A'O'$ cutting the arcs

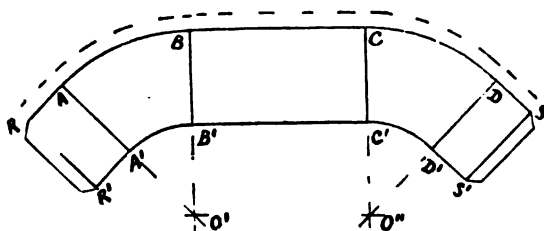
in the points A and A', and make A B equal to A B (Fig. 49) by marking off the same number of equal parts along A B (Fig. 50b) that we divide (arbitrarily) A B (Fig. 49) into. Join B O', cutting in B' the arc A' B'. Through B and B' draw B P, B' P' perpendicular to B B', make B P equal to B P (Fig. 49) and draw P P' perpendicular to B P. Through A and A' draw A R and A' R' perpendicular to A A', make A R equal to A R (Fig. 49) and draw R R' perpendicular to A R. Then R R' P' P will be the pattern of one round corner, with a half-end and a half-side pattern attached left and right.

CASE II.—Pattern when the body is to be made up of two pieces.

Now suppose the seams are to be at the middle of each end, at R and S (Fig. 49). The pattern required will then be of twice the amount shown in Fig. 50b. It will be found best to commence with the side pattern.

Draw in the plan (Fig. 49), any line X X perpendicular to the Q side-line; then X and X' will be plans of corresponding points (§ 26). Make X F equal to the given

FIG. 51.



height of the body and join X' F. Then X' F is the length of a slant of the body. Draw B B' (Fig. 51) equal to X' F (Fig. 49), and through B and B' draw B C and B' C' perpendicular to B B'. Make B C equal to B C (Fig. 49) and draw

CC' perpendicular to BC ; then $BC C' B'$ will be the pattern for the side.

We have now to join on to this, at BB' and CC' , the patterns for the round corners, which can be done thus. Produce BB' and CC' , and make BO' and CO'' each equal to OC (Fig. 50a) the radius of the larger arc of the corner pattern. With O' and O'' as centres and $O'B$ as radius, describe arcs BA and CD , and with the same centres and $O'B'$ as radius, describe arcs $B'A'$ and $C'D'$. Make BA and CD equal each to BA in the plan (Fig. 49), and join AC , cutting the arc $B'A'$ in A' , and DO'' cutting the arc $C'D'$ in D' . Through A and A' draw AR and $A'R'$ perpendicular to AA' ; make AR equal to AR (Fig. 49), and draw RR' perpendicular to AR . Next through D and D' draw DS and $D'S'$ perpendicular to DD' ; make DS equal to DS (Fig. 49), and draw SS' perpendicular to DS . Then $RR'S'S$ will be the pattern required.

It should be noted that $O'B'$ and $O''C'$ should be each equal to OE (Fig. 50a), or the pattern will not be true. This gives a means of testing its accuracy.

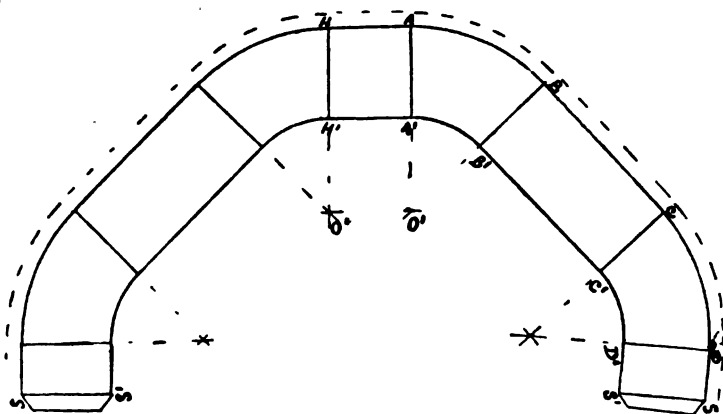
CASE III.—Pattern when the body is to be made up of one piece.

When the body is made in one piece it is usual to have the seam at S (Fig. 49). It is now best to commence with the end pattern.

Draw AA' (Fig. 52) equal to $X'F$ (Fig. 49), which, the body being equal-tapering, is the length of its slant anywhere, and draw AH and $A'H'$ perpendicular to AA' . Make AH equal to AH (Fig. 49) and draw HH' perpendicular to AH ; then $AA'H'H$ will be the end pattern. Next produce AA' , and make AO' equal to CO (Fig. 50a). Then O' will be the centre for the arcs of the corner pattern, which, drawn by Case II. of this problem, can now be attached at AA' . Through B and B' draw BC and $B'C'$ perpendicular to BB' ; make BC equal to BC (Fig. 49), and draw CC' perpendicular to BC . That completes the addition of the side pattern

$B B' C' C$ to the corner pattern. Next produce $C C'$, mark the necessary centre, and attach to $C C'$ the corner pattern $C D D' C'$, just as $A B B' A'$ was attached to $A A'$. Then through D and D' draw $D S$ and $D' S'$ perpendicular to $D D'$; make $D S$ equal to $D S$ (Fig. 49) and draw $S S'$ perpendicular

FIG. 52.



to $D S$; this adds half an end-pattern to $D D'$. By a repetition of the foregoing working on the $H H'$ side of the end-pattern $H H' A' A$, the portion $H H' S' S$ on the $H H'$ side may be drawn, and the one-piece pattern $S H A S S' A' H' S'$ of the body we are treating of completed.

PROBLEM XXVIII.

To draw, without long radii, the pattern for an equal-tapering body with top and bottom parallel, and having flat sides and ends, and round corners; the height and the dimensions of the top and bottom being given.

This is a fourth case of the preceding problem, and we will apply it to the first case of that problem, that is, when



the body is to be made up of four pieces; now, however, assuming that the pieces are so large that their patterns cannot be conveniently drawn by the method there given.

Draw (Fig. 53a) $PSS'P'$, one-quarter of the plan of the body. Divide CD the larger arc of the plan of the round corner, into three or more equal parts, and join each division-point to O the centre of the arcs of the plan. Draw DF perpendicular to DD' and equal to the given height of the body, and join $D'F$; then $D'F$ will be the true length of DD' . Next join $D2'$; draw DE perpendicular to it and

FIG. 53a.

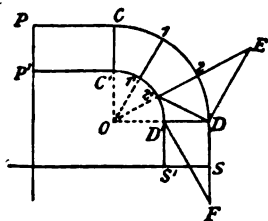
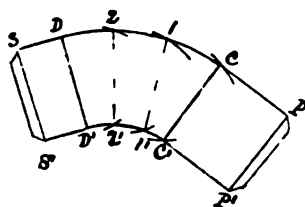


FIG. 53b.



equal to the given height, and join $2'E$; then $2'E$ may be taken as the true length of $D2'$. (In the fig. a portion of the line $2'E$ coincides with the line from 2 to $2'$. This is, however, merely a coincidence of the particular case.)

Now draw (Fig. 53b) DD' equal to $D'F$ (Fig. 53a), and with D and D' as centres and radii respectively $D2$ and $2'E$ (Fig. 53a) describe arcs intersecting in point 2 . Also with D' and D as centres and radii respectively $D'2'$ and $2'E$ describe arcs intersecting in point $2'$. Repeating this construction, but using points 2 and $2'$ just found as centres instead of D and D' , the points 1 and $1'$ can be found. Similarly using points 1 and $1'$ as centres and repeating the construction we can find the points C and C' . Join CC' ; draw a regular curve from C through 2 and 1 to D , and another regular curve from D' through $2'$ and $1'$ to C' ; then in $DCC'D'$ we have the pattern of the corner. To this

corner pattern, to get at the pattern we require, we have to attach a half-end and a half-side pattern.

Draw DS perpendicular to DD' and equal to DS (Fig. 53a). Also draw SS' perpendicular to DS , and through D' draw $D'S'$ parallel to DS . The half-end pattern is now attached.

Through C and C' draw CP and $C'P'$ perpendicular to CC' . Make CP equal to CP (Fig. 53a), and draw PP' perpendicular to CP . This attaches the half-side pattern, and completes the pattern required.

If the body is to be made in more than four pieces it will still be generally possible to have a complete corner in one of the pieces. The corner should always be marked out first, and whatever has to be attached should be added as described.

PROBLEM XXIX.

To draw the pattern for an oval equal-tapering body with top and bottom parallel, the height and the dimensions of the top and bottom being given.

Again we deal with four cases; three in this problem and one in the following.

CASE I.—Patterns when the body is to be made up of four pieces.

The plan of the body with lines of construction, drawn by Problem XVIII., is shown in Fig. 54. It will be evident from the plan and from *d*, p. 55, that the body is made up of two round equal-tapering bodies (frusta of cones); the ends $CD D' C'$, $FE E' F'$, being equal portions of a frustum of one cone, and the sides, $ECC'E'$, $DD'F'D'$, equal portions of a frustum of another cone. Such being the case, it is clear that we require two patterns, one for the two ends, and the other for the two sides; also that the seams should be at E , C , D , and F , where the portions meet.

To draw (Fig. 55b) the ends' pattern.

From any point A draw two lines OA , BA (Fig. 55a)

FIG. 54.

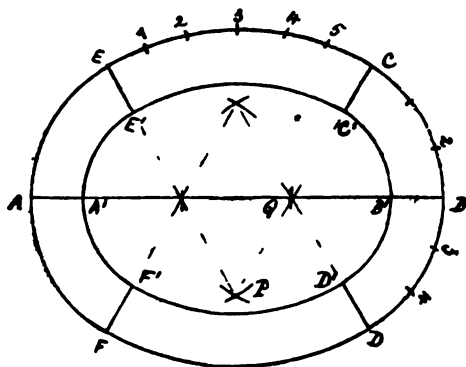


FIG. 55a.

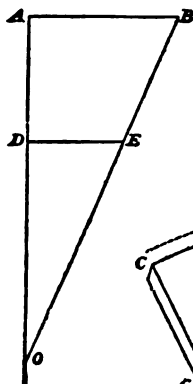


FIG. 55b.

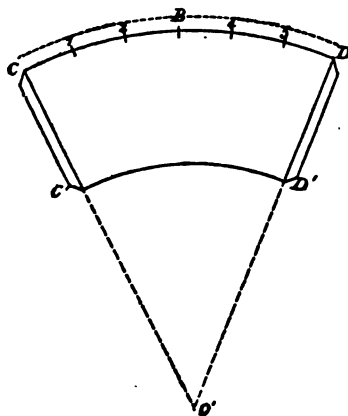


FIG. 56a.

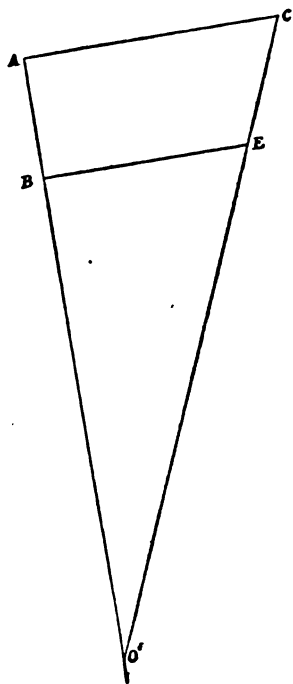
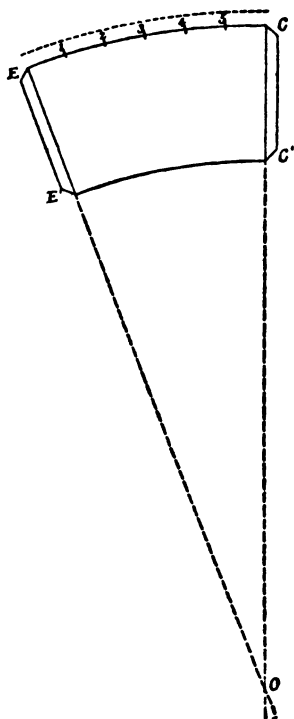


FIG. 56b.



perpendicular to each other ; make AB equal to QB (Fig. 54) the radius of the larger end-curve in the plan, and AD equal to the height of the body. From D draw DE perpendicular to AO and equal to QB' (Fig. 54) the radius of the smaller end-curve of the plan ; join BE and produce it, meeting AO in O . Then with any point O' (Fig. 55b) as centre, and OB (Fig. 55a) as radius, describe an arc CD ; and with same centre and OE (Fig. 55a) as radius describe an arc $C'D'$. Join any point C in the arc CD to O' by a line cutting the arc $C'D'$ in C' ; divide the arc CD (Fig. 54) into any number of equal parts, and set off from C (Fig. 55b) along CD parts equal to and as many as CD (Fig. 54) is divided into. Join $D O'$ cutting the arc $C'D'$ in D' ; then $D'D'C'C$ will be the pattern required.

To draw the pattern for the sides.

From any point A draw two lines AC , AO perpendicular to each other, and make AC equal to PC (Fig. 54), the radius of the larger side-curve and AB equal to the given height of the body. From B draw BE perpendicular to AB and equal to PC' , (Fig. 54) the radius of the smaller side-curve. Join CE and produce it, meeting AO in O . With any point O' (Fig. 56b) as centre and OC (Fig. 56a) as radius describe an arc CE ; and with same centre and OE (Fig. 56a) as radius describe an arc $C'E'$. Now join CO' , and, proceeding exactly similarly as in drawing the ends' pattern, complete $E'E'C'C$ the pattern for the sides.

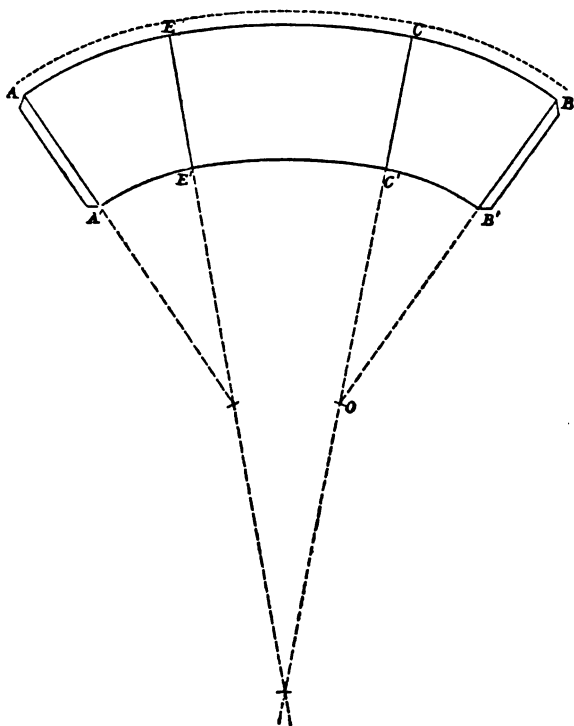
CASE II.—Patterns when the body is to be made up of two pieces.

In this case the seams are usually put at the ends A and B (Fig. 54). It is evident that one pattern only is now required, and that this is made up of a side pattern, with, right and left attached to it, a half-end pattern.

Draw (Fig. 57) a side pattern $ECC'E'$ in the way described in Case I. Then with C as centre and the radius of the ends' pattern, that is BO (Fig. 55a) as radius, set off the distance CO . With O as centre and radii OC and OC' describe arcs CB and $C'B'$. Make CB equal to CB (Fig. 54)

and join $B O$, cutting $C' B'$ in B' . Then $C B B' C'$ is a half-end pattern attached to $C C'$. The half-end pattern $E A A' E'$ is

FIG. 57.

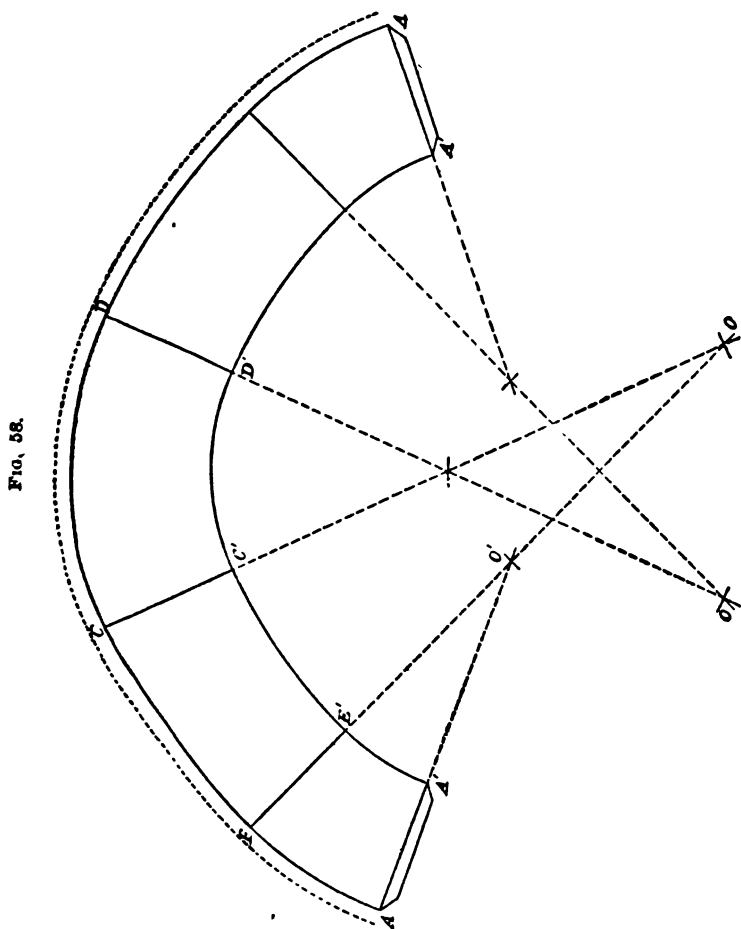


added at $E E'$ by repeating the construction just described. This completes $A B B' A'$ the pattern required.

CASE III.—Pattern when the body is to be made up of one piece.

We will put the seam at A (Fig. 54), the middle of one end. Draw $C D D' C'$ (Fig. 58), an end pattern in the way described in Case I. Produce $C C'$ and make $C O$ equal to

CO (Fig. 56a), the radius for a side pattern. With O as centre and radii OC and OC' describe arcs CE, C'E'. Make



CE equal to CE (Fig. 54) and join EO cutting C'E' in E'. Make E'O' equal to BO (Fig. 55a) and with O'E as radius

describe an arc $E A$. With same centre and $O' E'$ as radius describe an arc $E' A'$. Make $E A$ equal to $E' A'$ (Fig. 54) and join $A O'$, cutting $E' A'$ in A' . The remainder $D A A' D'$ of the pattern can be drawn by repeating the foregoing construction. The figure $A C D A' D' C' A'$ will be the pattern required.

PROBLEM XXX.

To draw, without long radii, the pattern for an oval equal-tapering body with top and bottom parallel, the height and the dimensions of the top and bottom being given.

This is a problem which will be found very useful for large work, especially with the pattern for the sides, the radius for which is often of most inconvenient length.

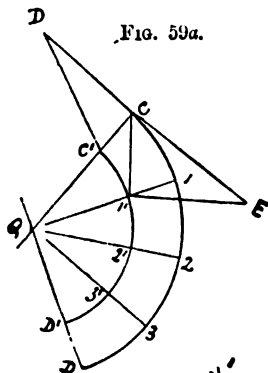
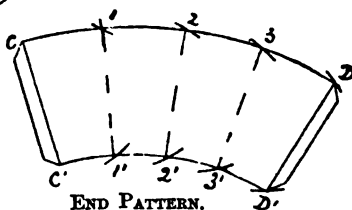


FIG. 59b.



To draw the ends' pattern.

Draw (Fig. 59a) the plan $C D D' C'$ of the end of the body. Divide $C D$ into four or more equal parts, and join, 1, 2, &c.,

to Q, by lines cutting $C'D'$ in $1', 2', \&c.$ From C draw CD perpendicular to CC' and equal to the given height. Join $C'D$, then $C'D$ is the true length of CC' . Join $C1'$; draw $1'E$ perpendicular to it and equal to the given height; and join CE. Then CE may be taken as the true length of $C1'$. Draw CC' (Fig. 59b) equal to $C'D$ (Fig. 59a). With C and C' as centres and radii respectively equal to $C1$ and CE (Fig. 59a) describe arcs intersecting in point 1. With C' and C as centres and radii respectively equal to $C'1'$ and CE (Fig. 59a) describe arcs intersecting in point $1'$. By using points 1 and $1'$ as centres and repeating the construction, points 2 and $2'$ can be found. Similarly find points 3 and $3'$, and D and D' . Join DD' , draw a regular curve from C through 1, 2, and 3, to D, and another regular curve from C' through $1', 2',$ and $3'$, to D' . The figure $CC'D'D$ is the pattern required.

Fig. 60a.

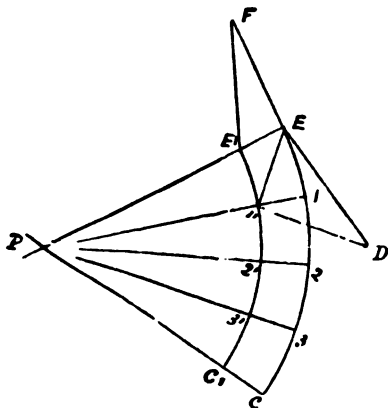
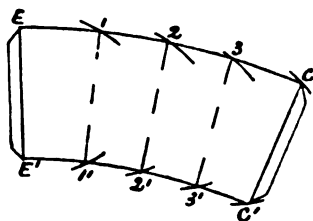


Fig. 60b.



SIDE PATTERN.

To draw the sides' pattern.

Draw (Fig. 60a) the plan $EC C'E'$ of the side of the body. Divide EC into four or more equal parts, and join 1, 2, &c., to P, by lines cutting $E'C'$ in $1', 2', \&c.$ From E draw EF

perpendicular to $E E'$ and equal to the given height. Join $E' F$; then $E' F$ is the true length of $E E'$. Next join $E 1'$; draw $1' D$ perpendicular to $E 1'$ and equal to the given height, and join $E D$; then $E D$ may be taken as the true length of $E 1'$. Now draw (Fig. 60*b*) $E E'$ equal to $E' F$ (Fig. 60*a*), and with E and E' as centres and radii respectively equal to $E 1$ and $D E$ (Fig. 60*a*) describe arcs intersecting in point 1. With E' and E as centres and radii respectively equal to $E' 1'$ and $D E$ (Fig. 60*a*) describe arcs intersecting in point 1'. Repeating the construction with points 1 and 1' as centres, the points 2 and 2' can be found; and similarly the points 3 and 3' and the points C and C' . Join $C C'$; draw a regular curve from E through 1, 2, and 3, to C , and another from E' through 1', 2', and 3', to C' . The figure $E E' C' C$ will be the pattern required.

BOOK II.

CHAPTER I.

PATTERNS FOR ROUND ARTICLES OF UNEQUAL TAPER OR INCLINATION OF SLANT.

(CLASS II. *Subdivision a.*)

(42.) WE stated at the commencement of the preceding division of our subject that it was advisable, in order that the rules for the setting out of patterns for articles having equal slant or taper should be better understood and remembered, to consider the principles on which the rules were based; and the remark is equally true and of greater importance in respect of the rules for the setting out of patterns for articles having unequal slant or taper. We have shown that the basis of the rules for the setting out of patterns for equal tapering bodies is the right cone; we purpose showing that the basis of the rules for the setting out of patterns for articles of unequal slant or taper is what is called the *oblique* cone. The consideration of the cone apart from its species, that is, apart from whether it is right or oblique, which becomes now necessary, immediately follows.

DEFINITION.

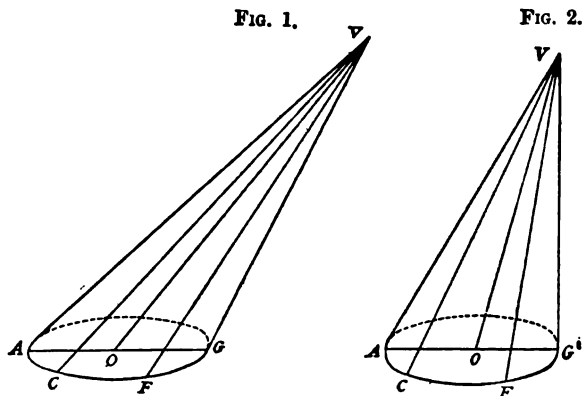
(43.) *Cone*.—A cone is a solid of which one extremity, the base, is a circle, and the other extremity is a point, the apex. The line joining the apex and the centre of the base is the axis of the cone.

(44.) Given a circle and a point in the line passing through the centre of the circle at right angles to its plane. If an indefinite straight line, passing always through the given point, move through the circumference of the given circle,

there will be thereby generated between the point and the circle, a solid; this solid is the *right* cone.

(45.) If, all other conditions remaining the same, the given point is *out of* the line passing through the centre of the circle at right angles to its plane, the solid then generated will be the *oblique* cone.

(46.) Figs. 1 and 2 represent oblique cones. The lines



VA, VC, VF, and VG (either fig.) drawn from the apex V to the circumference of the base of the cone, are portions of the generating line at successive stages of its revolution. It is but a step from this and will be a convenience, to regard these lines, first, as each of them part of an independent generating line, and then as each of them a complete generating line. Shaded representations of various oblique cones will be found later on.

(47.) Comparing now the right and oblique cones. A right cone may be said to be made up of an infinite number of equal generating lines, and an oblique cone of an infinite number of unequal generating lines.

(48.) If a right cone is placed on a horizontal plane, the apex is vertically over the centre of the base.

(49.) If when a cone is placed on a horizontal plane the apex is not vertically over the centre of the base, the cone is

oblique. Hence all cones not right cones are oblique cones. Hence also a cone is oblique if its axis is not at right angles to every diameter of its base.

(50.) In the right cone, any plane containing the axis is perpendicular to the base of the cone, and contains two generating lines (see Figs. 1a and 2a, p. 25).

(51.) In the oblique cone, only *one plane* containing the axis is perpendicular to the base of the cone, and this plane contains its longest and shortest generating lines. In Fig. 1 or 2 the lines VA and VG are respectively the longest and shortest generating lines, and the plane that contains these lines contains also VO the axis of the cone.

(52.) The obliquity of a cone is measured by means of the angle that its axis makes with that radius of the base that terminates in the extremity of the shortest generating line. Thus the angle VOG gives the obliquity of either cone VAG . The angle VOG also gives the inclination of the axis. As the angle VAG is in the same plane as VOG and smaller than that angle, it will be seen that not only is VA the longest generating line, but it is also the line of greatest inclination on the cone. Similarly as the angle that VG makes with AG produced is greater than VOG , the line VG is not only the shortest generating line, but also the line of least inclination on the cone.

(53.) The plane that contains the longest and shortest generating lines bisects the cone; consequently the generating lines of either half are, pair for pair, equal to one another.

(54.) If the elevation (see § 21, p. 48) of an oblique cone be drawn on a plane parallel to the bisecting plane, the elevations of the longest and shortest generating lines will be of the same lengths respectively as those lines. Thus if the triangle VAG (either fig.) be regarded as the elevation of the cone represented, then VA will be the true length of the longest generating line of the cone, and VG of the shortest. In speaking in the pages that follow, of elevation with regard to the oblique cone, we shall always suppose it to be on a plane parallel to the bisecting plane.

(55.) If the hypotenuse of a right-angled triangle represent

a generating line of any cone, right or oblique, then one of the sides containing the right angle is equal in length to the plan of that line, and the other side is equal to the height (distance between the extremities) of any elevation of it. See, p. 25, the triangles OBA , OGA , OEA , ODC , OKC , $OF C$, and, p. 109, the triangles $V'AV$, $V'BV$, $V'CV$, &c. See also § 22a.

(56.) As the generating lines of the oblique cone vary in length, the setting out of patterns of articles whose basis is the oblique cone (that is to say, the development of the curved surfaces of such articles) differs from that appertaining to articles in which the right cone is involved. The principles, however, are the same in both cases. In developments of the right cone, if a number of its generating lines are laid out in one plane, then as they are all equal and have a point (the apex) in common, the curve joining their extremities is an arc of a circle of radius equal to a generating line; while in developments of the oblique cone, as the generating lines, though they still have a point in common, vary in length, the individual lengths of a number of them must be found, as well as the distances apart of their extremities, before, by being laid out in one plane at their proper distances apart, a curve can be drawn through their extremities, and the required pattern ascertained.

The problem which follows gives the working of this in full.

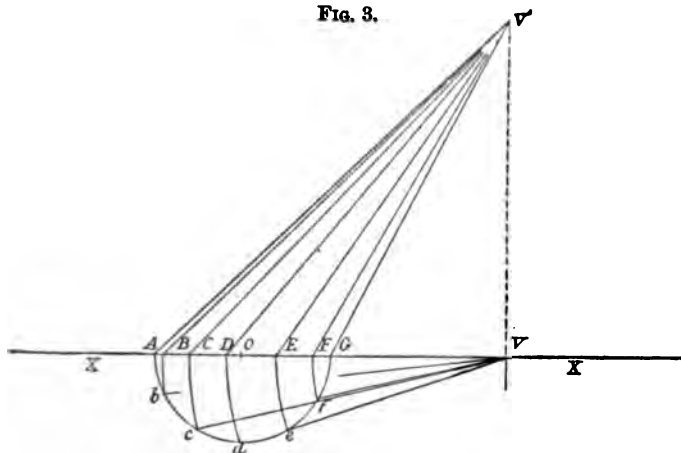
PROBLEM I.

To draw the pattern (develop the surface) of an oblique cone, the inclination of the axis (§ 52), its length and the diameter of the base of the cone being given.

First (Fig. 3) draw $V'AG$ the elevation of the cone, and AdG half the plan of its base; thus. Draw any line XX , and at any point O in it, make the angle $V'OX$ equal to the given inclination of the axis (a line OV' is omitted in the fig. to avoid confusion), and make OV' equal to the given length of the axis. With O as centre and half the

given diameter of the base as radius describe a semicircle $A d G$ cutting XX in A and G . Join $A V'$, $G V'$, then $V' A G$ is the elevation required, and $A d G$ is the half-plan of base. Next divide $A d G$ into any convenient number of parts, equal or unequal. The division here is into six, and the parts are equal; to make them so being an advantage. Now let fall $V' V$ perpendicular to XX ; and join V to the division points b, c, d, e, f (to save multiplicity of lines this is only partially shown in the fig.). The lines $V A$, $V b$, $V c$, &c., will be the lengths in plan of seven lines from apex

FIG. 3.

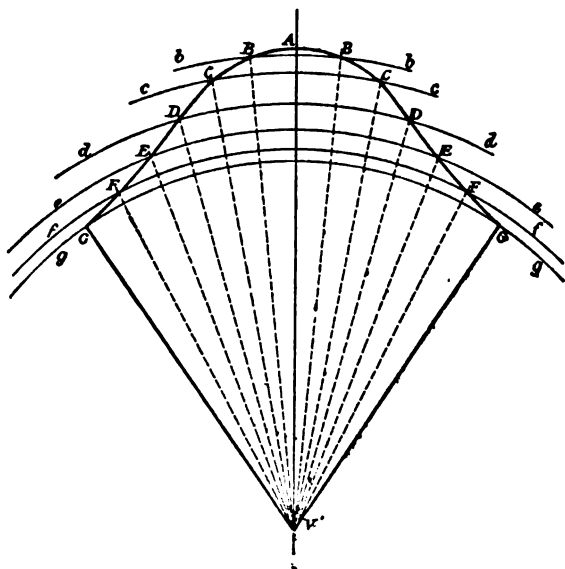


to base of cone, that is, of seven generating lines. Now with V as centre and radii successively $V b$, $V c$, $V d$, $V e$, and $V f$, describe arcs respectively cutting XX in B, C, D, E , and F . Join $V' B$, $V' C$, &c.; then as $V' V$ is the height of any elevation of either of the seven generating lines (see § 22a, p. 48, and § 55, p. 107) and as $V A$, $V B$, &c., are their plan lengths, we have in $V' A$, $V' B$, $V' C$, &c., their true lengths. $V' G$ is not only so however, it is (§ 54) the shortest of all the generating lines of the cone.

To draw the pattern of the cone with the seam to corre-

spond with $V'G$ the shortest generating line. Draw (Fig. 4) $V'A$ equal to $V'A$ (Fig. 3), and with V' as centre and $V'B$, $V'C$, $V'D$, $V'E$, $V'F$, and $V'G$ (Fig. 3) successively as radii describe arcs, respectively, bb , cc , dd , ee , ff , and gg . With A as centre and radius equal to Ab (Ab being the distance apart on the round of the cone at its base of the generating lines $V'A$ and $V'B'$) (Fig. 3) describe an arc cutting the arc bb right and left of $V'A$ in B and B . With

FIG. 4.



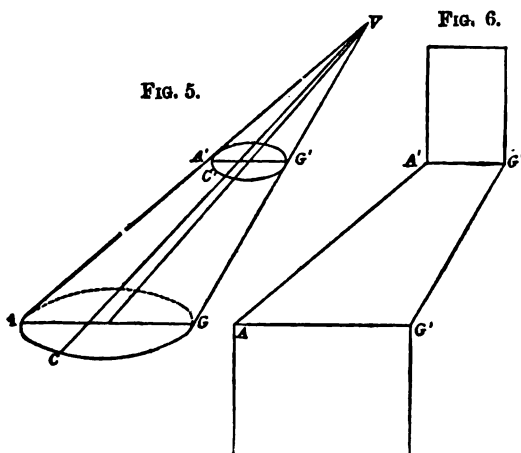
these points B and B as centres and radius as before (the distances apart of the extremities of the generating lines at the base of the cone being all equal), describe arcs cutting the arc cc right and left of $V'A$ in C and C . With same radius and the last-named points as centres describe arcs cutting dd right and left of $V'A$ in D and D . With D and D as centres and same radius describe arcs cutting ee right and left of $V'A$ in E and E , and with E and E as

centres and same radius describe arcs cutting ff right and left of $V'A$ in F and F . Similarly with same radius and F and F as centres find points G and G . Draw through A and the points B, C, D, E, F and G , right and left of $V'A$ an unbroken curved line. Also join $G V'$ right and left of $V'A$, then $G A G V'$ will be the pattern required.

The dotted lines $V'B, V'C, V'D$, &c., are not necessary for the solution of the problem, and are only drawn to show the position of the seven generating lines on the developed surface.

(57.) As the plane of the elevation $V'A G$ bisects the cone (§§ 53 and 54), it is clear that the seven generating lines found, correspond with seven other generating lines on the other half of the cone; so that in finding them for one half of the cone, we find them for the other, as the halves necessarily develop alike.

(58.) Round articles of unequal slant or taper having their ends parallel are portions (frusta) of oblique cones.



DEFINITION.

(59.) *Frustum*.—If an oblique cone be cut by a plane parallel to its base, then the part containing the apex is still an oblique cone as $V A' C' G'$ (Fig. 5), and the

part $A'ACGG'$ containing the base is a *frustum* of an oblique cone. A shaded representation of such frustum will be found in Fig. 15. This frustum, however, differs from that of the right cone (§ 12, p. 38), in that, though having circular ends, its sides are not of equal, but of unequal slant. Conversely, a round unequally tapering body, having its top and base parallel, is a frustum of an oblique cone. A tapering piece of pipe $A'AGG'$ (Fig. 6) joining two cylindrical pieces which are not in line with each other is a frustum of an oblique cone.

(60.) Referring to Fig. 5, it is evident that if the pattern for the larger cone, VAG , and the pattern for the smaller cone, $VA'G'$, be drawn from a common centre V' (Fig. 9), the figure $G'GAGG'$ (Fig. 9) will be the pattern for the portion $AGG'A'$ (Fig. 5) of the cone VAG . The line CC' shows a generating line of the frustum (b, p. 126).

FIG. 7b.

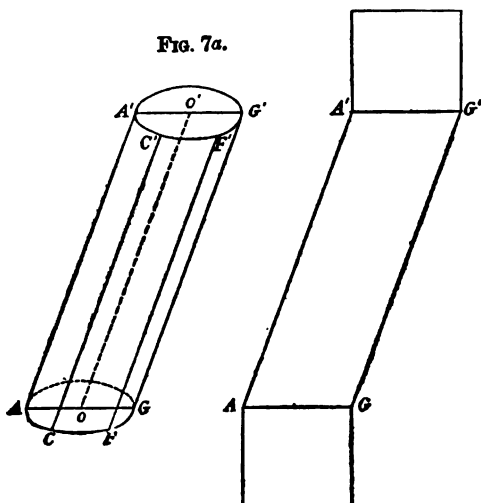


FIG. 7a.

(61.) A case that not unfrequently occurs needs mention here. Suppose the diameters of the top and base of a round

unequal-tapering body of parallel ends are very nearly equal, then it is evident that the apex of the oblique cone of which the body is a portion will be a long distance off, and, if the diameters be equal, then the body becomes what is called an oblique cylinder, (a cylinder is a round body without any taper at all). Of course this is an extreme case, but it is quite an admissible one. For, if the diameters of the ends of the body differ by only $\frac{1}{10000}$ inch, then, clearly, however short the body may be, we are dealing with a frustum of an oblique cone, although so nearly a cylinder that, for almost any purpose occurring in practice, it could be treated as a cylinder. Later on the advantages of looking upon the oblique cylinder as a special case of frustum of an oblique cone, and considering its generating lines as parallel, will be seen. Such a frustum is represented in $A A' G' G$ in Figs. 7a and 7b, the line $O O'$ being the axis of the frustum and CC' and FF' generating lines. A construction easily dealing with its development will be given presently.

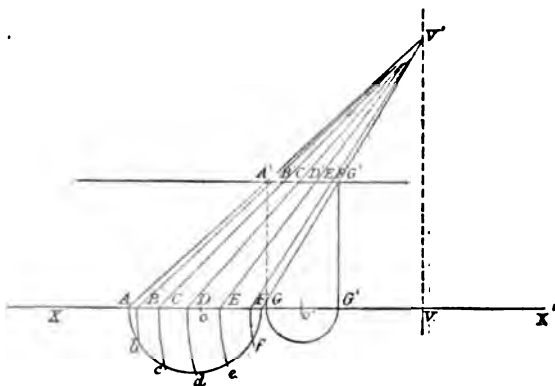
PROBLEM II.

To draw the pattern of a round unequal-tapering body with top and base parallel (frustum of an obliquecone, as in Fig. 5), the diameters of top and base, the height, and the inclination of the longest generating line being given.

First (Fig. 8) draw the elevation and half the plan of the body, thus: Draw a line XX' , and at any point A in it, make the angle $X' A A'$ equal to the given inclination of the longest generating line. At a distance from XX' equal to the given height, draw a line $A' G'$ parallel to XX' . From the point A' where $A' G'$ cuts AA' , make $A' G'$ equal to the diameter of the top, also make $A G$ equal to the diameter of the base; and join $G G'$. Then $A A' G' G$ is the elevation of the body. Now on $A G$ describe a semicircle $A d G$, this will be half the plan of the base. Produce $A A'$, $G G'$, to intersect in V' ; this point will be the elevation of

the apex of the cone of which the body is a portion. Through V' draw $V'V$ perpendicular to XX' ; divide the semicircle into any convenient number of equal parts, here six, in the

FIG. 8.

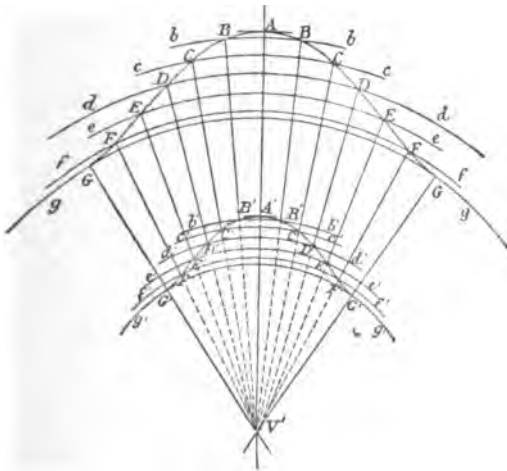


points b, c, d, e , and f ; with V as centre and radii successively Vb, Vc, Vd , &c., describe arcs respectively cutting XX in B, C, D , &c.; and join B, C, D , &c., to V' by lines cutting $A'G'$ in B', C', D', E' , and F' . Then BB', CC', DD' , &c., will be (see construction of last problem) the true lengths of various generating lines of the frustum (*b*, p. 126).

Now to draw the pattern (Fig. 9) so that the seam shall correspond with GG' (Fig. 8) the shortest generating line. Draw (Fig. 9) $V'A$ equal to $V'A$ (Fig. 8) and with V' (Fig. 9) as centre and radii successively equal to $V'B, V'C, V'D, V'E, V'F$, and $V'G$ (Fig. 8), describe, respectively, arcs bb, cc, dd, ee, ff , and gg . With A as centre and radius equal Ab (Fig. 8) (see preceding problem for the reason of this), describe arcs cutting the arc bb right and left of $V'A$ in B and B . With these points, B and B as centres and radius as before, describe arcs cutting the arc cc right and left of $V'A$ in C and C . With same radius and the last-

named points as centres, describe arcs cutting dd right and left of $V'A$ in D and D . With D and D as centres and same radius, describe arcs cutting ee right and left of $V'A$ in E and E , and with E and E as centres and same radius describe arcs cutting ff in F and F . Similarly, with same radius and F and F as centres find points G and G . Join the points B, C, D, E , &c., right and left of $V'A$ to V' . With V' as centre and $V'A'$ (Fig. 8) as radius describe an arc

FIG. 9.



cutting $V'A$ in A' . With same centre and $V'B'$ (Fig. 8) as radius describe an arc $b'b'$ cutting $V'B$ right and left of $V'A$ in B' . With same centre and $V'C'$ (Fig. 8) as radius describe an arc $c'c'$ cutting $V'C$ right and left of $V'A$ in C' . Similarly, with the same centre and $V'D'$, $V'E'$, $V'F'$, and $V'G'$ (Fig. 8) successively as radii describe, respectively, arcs $d'd'$, $e'e'$, $f'f'$, and $g'g'$, cutting $V'D$, $V'E$, $V'F$, and $V'G$, right and left of $V'A$ respectively in D' , E' , F' , and G' . Draw through A and the points B, C, D, E, F , and G , right-

and left of $V'A$ an unbroken curved line. Also draw through A' and the points B', C', D', E', F' , and G' , right and left of $V'A$ an unbroken curved line. Then $G'A G G' A' G'$ will be the pattern required.

The small semicircle (Fig. 8) and perpendicular lines from its extremities are not needed in this problem, but are introduced in illustration of that next following.

(62.) In the applications of the oblique cone, it is generally in the form next following that the problem presents itself.

PROBLEM III.

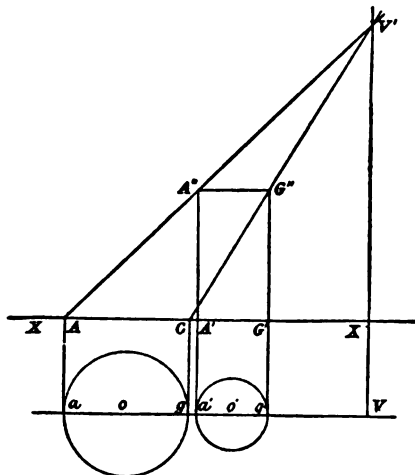
To draw the pattern of a round unequal-tapering body with top and base parallel (frustum of oblique cone), its plan and the perpendicular distance between the top and base (the height of the body) being given.

The working of this problem should be carefully noted, for the reason just above stated.

Let $ag a' g'$ (Fig. 10) be the given plan of the body. The side lines of the plan are not drawn, but only the circles of its top and base, as we do not make use of the side lines. In fact, all that we really make use of is, as will be presently seen, the halves of the plan-circles of the top and base of the body, not, however, placed anyhow, but *in their proper positions relatively to one another in plan*. This is the all-essential point. If the plan-circle $a' g'$ (Fig. 10) of the top of the frustum were further removed than represented from the plan-circle ag of its base, we should, by the end of our working, get at the pattern of some other oblique-cone frustum than that the pattern of which we require. Through OO' the centres of the circles draw an indefinite line aV , and draw any line XX parallel to OO' . Through a and g draw aA and gG perpendicular to XX , and through a' and g' draw indefinite lines $a'A''$ and $g'G''$ perpendicular to XX , and cutting it in

points A' , G' . Make $A'A''$ and $G'G''$ each equal to the given height or perpendicular distance that the top and base are apart. Join $A'A''$ and $G'G''$, and produce these lines to intersect in V' . The problem can now be completed by Problem II.

FIG. 10.



It should be noted that it is only necessary to draw half the plans of the top and base as shown by the semicircles AG and $A'G'$ in Fig. 8.

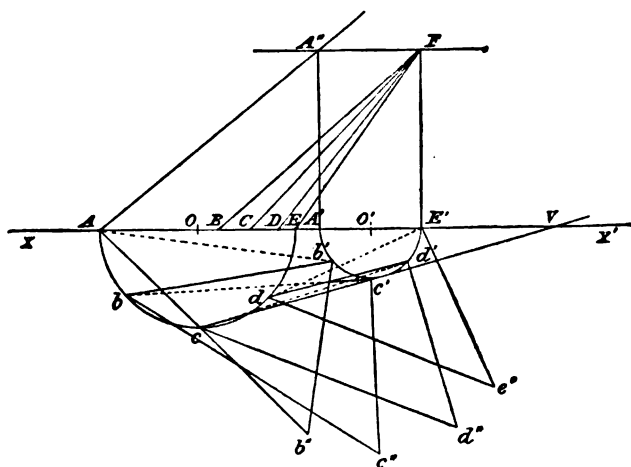
The case of large work, where long radii would be inconvenient, we treat as a separate problem, and we will suppose the dimensions given to be those of Problem II. The method is also suitable where there is little difference between the diameters of top and base of the body.

PROBLEM IV.

To draw without long radii the pattern of a round unequal-tapering body with top and base parallel, the diameters of the top and base, the height, and the inclination of the longest generating line being given.

Draw any line XX' (Fig. 11) and at point A in it make the angle $X'AA''$ equal to the given inclination of the longest generating line. At a distance from XX' equal to the given height draw a line $A''F$ parallel to XX' . Make AE equal to the longer of the given diameters, and on it describe the semicircle $A c E$. From the point A'' where

FIG. 11.



$A''F$ cuts AA'' let fall $A''A'$ perpendicular to XX . Make $A'E'$ equal to the shorter of the given diameters, and on it describe the semicircle $A'c'E'$. Next find c and c' the mid-points of the semicircular arcs. These are easily found by drawing lines (not shown in the fig.) through O and O' (the

centres from which the semicircles are described) perpendicular to XX' and cutting the semicircular arcs in the required points c and c' . Join cc' and produce it meeting XX' in V ; this point will be the plan of the apex of the cone of which our tapering body is a frustum. This point is by no means always within what may be termed workable reach, as for instance where the two semicircles are nearly equal the lines XX' and cc' therefore very nearly parallel, and the producing cc' to V impracticable. We will work under both suppositions.

(63.) If V is accessible, then divide the larger semicircle into any convenient number of parts (four parts only are taken in the figure in order to keep it clear), as Ab, bc, cd, dE . Join b and d to V (c is already thus joined, the lines from b and d to V are only drawn in the fig. as far as the smaller semicircle) by lines cutting the smaller semicircle in b' and d' . Then $AA', bb', cc', \&c.$, are the plans of generating lines of the frustum (tapering body), and in order to draw its pattern their true lengths must be found.

(64.) If V is inaccessible, then divide the smaller semicircle into four parts (the same 'convenient number' of parts that the larger semicircle was divided into), in the points b', c', d' , and join $bb', cc',$ and dd' .

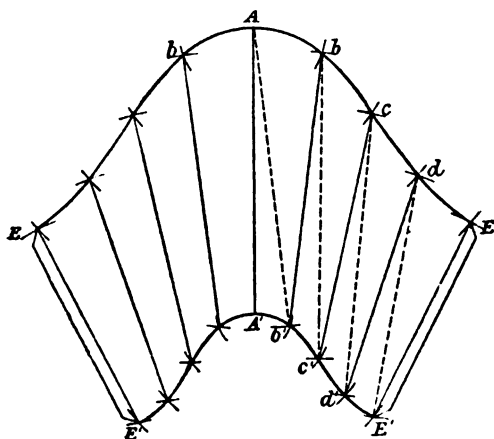
The true lengths of $bb', cc', \&c.$, are found as follows: From E' draw a line perpendicular to XX' and cutting $A''F$ in F , and join EF . Then EF is the true length of EE' . Now make $E'D$ equal to dd' and join DF ; then DF is the true length of dd' . Next set off $E'C$ equal to cc' , and $E'B$ equal to bb' ; and join CF and BF . Then CF and BF are the true lengths respectively of cc' and bb' . The true length of AA' we already have in AA'' , and as this is the longest generating line of the frustum, EF will be the shortest.

We proceed now to find the distance the points A and b' , b and c' , c and d' , $\&c.$, are apart, which we do by finding the true lengths of the lines $Ab', bc', cd',$ and dE' , joining the points. Through b' draw $b'b''$ perpendicular to Ab' , and equal to the given height. Join Ab'' ; then Ab'' may be

taken as the true length of $A b'$. Similarly, through c' , d' , and E' draw lines equal to the given height, and perpendicular to $b c'$, $c d'$, and $d E'$ respectively. Join $b c''$, $c d''$, and $d e''$; then $b c''$, $c d''$, and $d e''$ may be taken as the true lengths required.

To draw the pattern (Fig. 12) the seam to correspond with the shortest generating line. Draw $A A'$ equal to $A A''$ (Fig. 11) and with A and A' as centres and radii respectively $A b''$ and $A' b'$ (Fig. 11) describe arcs intersecting in b' . Next with b' and A as centres and radii respectively $B F$ and $A b$ (Fig. 11) describe arcs intersecting in b . Then A, b, A', b' , are points in the curves of the pattern. With b and b' as

FIG. 12.



centres and radii respectively $b c''$ and $b' c'$ (Fig. 11) describe arcs intersecting in c' . With c' and b as centres and radii respectively $C F$ and $b c$ (Fig. 11) describe arcs intersecting in c ; and with c and c' as centres and radii respectively $c d''$ and $c' d'$ (Fig. 11) describe arcs intersecting in d' . With d' and c as centres and radii respectively $D F$ and $c d$ (Fig. 11) describe arcs intersecting in d . Similarly find E' and E . Draw unbroken curved lines through $A b c d E$ and $A' b' c' d' E$

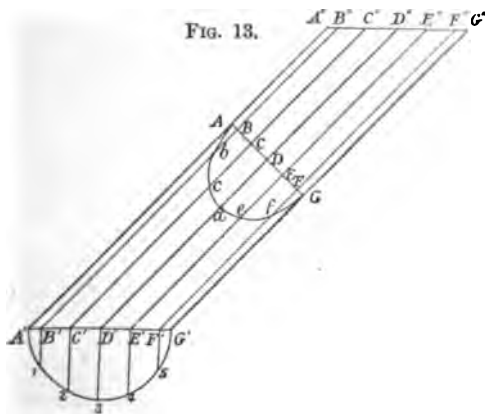
and join EE' ; that will give us half the pattern. By like procedure we find the other half of the pattern, that is to the left of AA' .

(65.) The lines $bb', cc', \&c.$, and the dotted lines $A b', b c', \&c.$, are drawn in Fig. 12 simply to show the position that the lines which correspond to them in Fig. 11 ($b b', A b', \&c.$) take upon the developed surface of the tapering body. It is evident that it is not a necessity to make distinct operations of the two halves of the pattern; for as the points $b', b, c', c, \&c.$, are successively found, the points on the left of AA' corresponding to them can be set off.

PROBLEM V.

To draw the pattern of an oblique cylinder (inclined circular pipe for example), the length and inclination of the axis and the diameter being given.

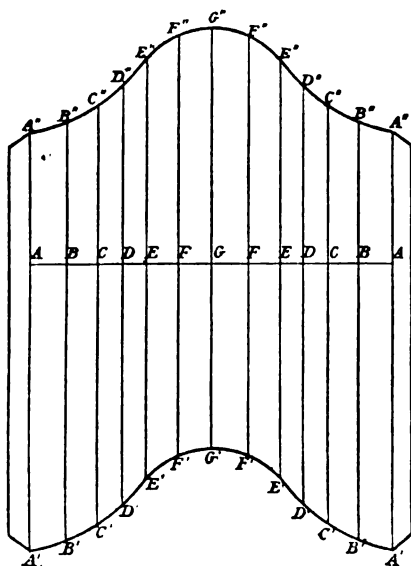
Draw (Fig. 13) any line $A'G'$, and at the point A' in it



make the angle $G'A'A''$ equal to the given inclination of the axis. Make $A'G'$ equal to the given diameter, and draw a line $G'G''$ parallel to $A'A''$. Make $A'A''$ and $G'G''$ each equal

to the length of the cylinder (the length of a cylinder is the length of its axis) and join $A''G''$. Then $A'A''G'G''$ is the elevation of the cylinder. Now on $A'G'$ describe a semicircle, and divide it into any number of equal parts, in the points 1, 2, 3, &c.; through each point draw lines perpendicular to $A'G'$, meeting it in points $B', C', D', \&c.$, and through $B', C', D', \&c.$, draw lines parallel to $A'A''$. Draw any line AG perpendicular to $A'A''$ and $G'G''$, cutting the lines $B'B'', C'C'', \&c.$, in points $B, C, \&c.$ Next make Bb equal to $B'1$, Cc equal to $C'2$, Dd equal to $D'3$, Ee equal to $E'4$, and Ff equal to $F'5$, and draw a curve from A through the points $b, c, d, \&c.$, to G . It is necessary to remark that this curve is not a semicircle, but a semi-ellipse (half an ellipse).

FIG. 14.



To draw the pattern (Fig. 14). Draw any line AA' , and at about its centre draw any line $G''G'$ perpendicular to

it and cutting it in G. From G, right and left of it, on the line A A mark distances G F, F E, E D, D C, C B, and B A equal respectively to the distances *Gf, fe, ed, dc, &c.* (Fig. 13). Through the points F, E, D, &c., right and left of G, draw lines parallel to G" G'. Make G G', G G" equal to G G', G G" (Fig. 13) respectively. Similarly make F F', F F", E E', E E", D D', D D", &c., right and left of G' G" equal respectively to F F', F F", E E', E E", D D', D D" &c. (Fig. 13). Draw an unbroken curved line from G" through F", E", D", &c., right and left of G" and an unbroken curved line through F', E', D', &c., right and left of G'. The figure A" G" A" A' G' A' will be the pattern required. The two parts G" A" A' G' of the pattern are alike in every respect.

CHAPTER II.

UNEQUAL-TAPERING BODIES, OF WHICH TOP AND BASE ARE
PARALLEL, AND THEIR PLANS.

(66.) BEFORE going into problems showing how to draw the patterns of unequal-tapering bodies with parallel ends, bodies which are (as the student will realise as he proceeds) partly or wholly portions of oblique cones, it will be necessary to enter into considerations in respect of the plans of frusta of such cones (see § 58, p. 111), similar to those appertaining to the plans of frusta of right cones treated of in Chap. V., Book I.; but to us of greater importance, because the constructions in problems for the setting out of patterns of bodies having unequal taper or inclination of slant are a little more difficult than those in problems for patterns of equal-tapering bodies. The chapter referred to may be now again read with advantage.

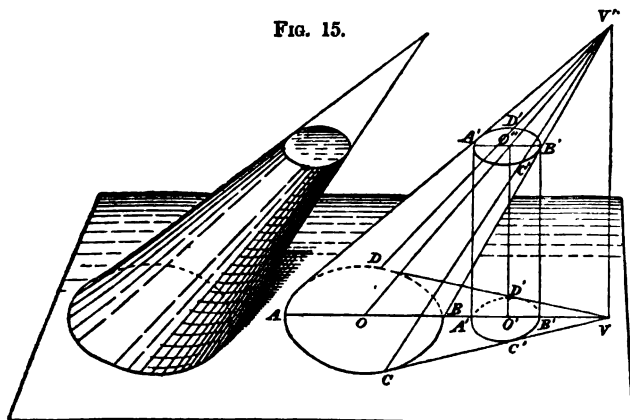
As much use will be hereafter made of the terms *Proportionate Arcs* and *Similar Arcs* we now define them. We also extend our explanation of *Corresponding Points*.

DEFINITIONS.

(67.) PROPORTIONAL ARCS: SIMILAR ARCS.—Arcs are *proportional* when they are equal portions of the circumferences of the circles of which they are respectively parts; they are *similar* when they are contained between the same generating lines. Similar arcs are necessarily proportional. In Fig. 15 the arcs A D and A' D' are proportional because each is a quarter of the circumference of the circle to which it belongs. They are similar because the generating lines V' A and V' D contain them both.

(68.) CORRESPONDING POINTS.—Points on the same generating line are *corresponding points* (compare § 24); thus,

the points **A** and **A'** are corresponding points, because they are on the same generating line $V'A$; also the points **C** and **C'** on the generating line $V'C$. The point **A'** on $V'A$ is the



plan of **A** on $V'A$; the point **C'** on $V'C$ is the plan of **C** on $V'C$; and so on.

(69.) From the figure, which shows a frustum of an oblique cone standing on a horizontal plane, it will be seen that the plan of a round unequal-tapering body (frustum of oblique cone) consists mainly of two circles $CADB$, $C'A'D'B'$, the plans of the ends of the body. In Fig. 16 is shown a complete plan of an oblique-cone frustum. With the connecting lines of the two sides we are not concerned, but may simply mention that they are *tangents* (lines which touch but do not cut) to the circles. Further from Fig. 15 it will be seen that, completing the cone of which the tapering body is a portion.

a. The plan of the axis (line joining the centres of the ends) of a frustum is a line joining the centres of the circles which are the plans of its ends; thus, the line OO' is the plan of the axis OO'' (see also OO' , Fig. 16).

Similarly, the plan of the axis of the complete cone, is the plan, produced, of the axis of its frustum; thus, OV is the plan of OV' .

b. The plan, produced both ways, of the axis of a frustum contains the plans of the lines of greatest and least inclination on the frustum (see § 52); that is to say, of the longest and shortest lines on it. Thus, OO' , produced both ways, contains the plans of AA' and BB' . It is convenient to regard lines joining corresponding points of a frustum (corresponding points of a frustum are points on one and the same generating line of the complete cone) as generating lines of the frustum. (See in connection with this, § 46). Then lines AA' , BB' , for instance, may be spoken of as generating lines of the frustum represented.

Similarly, the plans of the longest and shortest generating lines of a cone are contained in the plan, produced, of the axis of its frustum; thus, the plans of $V'A$ and $V'B$ are contained in OO' produced, both ways.

c. The line, produced, which joins the centres of the plans of the ends of a frustum, contains the plan of the apex of the cone; thus, OO' , produced, contains V the plan of V' .

d. The line, produced, which joins the plans of corresponding points of a frustum (see definition, § 68) contains the plan of the apex of the complete cone, and, produced only as far as the plan of the apex is the plan of a generating line of the cone; thus, C and C' being corresponding points on the cone, the plan, CC' , produced, of the joining C and C' , contains V ; and CV is the plan of the generating line CV' .

e. The plans produced of all generating lines of a frustum intersect the plan of its axis produced, in one point, and that point is the plan of the apex of the complete cone; for example, the plans produced, of the generating lines CC' and DD' of the frustum intersect OO' produced in V .

(70.) It follows from *e* that the plan of the apex of the complete cone of which a given frustum is a portion can easily be found if we have given the plans of the ends of the

frustum and the plans of two corresponding points not in the line passing through the centres of the plans of its ends. This is a valuable fact for us, as it spares us elevation drawing which in many cases is very troublesome, and indeed, sometimes practically impossible, as, for instance, where an unequal-tapering body is frustum of an exceedingly high cone the axis of which is but little out of the perpendicular. This is a case in which although the apex cannot be found in elevation because of the great length of the necessary lines, it can readily be found in plan, because, in plan, the requisite lines are short. An oblique cone may of course not only be exceedingly long, but also very greatly out of the perpendicular. In this case it is impracticable anyhow to find the plan of the apex. Problem IV., just solved, meets both cases. It was by *e* that we there found the plan of the apex when accessible, that is, where the lines of the plan are not unduly long (see Fig. 11) by joining the plans of corresponding points *c* and *c'* (*c* and *c'* are corresponding points in that they are mid-points on the half-plans of the ends of the frustum, and therefore necessarily on one and the same generating line), and producing *cc'* to intersect *OO'*, the line joining the centres of the plans of the ends, that is to intersect the plan produced of the axis of the frustum.

(71.) Passing the foregoing under review, it will be seen that if we have two circles which are the plans of the ends of a round unequal-tapering body (frustum of an oblique cone) standing on a horizontal plane, and the circles are in their proper relative positions as part plan of the frustum, then the line produced, one or both ways as may be necessary, which joins the centres of the circles, contains:—

The PLAN of the AXIS of the frustum (see *a*, p. 125).

The PLAN of the AXIS of the cone of which the frustum is a part (see *a*, p. 125).

The PLAN of the APEX of the cone (see *c*, p. 126).

The PLANS of the LONGEST and SHORTEST GENERATING lines of the frustum (see *b*, p. 126).

The PLANS of the LONGEST and SHORTEST GENERATING LINES of the cone, of which the frustum is a part (see *b*, p. 126).

The PLANS of the LINES of GREATEST and LEAST INCLINATION of the frustum (see *b*, p. 126).

The PLANS of the LINES of GREATEST and LEAST INCLINATION of the cone, of which the frustum is a part (see § 52, p. 107).

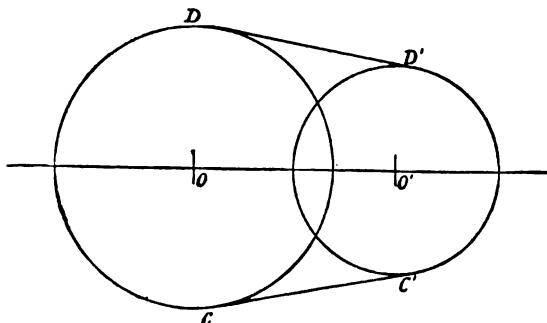
And this is a matter of very great practical importance, as will be seen later on.

(72.) As with circles under the conditions stated, so exactly with arcs which form the plans either of the ends or of portions of the ends, of an unequal-tapering body.

(73.) Further, referring to *c* and *d* of p. 55 as to round equal-tapering bodies we are now in a position to deduce (see Fig. 15) the following as to round unequal-tapering bodies.

f. The plan of a round unequal-tapering body with top and base parallel (frustum of oblique cone) consists essentially of two circles, not concentric, definitely situate relatively to one another. See Fig. 16.

FIG. 16.



Similarly the plan of a *portion* of such round unequal-tapering body (frustum of oblique cone) consists essentially of two arcs definitely situate relatively to one another, and

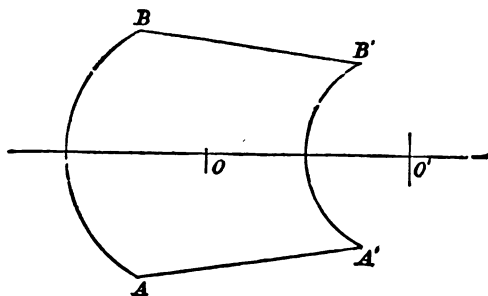
not concentric. See Fig. 17. OO' is the axis of the complete frustum.

g. Conversely.—If two circles, not having the same centre, definitely situate relatively to one another, form essentially the plan of a tapering body having parallel ends, that body is a round unequal-tapering body (frustum of oblique cone).

In Fig. 16, if the two circles represent essentially the plan of a tapering body having parallel ends, then the body, of which the circles are the essential plan, is a round unequal-body (frustum of oblique cone).

Similarly if two arcs definitely situate relatively to one another, and not having a common centre, form the essential part of the plan either of a tapering body or of a portion of

FIG. 17.



a tapering body having parallel ends, then that body or portion is a portion of a round unequal-tapering body (frustum of oblique cone).

In Fig. 17 if the arcs form the essential part of the plan, either of a tapering body or of a portion of a tapering body having parallel ends; then the body or portion of body, of which that fig. is the plan, is a portion of a round unequal-tapering body (frustum of oblique cone). In the particular plan represented the arcs are similar; the points B and B', and A and A' are therefore corresponding points.

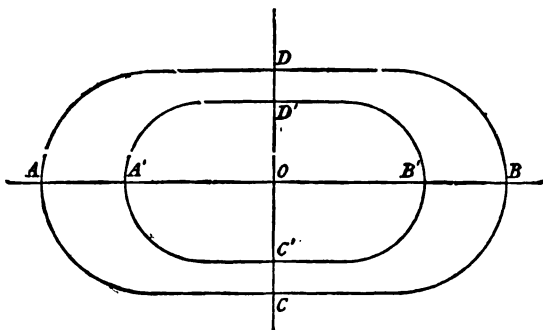
We will now proceed to draw the plans of some unequal-tapering bodies, of which patterns will be presently set out.

PROBLEM VI.

To draw the plan of an unequal-tapering body with top and base parallel and having straight sides and semicircular ends (an "equal-end" bath with semicircular ends), from given dimensions of top and bottom.

Draw $A'B'C'D'$ (Fig. 18) the plan of the bottom by Problem XVI, p. 20. Bisect $A'B'$ in O , and through O draw CD perpendicular to $A'B'$. Make OA and OB each

FIG. 18.



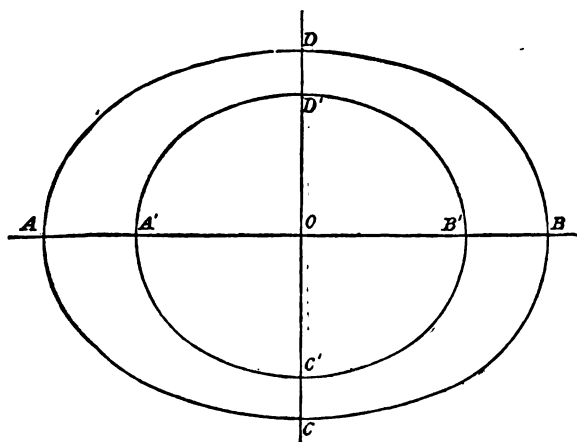
equal to half the length, and OC and OD each equal to half the width of the top. The plan of the top can now be drawn in the same manner as that of the bottom, completing the plan required.

PROBLEM VII.

To draw the plan of an oval unequal-tapering body with top and base parallel (an oval bath), from given dimensions of top and bottom.

Draw (Fig. 19) $A'B'C'D'$ the plan of the bottom, the given length and width, by Problem XII., p. 13; and make

FIG. 19.



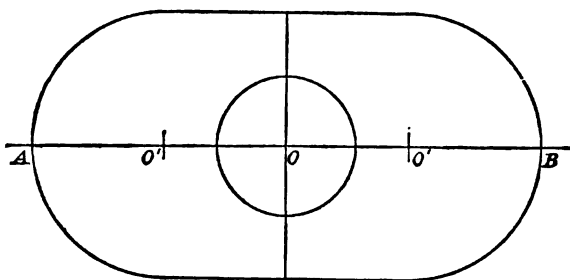
OA and OB each equal to half the length, and OC and OD each equal to half the width of the top. The plan of the top can now be drawn in the same manner as that of the bottom; this completes the plan required.

PROBLEM VIII.

To draw the plan of a tapering body with top and base parallel and having oblong bottom with semicircular ends and circular top (tea-bottle top), from given dimensions of top and bottom.

Draw (Fig. 20) the plan of the oblong bottom by Problem XVI., p. 20, and with *O* the intersection of the axes of the oblong as centre and half the diameter of the top as radius, describe a circle. This completes the plan.

FIG. 20.



We here, for the first time, extend the use of the word 'axes' (see Problems XII. and XIV., pp. 13 and 15). It is convenient to do so, and the meaning is obvious.

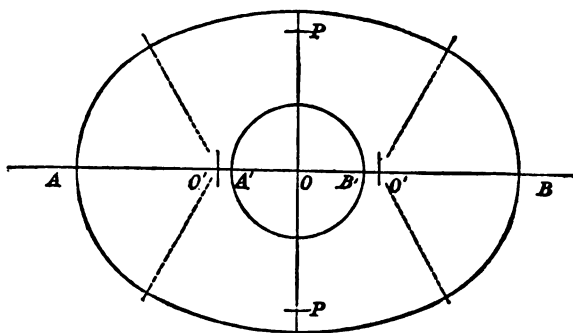
PROBLEM IX.

To draw the plan of a tapering body with top and base parallel, the top being circular and the bottom oval (oval canister-top), from given dimensions of top and bottom.

Draw (Fig. 21) the plan of the oval bottom by Problem XII., p. 13, and with *O* the intersection of the axes as centre,

and half the given diameter of the top as radius, describe a circle. This completes the plan.

FIG. 21.

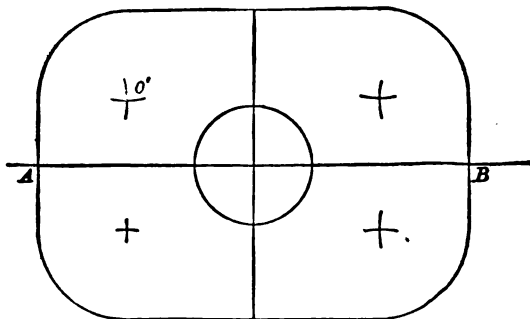


PROBLEM X.

To draw the plan of a tapering body with top and base parallel and having oblong base with round corners and circular top, from given dimensions of the top and bottom.

Draw (Fig. 22) the plan of the oblong bottom by Problem XV., p. 19, and with O the intersection of the axes of the

FIG. 22.



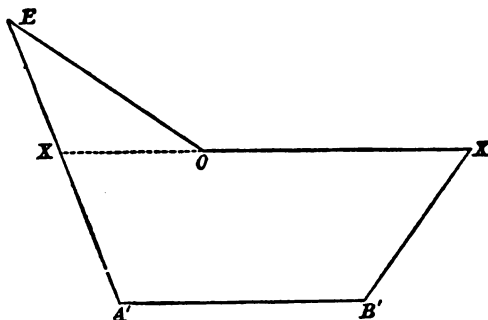
oblong bottom as centre and half the diameter of the top as radius, describe a circle. This completes the plan.

PROBLEM XI.

To draw the plan of an Oxford hip-bath.

Fig. 23 is a side elevation of the bath, drawn here only to make the problem clearer, not because it is necessary for the working. No method that involves the drawing of a full-size side elevation is practical, on account of the amount of space that would be required.

FIG. 23.



The bottom of an Oxford hip-bath is an egg-shaped oval. The portion O X' of the top is parallel to the bottom A' B', and the whole X X' top, the portion O X E of the bath being removed, is also an egg-shaped oval. In speaking of the plan of the 'bath,' we mean the plan of the X X' B' A' portion of it, as the plan of this portion is all that is necessary to enable us to get at the pattern of the bath.

We will first suppose the following dimensions given:—The length and width of the bottom, and the length of the X X' top, the height of the bath in front, and the inclination of the slant at back.

First draw (Fig. 25) the plan of the bottom A' D' B' C' by Problem XIII., p. 14. To draw the horizontal projection of

the XX' top, make (Fig. 24a) the angle $AA'E$ equal to the given inclination of the slant at the back. Through A' draw $A'H$ perpendicular to AA' , and equal to the height of the bath in front; through H draw HX parallel to AA' and cutting $A'E$ in X ; and draw XA perpendicular to XH ; then AA' will be the distance, in plan, at the back, between the curve of the bottom and the curve of the XX' top (Fig. 23). Make $A'A$ (Fig. 25) equal to AA' (Fig. 24a), and make AB equal to the length of the XX' top. With O as centre and OA as radius describe a semicircle; the remainder of the oval of the XX' top can now be drawn as was that of the bot'om. This completes, as stated above, all that is necessary of the plan of the bath to enable its pattern to be drawn.

FIG. 24a.

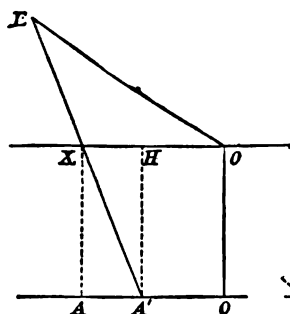
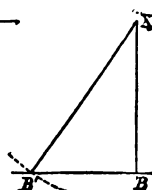


FIG. 24b.



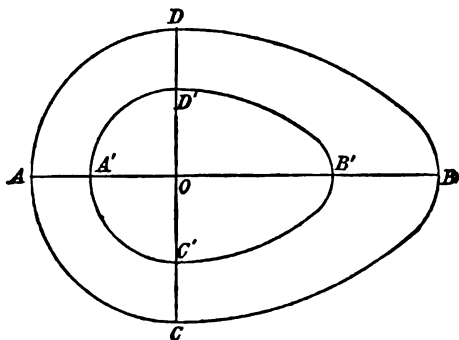
If the width of the XX' top is given, and not the inclination of the bath at back, make OA (Fig. 25) equal to half that width, and proceed as before. The seam in an Oxford hip-bath, at the sides, is on the lines of which CC' and DD' are the plans.

If the length of the XX' top (Fig. 23) is not given, it can be determined in the following manner:—

Let the angle $XB'B$ (Fig. 24b) represent the inclination of the slant of the front, and $B'X$ its length. Through X

draw XB perpendicular to $B'B$; then BB' will be the distance in plan, at the front, between the curve of the bottom and the curve of the XX' top (Fig. 23), and this distance, marked from B' to B (Fig. 25), together with the distance $A'A$ at the back end of the plan of the bottom, fixes the length required.

FIG. 25.



If the lengths only of the slants of the bath at back and front are given and not their inclinations, the plan of the XX' top can be drawn as follows:—

Draw two lines XB , $B'B$ (Fig. 24b) perpendicular to one another and meeting in B ; make BX equal to the given height of the bath in front, and with X as centre and radius equal to the given length of the slant at the front, describe an arc cutting $B'B$ in B' . Make $B'B$ (Fig. 25) equal to BB' (Fig. 24b), and BA equal to the given length of the XX' top; this will give the distance $A'A$. Now make AA' (Fig. 24a) equal to $A'A'$ (Fig. 25), draw AX and $A'H$ perpendicular to AA' and equal to the given height of the bath in front; join HX and draw $A'E$, through X , equal to the length of the slant at the back; the remainder of the plan of the XX' top can then be drawn as already described.

It will be useful to show here in this problem how to

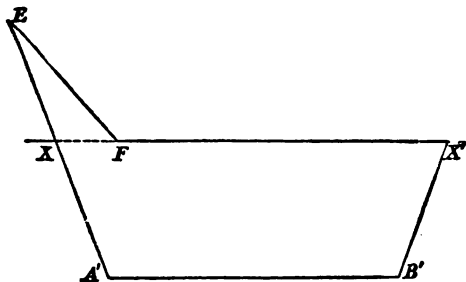
complete the back portion already commenced in Fig 24a of the side elevation of the bath. Make $A'E$ equal to the slant (§ 4, p. 24) at back, which must of course be given, and make XO equal to half the given width of the XX' top; join OE , and draw OO perpendicular to AA' produced; then $A'E O O$ is the elevation required.

PROBLEM XII.

To draw the plan of an Athenian hip-bath or of a sitz-bath.

Fig. 26 is a side elevation of the bath, drawn for the reason mentioned in the preceding problem.

FIG. 26.



The bottom of an Athenian hip-bath or a sitz-bath is an ordinary oval. The portion $X'F$ of the top is parallel to the bottom $A'B'$, and the whole XX' top, the portion FXE of the bath being removed, is also an ordinary oval. Similarly as with the bath of the last problem; we mean by plan of the 'bath,' the plan of the $XX'B'A'$ portion of it; no more being required for the drawing of the pattern of the bath.

We will first suppose the given dimensions to be those of

bottom and the curve of the XX' top. Make AA' (Fig. 27) and BB' each equal to AA' (Fig. 28a); then AB will be the length required.

FIG. 28a.

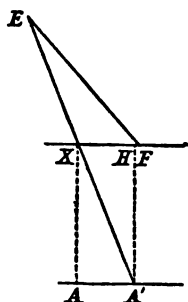


FIG. 28b.



If the length of the XX' top of the bath (Fig. 26) is not given, nor the inclination of the slant at front and back, but only the length of the slant at front, the required length can be thus ascertained.

Draw two lines XB , $B'B$ (Fig. 28b) perpendicular to one another and meeting in B ; make BX equal to the given height of the bath in front, and with X as centre, and radius equal to the length of the slant at the front, describe an arc cutting $B'B$ in B' . Make $A'A$ and $B'B$ (Fig. 27) each equal to $B'B'$ (Fig. 28b), then AB is the length wanted. The remainder of the plan can be drawn as described above.

By a little addition to Fig. 28a we get at the back portion of the side elevation of the bath. It will be useful to do this. Produce $A'X$ and make $A'E$ equal to the slant at back, which must of course be given. Then, on the plan (Fig. 27), E being the meeting point of the end and side curves of the oval $ADBC$, draw EF perpendicular to AB . Make XF (Fig. 28a) equal to AF (Fig. 27); join FE ; this completes the elevation required.

PROBLEM XIII.

To draw the plan of an oblong taper bath, the size of the top and bottom, the height, and the slant at the head being given.

To draw D E F C (Fig. 30) the plan of the top. Draw A B equal to the given length of the top, and through A and B draw lines perpendicular to A B. Make A E and A D each equal to half the width of the top at the head of the bath, and B F and B C each equal to half the width of the top at the toe; and join E F and D C. Next from E mark off along E F and E D equal distances, E G and E H, according to the size of the round corner required at the head. (It will be useful practice for the student to work this problem, commencing with the plan of the bottom, and its smaller corners, for the reason given in § 27a, p. 63). Through G and H draw

FIG. 29a.

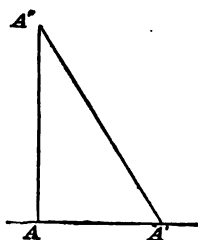


FIG. 29b.

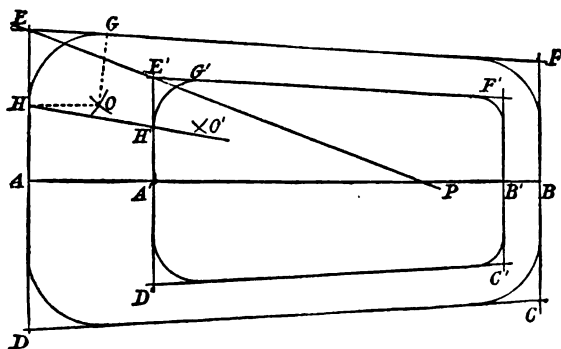


lines perpendicular to E F and E D respectively, intersecting in O; and with O as centre and O G as radius describe an arc H G to form the corner. The round corners at D F C, &c., are drawn in like manner.

To draw the plan of the bottom. Let the angle A''A'A (Fig. 29a) be the angle of the inclination of the slant at the head, and A'A'' the length of the slant. Through A'' draw

A" A perpendicular to A A', then A A' will be the distance between the lines, in plan, of the top and bottom at the head. Make A A' (Fig. 30) equal to A A' (Fig. 29a), and A' B' equal to the length of the bottom. Through A' and B' draw lines each perpendicular to A B; make A' E' and A' D' each equal to half the width of the bottom at the head, and B' F' and B' C' each equal to half the width of the bottom at the toe. Join E' F' and D' C'. The round corner of the bottom at the head must be drawn in proportion to the round corner of the top at the head, and this is done in the following manner. Join E E' and produce it, to meet

FIG. 30.



A B in P, and join H P by a line cutting D' E' in H'; make E' G' equal to E' H', and complete the corner from centre O' obtained as was the centre O. Draw the other corners in similar way, and this will complete the plan required. The D corner is like the E corner; the corners also at F and C correspond. Similarly with the E' and D', and F' and C' corners.

If the length of the bath is given and the length of slant at (but not its inclination) head or toe, the distance A A' can be

found by drawing two lines $A''A$, $A'A$ (Fig. 29a) perpendicular to one another and meeting in A , and making AA'' equal to the given height; then, with A'' as centre and $A''A'$, the given length of the slant at the head, as radius, describe an arc cutting AA' in A' . Then AA' is the distance required. Similarly (Fig. 29b) the distance BB' can be found.

CHAPTER III.

PATTERNS FOR ARTICLES OF UNEQUAL TAPER OR INCLINATION OF SLANT, AND HAVING FLAT (PLANE) SURFACES.

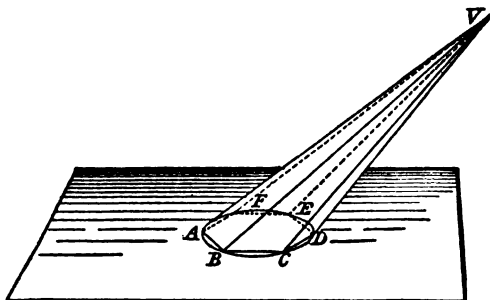
(CLASS II. *Subdivision b.*)

ARTICLES of unequal taper or inclination of slant, and having plane or flat surfaces (hoppers, hoods, &c.), are frequently portions (*frusta*) of oblique pyramids, or parts of such *frusta*.

DEFINITIONS.

(74.) Oblique Pyramid: Frustum of Oblique Pyramid:—Oblique pyramids have not yet been defined. For our purpose it will be sufficient to define an oblique pyramid negatively, that is, as a pyramid which is not a right pyramid; and when cut by a plane parallel to its base (that

FIG. 31.

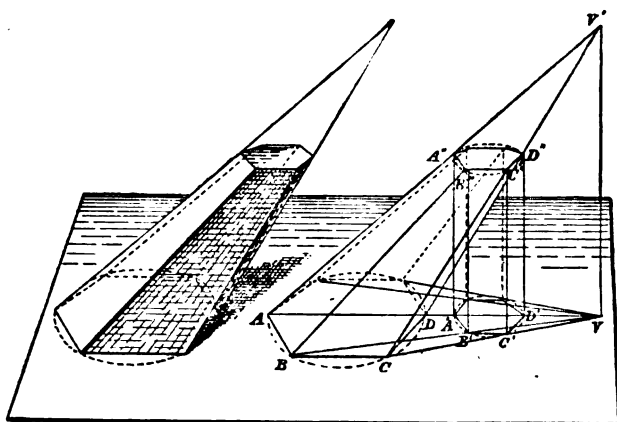


is, when truncated), to define its frustum (§ 33, p. 69) as the frustum of a pyramid which is not a right pyramid. In the oblique pyramid the faces are not all equally inclined. Articles of which the faces are not all equally inclined are

not necessarily portions of oblique pyramids. One such case will be given later on. The problems immediately following deal with articles of which the faces are not all equally inclined, but which are portions of oblique pyramids.

(75.) Further, an oblique pyramid, when it has a base through the angular points of which a circle can be drawn, can be inscribed in an oblique cone like as a right pyramid in a right cone, and this property gives constructions for solving most of our oblique-pyramid problems, somewhat similar to those in Book I., Chapter VI., where the right pyramid is concerned. Fig. 31 represents an oblique hexagonal pyramid inscribed in an oblique cone. This fig. should be compared with Fig. 31, p. 67. The edges of the oblique pyramid are generating lines of the cone.

FIG. 32.



(76.) Also from Fig. 32 it will be seen that the plan of a frustum of an oblique hexagonal pyramid standing on a horizontal plane consists of two hexagons $ABCD$ and $A'B'C'D'$ (the plans of the ends), whose similarly situated sides, AB and

$A'B'$, BC and $B'C'$, CD and $C'D'$ for instance, are parallel, and whose corresponding points (§ 68, p. 124) A , A' and B , B' , for instance, are joined by lines AA' , BB' , which are the plans of the edges of the frustum. Just as in the case of the frustum of the oblique cone (see d and e , p. 126), if a line joining corresponding points in plan be produced, it will contain the plan of the apex of the complete pyramid of which the frustum is a portion; and if another such line be produced to intersect the first line, the point of intersection will be that plan of apex. For example, the lines AA' , BB' and CC' produced meet in a point which is the plan of the apex of the pyramid of which the frustum $ABDD''B''A''$ is a portion.

(77.) From this it follows that if the plan of a tapering body with top and base parallel and having plane or flat surfaces be given, we can at once determine whether the tapering body is or not a frustum of an oblique pyramid by producing the plans of the edges. If these meet in one point, then the given plan is that of a frustum of an oblique pyramid.

PROBLEM XIV.

To draw the pattern of an oblique pyramid.

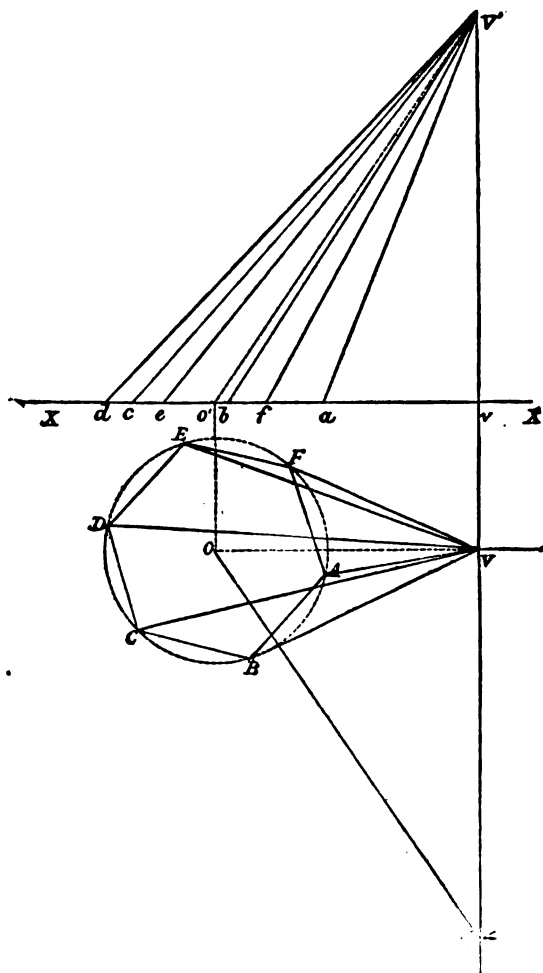
CASE 1.—Given the plan of the pyramid and its height.

Let $ABCDEFV$ (Fig. 33) be the plan of the pyramid (here a hexagonal pyramid), V being the apex, and OV the plan of the axis. Draw XX parallel to OV , and through V draw VV' perpendicular to XX , and cutting it in v ; make vV' equal to the given height of the pyramid. Next make va , vb , vc , vd , ve , and vf equal respectively to VA , VB , VC , VD , VE , and VF , the plans of the edges of the pyramid. Joining $V'a$, $V'b$, $V'c$, &c., will give the true lengths of these edges.

To draw the pattern of the pyramid with the seam at the edge VA . Draw VA (Fig. 34) equal to $V'a$ (Fig. 33); with

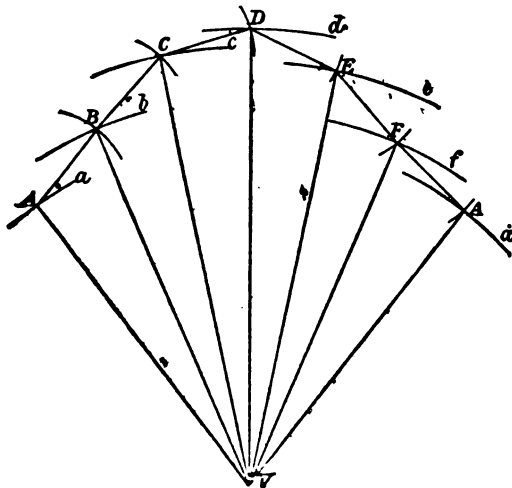
V as centre and $V'b$ (Fig. 33) as radius describe arc b , and with A as centre and AB (Fig. 33) as radius describe an arc intersecting the arc b in B. The other points C, D, E, F, A, are

FIG. 33.



found in similar manner. Thus, with V (Fig. 34) as centre and V'c, V'd, V'e, V'f, and V'a (Fig. 33) successively as radii, describe arcs *c*, *d*, *e*, *f*, and *a* (Fig. 34). Next, with B (Fig. 34) as centre and BC (Fig. 33) as radius, describe an arc intersecting arc *c* in C; with CD (Fig. 33) as radius and C (Fig. 34) as centre describe an arc cutting arc *d* in D; with D (Fig. 34) as centre and DE (Fig. 33) as radius describe an arc intersecting arc *e* in E; and so on for points F and A. Join AB, BC, CD, DE, EF, FA and AV, and this will complete the pattern required.

FIG. 34.



Joining the points B, C, D, &c., to V, it will be seen that the pattern is made up of a number of triangles, each triangle being of the shape of a face of the pyramid, also that the construction of the pattern is very similar to the construction of that of an oblique cone.

Should it be inconvenient to draw XX in the position shown in Fig. 33, the true lengths of the edges of the pyramid

can be found in the following manner. Draw XX quite apart from the plan of the pyramid, and from any point v in it draw vV' perpendicular to XX , and equal to the height of the pyramid, and proceed as just described.

CASE II.—Given the plan of the pyramid and the length of its axis.

Draw XX (Fig. 33) parallel to OV , the plan of the axis; through V draw VV' perpendicular to XX , and through O draw OO' perpendicular to XX . With O' as centre and the given length of the axis as radius describe an arc cutting VV' in V' ; then vV' will be the height of the pyramid, and we now proceed as in Case I.

Or, draw Vx perpendicular to OV , and with O as centre and radius equal to the length of the axis describe an arc cutting Vx in x ; Vx will be the height of the pyramid.

PROBLEM XV.

To draw the pattern of a frustum of an oblique pyramid.

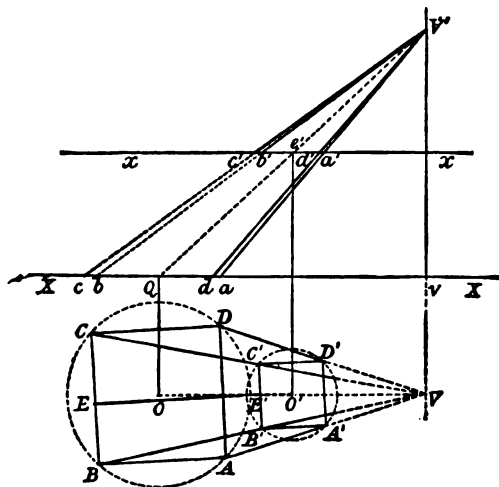
CASE I.—Given the plan of the frustum and its height.

Let $ABCD D' A' B' C'$ (Fig. 35) be the plan of the frustum (here of a square pyramid). Produce AA' , BB' , &c., the plans of the edges to meet in a point V ; this point is the plan of the apex of the pyramid of which the frustum is a part. Join O , the centre of the square which is the plan of the large end of the frustum, to V . The line OV will pass through o' , the centre of the plan of the small end; OO' will be the plan of the axis of the frustum, and OV the plan of the axis of the pyramid of which the frustum is a portion.

Draw XX parallel to OV ; through V draw VV' perpendicular to XX , and cutting it in v . Make vx equal to the given height of the frustum, and through x draw xx parallel to XX ; through O draw OQ perpendicular to XX and

meeting it in Q and through O' draw $O'Q'$ perpendicular to XX and cutting xx in e' . Join Qe' and produce it to intersect vV' in V' . Next make va, vb, vc, vd equal to VA, VB, VC, VD respectively; join a, b, c , and d to V' by lines cutting xx in points a', b', c' , and d' ; $aa', bb', &c.$, are the lengths of the edges of the frustum.

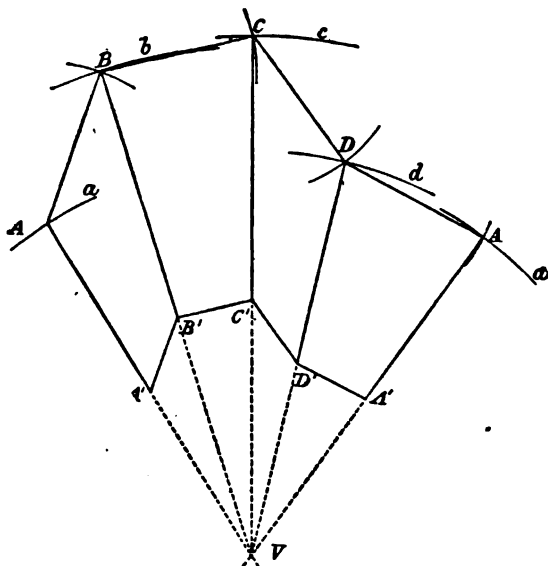
Fig. 35.



To draw the pattern with the seam at AA' . Draw VA (Fig. 36) equal to $V'a$ (Fig. 35); with V as centre and $V'b$ (Fig. 35) as radius describe an arc b , and with A as centre and AB (Fig. 35) as radius describe an arc intersecting arc b in B ; with $V'c$ (Fig. 35) as radius and V as centre describe arc c , and with BC (Fig. 35) as radius and B as centre describe an arc intersecting the arc c in C . Next with $V'd$ and $V'a$ (Fig. 35) as radii and V as centre describe arcs d and a ; with C as centre and radius CD (Fig. 35) describe an arc intersecting arc d in D ; and with DA (Fig. 35) as radius and D as centre describe an arc intersecting the arc a in A . Join A, B, C, D ,

and A to V ; make AA' , BB' , CC' , DD' respectively equal to aa' , bb' , cc' , dd' (Fig. 35), and join AB , BC , CD , DA , $A'B'$, $B'C'$, $C'D'$, &c. Then $ABCDAA'D'C'BA'$ is the pattern required.

FIG. 36.



(78.) The dotted circles (Fig. 35) through the angular points of the plans of the ends show the plans of the ends of the frustum of the oblique cone which would envelop the frustum of the pyramid. From the similarity of the construction above to that for the pattern of a frustum of an oblique cone, it will be evident that we have treated the edges of the frustum as generating lines (see *b*, p. 126) of the frustum of the oblique cone in which the frustum of the pyramid could be inscribed.

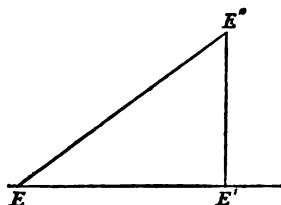
Should it be inconvenient to draw XX in conjunction with the plan of the pyramid draw XX quite apart, and from any

point v in it draw vV' perpendicular to XX ; make vx equal to the height of the frustum and draw xx parallel to XX . Make va, vb, vc, vd equal to VA, VB, VC, VD (Fig. 35) respectively; and make xa', xb', xc', xd' equal to VA', VB', VC', VD' (Fig. 35) respectively. Join $aa', bb', cc',$ and dd' by lines produced to meet vV' in V' , and proceed as stated above.

CASE II.—Given the dimensions of the two ends of the frustum, the slant of one face and its inclination (the slant of the face of a frustum of a pyramid is a line meeting its end lines and perpendicular to them).

Draw (Fig. 37) a line EE'' equal to the given slant, make the angle $E''E'E'$ equal to the given inclination, and let fall $E''E'$ perpendicular to EE' . Draw $ABCD$ (Fig. 35), the plan of the large end of the frustum, and let BC be the plan of the bottom edge of the face whose slant is given. Bisect BC in E and draw EE' perpendicular to BC and equal to

FIG. 37.



EE'' (Fig. 37). Through E' draw $B'C'$ parallel to BC ; make $E'C'$ and $E'B'$ each equal to half the length of the top edge of the BC face, through C' and B' draw $C'D'$ and $B'A'$ parallel to CD and BA ; make $C'D'$ and $B'A'$ each equal to $B'C'$; join $D'A'$, also $AA', BB', CC',$ and DD' ; this will complete the plan of the frustum. EE'' (Fig. 37) is the height of the frustum. The remainder of the construction is now the same as that of Case I.

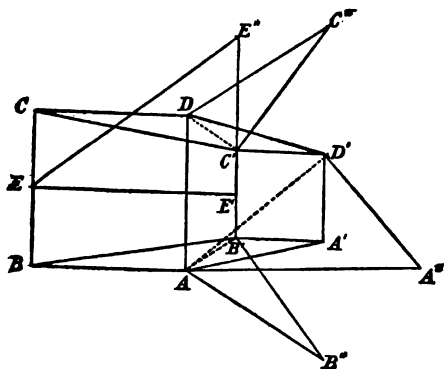
For large work and where the ends of a frustum are of nearly the same size, it would be inconvenient to use long radii. For unequal-tapering bodies which are not portions of oblique pyramids, as in Problem XVII., the method now given, or a modification of it, must be used.

PROBLEM XVI.

To draw, without long radii, the pattern for a frustum of an oblique pyramid. The plan of the frustum and its height being given.

Let $ABCD D' A' B' C'$ (Fig. 38) be the plan of the frustum. From any point E in BC draw EE' perpendicular to BC and $B' C'$, the plans of the bottom and top edges of the face $BC B' C'$ of the frustum. Draw $E' E''$ perpendicular to EE' and

FIG. 38.

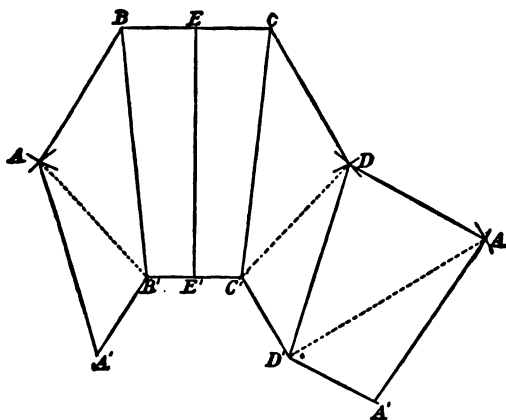


equal to the height (which either is given or can be found as in Case II. of last problem), and join EE'' , then EE'' is the true length of a slant of the face $BC B' C'$, of the frustum. Join DC' , and find its true length (DC'') by drawing $C' C''$

perpendicular to $D C'$ and equal to the height of frustum and joining $D C''$. Next join $D' A$ and $B' A$; through D' and B' draw lines $D' A''$, $B' B''$ perpendicular to $D' A$ and $B' A$ respectively, and make $D' A''$ and $B' B''$ each equal to the given height of the frustum; join $A A''$ and $A B''$, then $A A''$ and $A B''$ are the true lengths of $D' A$ and $B' A$ respectively.

To draw the pattern of the face $B C B' C'$, draw $E E'$ (Fig. 39) equal to $E E''$ (Fig. 38), and through E and E' draw $B C$ and $B' C'$ perpendicular to $E E'$. Make $E C$, $E B$, $E' C'$, and $E' B'$ equal to $E C$, $E B$, $E' C'$, and $E' B'$ (Fig. 38) respectively; join $C C'$ and $B B'$; this completes the pattern of the face. The patterns of the other faces are found in the following manner:—With C' (Fig. 39) and C as centres and

FIG. 39.



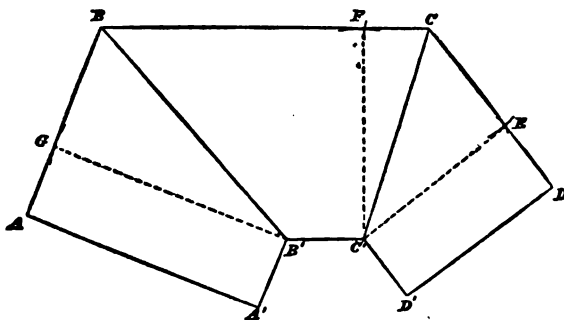
$D C''$ and CD (Fig. 38) as radii respectively, describe arcs intersecting in D ; join CD , draw $C' D'$ parallel to CD and equal to $C' D'$ (Fig. 38); and join $D D'$. With D' and D (Fig. 39) as centres and $A A''$ and $D A$ (Fig. 38) as radii respectively describe arcs intersecting in A ; join $D A$, draw $D' A'$ parallel to $D A$ and equal to $D' A'$ (Fig. 38), and join A

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equal to the given length, describe an arc cutting $C'C''$ in C'' . Join $F'C''$; then $C'C''$ is the height of the hood, which we need. If the height of the hood is given instead of the length $F'C''$, make $C'C''$ equal to the height and join $F'C''$, which will be the true length of $F'C'$. Next, through C' draw $C'E$ perpendicular to CD ; draw $C'C''$ perpendicular to $C'E$, make $C'C''$ equal to the height and join $E C''$. Now produce $C'B'$ to meet AB in G ; draw $B'B''$ perpendicular to $B'G$ and equal to the height, and join $G B''$.

To draw the pattern of the hood. Draw $F C'$ (Fig. 41) equal to $F C''$ (Fig. 40); through F and C' draw $B C$ and $B' C'$, each perpendicular to $F C'$; make $F B$ equal to $F B$ (Fig. 40); make $F C$ equal to $F C$ (Fig. 40), and $C' B'$ equal to $C' B'$ (Fig. 40). Join $B B'$ and $C C'$, then $B B' C' C$ will be the pattern of the face of which $B B' C' C$ (Fig. 40) is the plan. To draw the pattern of the face $C' D' D C$ (Fig. 40). With C' and C (Fig. 41) as centres and $E C''$ and $C E$

FIG. 41.



(Fig. 40) as radii respectively, describe arcs intersecting in E. Join CE and produce it, making CD equal to CE (Fig. 40), and through C' draw C'D' parallel to CE and equal to CE (Fig. 40). Join DD', then C'CEDD' is the pattern of the face of which C'CEDD' (Fig. 40) is the plan. With B' and B as

centres and radii respectively equal to $B''G$ and BG (Fig. 40), describe arcs intersecting in G . Join BG and produce it, making BA equal to BA (Fig. 40), and through B' draw $B'A'$ parallel to BA and equal to $B'A'$ (Fig. 40). Join AA' ; and the pattern for the hood is complete.

CHAPTER IV.

PATTERNS FOR UNEQUAL-TAPERING ARTICLES OF FLAT AND CURVED SURFACE COMBINED.

CLASS II. (*Subdivision c.*)

FROM what has been stated about the plans of unequal-tapering bodies and from *g*, p. 129, it will be evident that the curved surfaces of the articles now to be dealt with are portions of frusta of oblique cones.

(79.) The advantages referred to in § 61 of looking upon the oblique cylinder as frustum of an oblique cone will be evident in this chapter. For there is to each of the problems a Case where the plan arcs of the curved portions of the body treated of have equal radii. To deal with these as problems exceptional to a general principle would be most inconvenient. As extreme cases, however, of the one principle that the curved portions of the bodies before us are portions of frusta of oblique cones, their solution presents no difficulty. It will be sufficient to take one such Case in connection with only one of the bodies. This we shall do in Case IV. of the next problem.

PROBLEM XVIII.

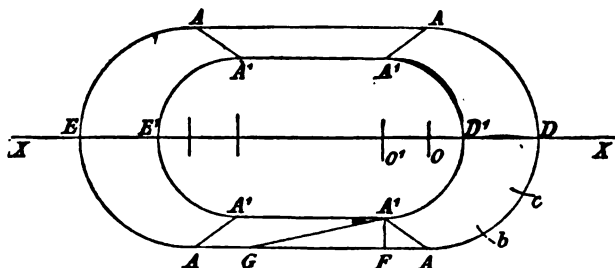
To draw the pattern for an unequal-tapering body with top and base parallel and having flat sides and semicircular ends (an 'equal-end' bath, for instance), the dimensions of top and bottom of the body and its height being given.

Five cases will be treated of; four in this problem and one in the problem following.

CASE I.—Patterns when the body is to be made up of four pieces.

Draw (Fig. 42) the plan of the body (see Problem VI., p. 180), preserving of its construction the centres O, O' and the points A, A' in which the plan lines of the sides and curves

FIG. 42.



of the ends meet each other. Join $A A'$, as shown (four places) in the fig. The ends $A D A' D' A'$ and $A E A' E' A'$ of the body (see *g*, p. 129) are portions of frusta of oblique cones. Let us suppose that the seams are to be at the four A corners where they are usually placed, and to correspond with the four lines $A A'$. Then we shall require one pattern for the flat sides, and another for the semicircular ends.

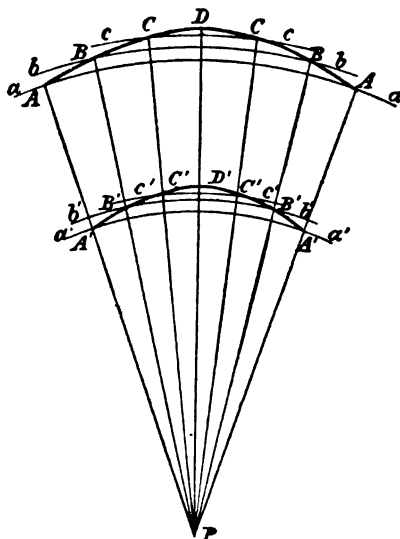
To draw the end pattern.

(80.) Draw $A b D D' A'$ (Fig. 43) the $A b D D' A'$ portion of Fig. 42 separately, thus. Draw any line $X X$ and with any point O (to correspond with O , Fig. 42) in it as centre and $O D$ (Fig. 42) as radius describe an arc (here a quadrant) $D b A$ equal to the arc $D b A$ (Fig. 42). Make $D O'$ equal to $D O'$ (Fig. 42), and with O' as centre and $O' D'$ (Fig. 42) as radius describe an arc (here a quadrant) $D' A'$ equal to the arc $D' A'$ (Fig. 44). Joining $A A'$ completes the portion of Fig. 42 required. Now divide $D A$ into any number of equal parts, here three, in the points b and c . From D' draw $D' D''$ perpendicular to $X X$ and equal to the given height. Then D, D'' are, in elevation, the corresponding points of which

cessively as radii, describe arcs cutting XX in C , B , and A'' . Join these points to P by lines cutting $O''D''$ in C' , B' , and A' .

Next draw a line PD (Fig. 44) equal to PD (Fig. 43), and with P as centre and PC , PB , and PA'' (Fig. 43) successively as radii describe arcs cc , bb , and aa . With D as centre and radius equal Dc (Fig. 43) describe arcs cutting arc cc in C and C right and left of PD . With same radius and these points C and C successively as centres describe arcs cutting arc bb in B and B right and left of PD . With B and B

FIG. 44.



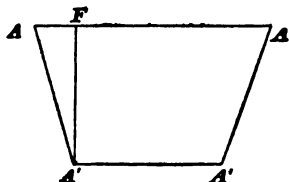
successively as centres and same radius describe arcs cutting arc aa in A and A right and left of PD . Join the points C , B , and A right and left of PD to P . With P as centre and PD'' (Fig. 43) as radius describe an arc cutting PD in D'' . With same centre, and PC' (Fig. 43) as radius, describe arc $c'c'$ cutting lines PC right and left of PD in C' and C' . With same centre, and PB' (Fig. 43) as radius, describe arc

$b'b'$ cutting lines PB right and left of PD in B' and B' . Similarly find points A' and A' . Through the successive points A, B, C, D, C, B, A , draw an unbroken curved line. Also through the successive points $A', B', C', D', C', B', A$, draw an unbroken curved line. Then $ADAA'D'A'$ will be the required pattern for ends of the body.

To draw the pattern for the sides.

Through A' (Fig. 42) draw $A'F$ perpendicular to $A'A$ make FG equal to the given height and join $A'G$. Then $A'G$ is the slant of the body at the side. Next draw (Fig. 45) AA equal to AA (Fig. 42), and make AF equal

FIG. 45.



to AF (Fig. 42); through F draw FA' perpendicular to AA and equal to $A'G$ (Fig. 42), and through A' draw $A'A'$ parallel to AA . Make $A'A'$ equal to $A'A'$ (Fig. 42). Join AA' , AA' , then $AA'A'A$ is the pattern for the sides.

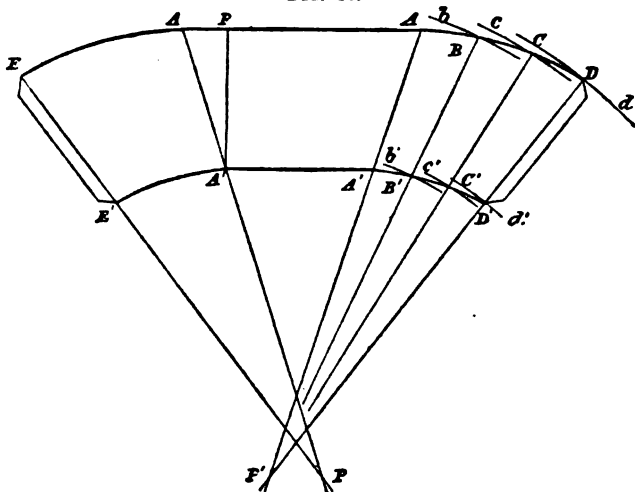
CASE II.—Pattern when the body is to be made up of two pieces.

We will take it that the seams are to be at DD' and EE' (Fig. 42). It is evident that we want but one pattern, which shall include a side of the body and two half-ends.

First draw as just explained $A'AF A'A'$ (Fig. 46) a side-pattern of the body. Produce one of the lines AA' of this pattern, and make AP' equal to $A''P$ (Fig. 43). With P' as centre and PB, PC , and PD (Fig. 43) successively as radii describe arcs b, c , and d , and with A as centre and Ab (Fig. 43) as radius describe an arc cutting arc b in B . With same radius and B as centre describe an arc cutting arc c in C ; similarly with C as centre and same radius find D .

Join BP' , CP' , DP' . Now with PB' (Fig. 43) as radius and P' as centre describe an arc b' cutting PB in B' , and with PC' , PD' (Fig. 43) successively as radii describe arcs c' and d' cutting $P'C$ and $P'D$ in C' and D' . Through the points

FIG. 46.



A , B , C , and D draw an unbroken curved line. Also through the points A' , B' , C' , and D' draw an unbroken curved line. Then $ADD'A'$ will be a half-end pattern attached to the right of the side pattern. Draw the other half-end pattern $AE'E'A'$ in the same manner; then $EFDD'A'A'E'$ will be the complete pattern required.

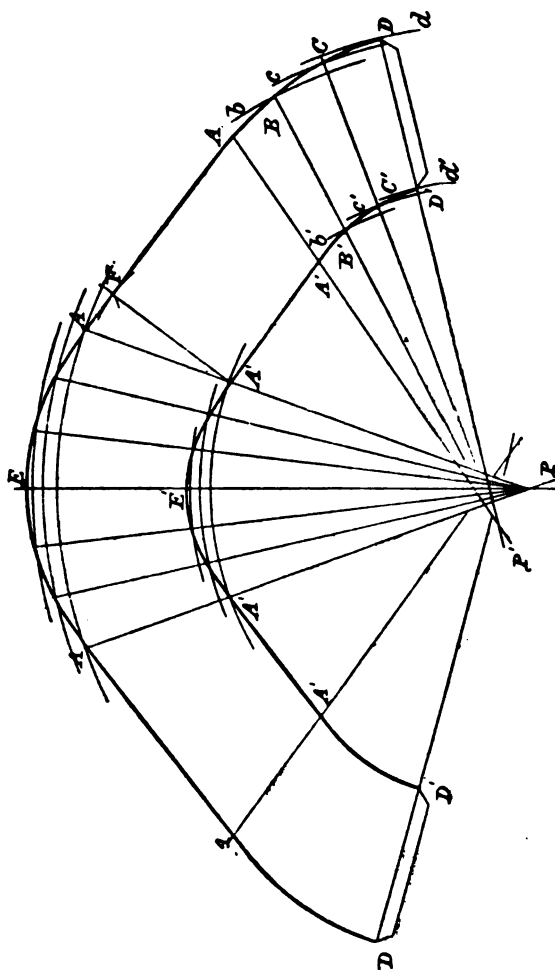
CASE III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with DD' (Fig. 42).

First draw $AEAA'E'A'$ (Fig. 47) an end pattern of the body in the same manner that $ADAA'D'A'$ (Fig. 44) was drawn. With A' and A (right of EE') as centres and $A'G$ and AF (the small length AF) (Fig. 42) respectively as

radii describe arcs intersecting in F; join A F and produce it, making A A equal to A A (Fig. 42). Through A' (extremity of F A') draw A' A' parallel to A A and equal to

FIG. 47

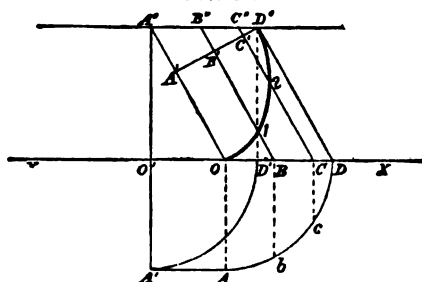


$A' A'$ (Fig. 42). Join $A A'$, the extremities of the lines just drawn, and produce it indefinitely; and make $A P'$ equal $A'' P$ (Fig. 43). With P' as centre and $P B$, $P C$, and $P D$ (Fig. 43) successively as radii describe arcs b , c , and d , and with A (of $A P'$) as centre and $A b$ (Fig. 43) as radius describe an arc cutting arc b in B . With same radius and B as centre describe an arc cutting arc c in C , and similarly with C as centre and same radius find D . Join $B P'$, $C P'$, and $D P'$. Now with P' as centre and radii successively equal to $P B'$, $P C'$, and $P D''$ (Fig. 43) describe arcs b' , c' , and d' cutting $P' B$, $P' C$, and $P' D$ in B' , C' , and D' . Through the points A , B , C , and D draw an unbroken curved line. Also through the points A' , B' , C' , and D' draw an unbroken curved line. We have now in $D F A A' A' D'$ attached to the right of the end pattern we started with, a side pattern and a half-end pattern. By a repetition of the foregoing construction we can attach $A A D D' A' A'$ to the left of the end pattern we started with. The figure $D E D D' E' D'$ will be the complete pattern required.

CASE IV.—Where the plan arcs $D A$, $D' A'$ (Fig. 42) have equal radii.

This is the extreme case above (§ 79, p. 157) referred to, where the cone becomes cylindrical. Problem V., p. 121, may

FIG. 48.



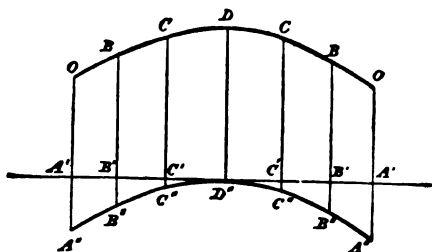
advantageously be compared with the work now given. The arcs (Fig. 48) $D A$ and $D' A'$ (here quadrants) being equal,

their radii OD , $O'D'$ are also equal. Through D' and O' draw $D'D''$, and $O'A''$ perpendicular to XX and each equal to the given height of the body. Join $D'D''$, $A''D''$; divide the arc DA into any number of equal parts, here three, in the points b and c ; and through c , b , and A draw cC , bB , and AO each perpendicular to XX and cutting it in C , B , and O respectively. The arc DA being here a quadrant the point where the line from A perpendicular to XX cuts XX is necessarily O , the centre whence the arc is drawn. Through C , B , and O draw $C'C''$, $B'B''$, and OA'' parallel to $D'D''$ and cutting $A''D''$ in points C'' , B'' , and A'' . Also through D' draw a line $D''A'$ perpendicular to $D'D''$ and cutting the lines just drawn in C' , B' and A' . Make $c2$ equal to Cc ; $B'1$ equal to Bb , and $A'O$ equal to OA . From D'' through 2 , 1 , to O draw an unbroken curved line.

To draw the pattern.

Draw $D'D''$ (Fig. 49) equal to $D'D'$ (Fig. 48) and through D' draw an indefinite line $A'A'$ perpendicular to DD'' .

FIG. 49.



Mark off on $A'A'$, right and left of D'' , $D''C'$, $C'B'$, and BA' respectively equal to the distances between D'' and 2 , 2 and 1 , and 1 and 0 (Fig. 48); and through C' , B' , and A' , right and left of D'' draw indefinite lines each parallel to $D'D''$. Make $C'C$ right and left of $D'D''$ equal to $C'C$ (Fig. 48); and make $B'B$ right and left of $D'D''$ equal to $B'B$ (Fig. 48). Also make $A'O$ right and left of $D'D''$ equal to $A'O$ (Fig. 48).

Next make $C'C''$ right and left of DD'' equal to $C'C'$ (Fig. 48). Similarly find points B'', A'' right and left of DD'' by making $B'B'', A'A''$ respectively equal to $B'B'$, and $A'A'$ (Fig. 48). Through the points O, B, C, D, C, B, O , draw an unbroken curved line. Also through the points $A'', B'', C'', D'', C'', B'', A''$, draw an unbroken curved line. Then $ODOA''D''A''$ will be the pattern required.

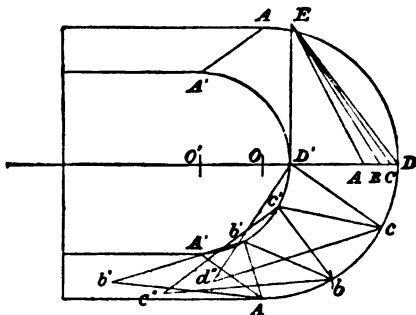
PROBLEM XIX.

To draw, without long radii, the pattern for an unequal-tapering body with top and base parallel and having flat sides and equal semicircular ends (an 'equal-end' bath, for instance). The dimensions of the top and bottom of the body and its height being given.

This problem is a fifth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern.

FIG. 50.

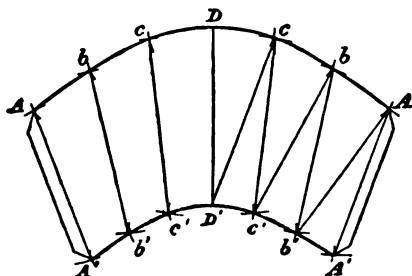


First draw half the plan (Fig. 50). It is evident that the drawing of the side pattern presents no difficulty, as long

radii are not involved. It can be drawn as in Case I. of preceding problem. Divide the quadrants DA , $D'A'$, each into the same number of equal parts, here three, in the points c, b, c', b' ; join cc', bb' . Through D' draw $D'E$ perpendicular to $D'D$ and equal to the given height of the body. From D' along $D'D$ mark off $D'A, D'B$, and $D'C$ respectively equal to $A'A, b'b$, and $c'c$; and join points D, C, B , and A to E , then EA, EB, EC , and ED , will be the true lengths of $A'A, b'b, c'c$, and $D'D$ respectively. Next join cD' , and draw $D'd''$ perpendicular to $D'c$ and equal to the given height. Join cd'' , then cd'' may be taken as the true length of $D'c$. Similarly join $b'c'$ and Ab' , and through c' and b' draw $c'c''$ and $b'b''$ perpendicular to $c'b$ and $b'A$ respectively, and each equal to the given height. Join $b'c''$ and Ab'' , then $b'c''$ and Ab'' may be taken as the true lengths of $b'c'$ and Ab' respectively.

Now draw (Fig. 51) a line DD' equal to DE (Fig. 50) and with D' and D as centres and radii respectively equal to $d''c$

FIG. 51.



and D c (Fig. 50) describe arcs right and left of DD', intersecting in c and c. With c, right of D, and D' as centres, and radii respectively equal to CE and D' c' (Fig. 50) describe arcs intersecting in c', right of D'. With c, left of D, and D' as centres, and radii as before, describe arcs intersecting in c', left of D'. With successively c' and c right and left of DD',

as centres and radii respectively equal to $c''b$ and cb (Fig. 50) describe arcs intersecting in b and b right and left of DD' . With successively b and c' right and left of DD' as centres and radii respectively equal to BE and $c'b'$ (Fig. 50) describe arcs intersecting in b' , and b' right and left of DD' , and with successively b' and b right and left of DD' as centres and radii respectively equal to Ab'' and bA (Fig. 50) describe arcs intersecting in A and A right and left of DD' . Similarly with AE and $b'A'$ (Fig. 50) as radii and centres respectively A and b' describe intersecting arcs to find points A' and A' , right and left of DD' . Through the points A, b, c, D, c, b, A draw an unbroken curved line. Also through the points $A', b', c, D', c', b', A'$, draw an unbroken curved line. Join AA', AA' , right and left of DD' , then $ADA'A'D'A'$ will be the pattern required.

The lines $cc', cD', bb',$ &c., are not needed in the working; they are drawn here to aid the student by showing him how the pattern corresponds with the plan, line for line of same lettering (see also § 65, p. 121).

PROBLEM XX.

To draw the pattern for an oval unequal-tapering body with top and base parallel (an oval bath, for instance). The height and dimensions of the top and bottom of the body being given.

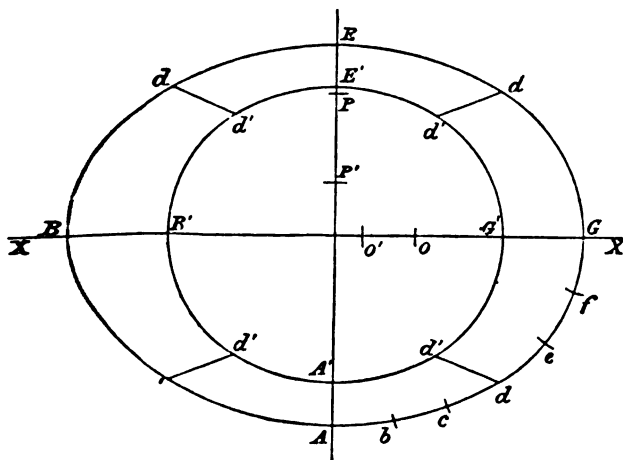
Four cases will be treated of, three in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 52) the plan of the body (see Problem VII., p. 131), preserving of its construction, the centres O, O', P, P' and the several points d and d' in which the side and end curves meet each other. Join dd' , as shown (four places) in the fig. From the plan we know (see *g*, p. 129) that $dGd, d'G'd', dBd, dB'd'$, the ends of the body are like portions of the frustum of an oblique cone; we also know that

$d A d d' A' d'$, $d E d d' E' d'$, the sides of the body are like portions of the frustum of an oblique cone.

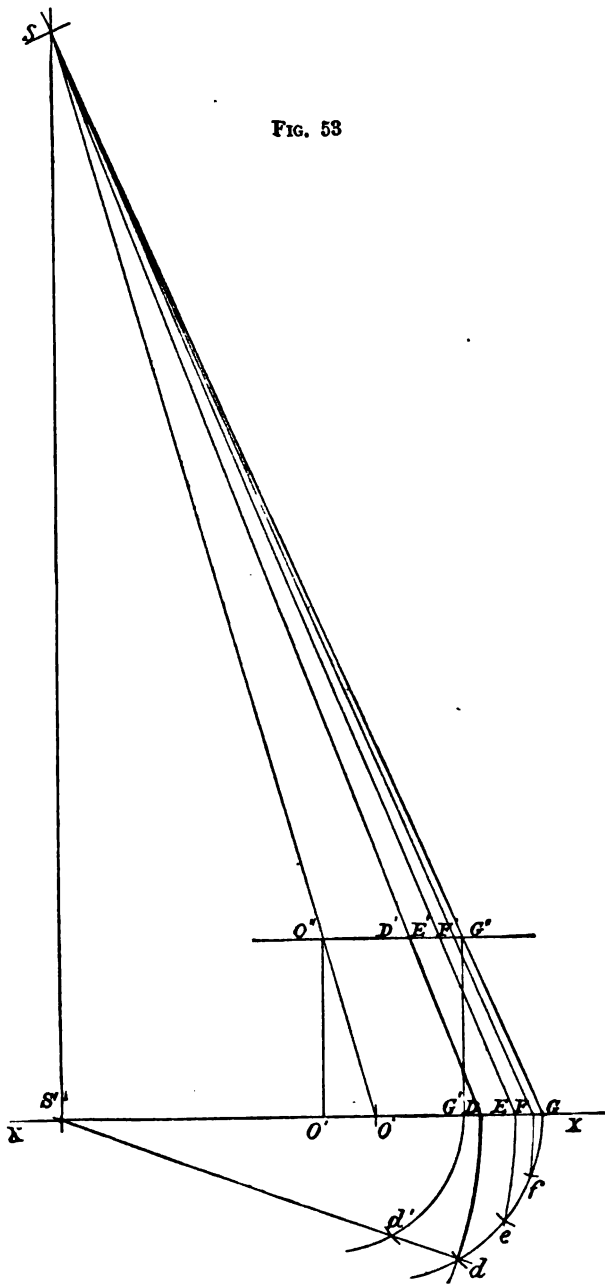
In Plate I. (p. 181), is a representation of the oval unequal-

FIG. 52.



tapering body for which patterns are required, also of two oblique cones (α and Z). The oblique cones show (except as to dimensions) to what portions of their surfaces the several portions of the surface of the oval body correspond. Thus the sides, A' , of the body correspond to the portion A of cone α , and the ends, B' , B' , correspond to the B portion cone Z . The correspondence will be more fully recognised as we proceed with the problem. The difference of obliquity between B' and B is seeming only, not real; and arises simply from Z being turned round so that the whole of the $d B d d' B' d'$ (Fig. 52) of the cone shall be seen. If the representation of Z showed its full obliquity, then the line on it from base to apex would be the right-hand side line of the cone, and only half of the B portion could be seen.

FIG. 53



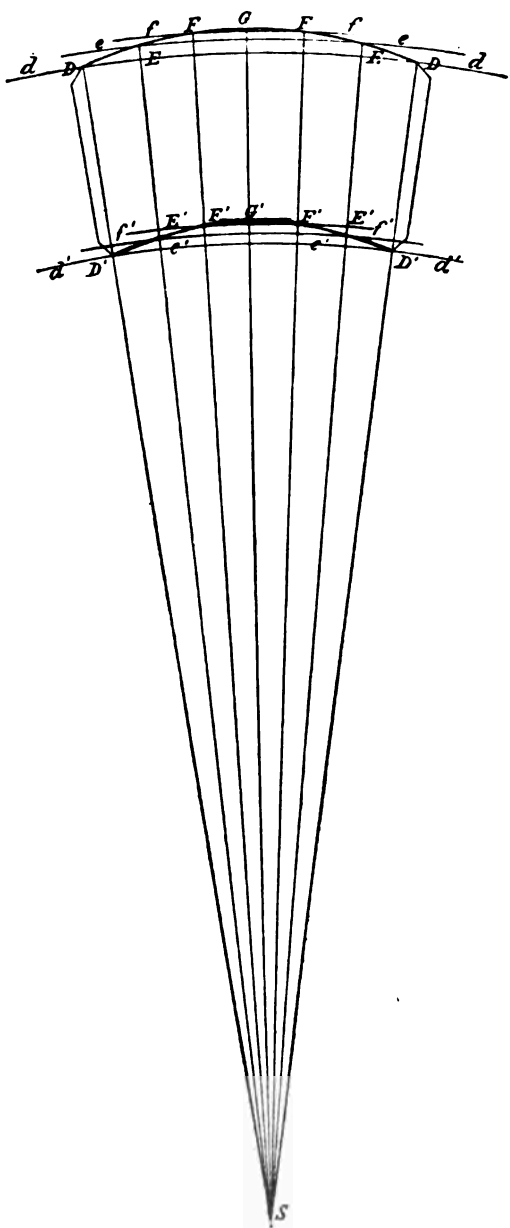
CASE I.—Patterns when the body is to be made up of four pieces.

It is clear that we require two patterns; one for the two ends, and one for the two sides; also that the seams should correspond with the four lines dd' , where the portions of the respective frusta meet each other.

To draw the pattern for the ends.

Draw separately (Fig. 53) the $G'Gfd'd'$ portion of Fig. 52, thus. Draw any line XX , Fig. 53, and with any point O (corresponding to O , Fig. 52) in it as centre and OG (Fig. 52) as radius, describe an arc Gd equal to Gd of Fig. 52. Make $G'O'$ equal to GO' (Fig. 52), and with O' as centre and $O'G'$ (Fig. 52) as radius describe an arc $G'd'$ equal to $G'd'$ of Fig. 52. Joining dd' completes the portion of Fig. 52 required. Now divide the arc Gd into any number of equal parts, here three, in the points f and e . At G' and O' draw $G'G''$, $O'O''$ perpendicular to XX , and each equal to the given height of the body. Join OO'' , GG'' ; produce them to their intersection in S (§ 80, p. 158); and from S let fall SS' perpendicular to XX . Join $O''G''$. With S' as centre and $S'f$, $S'e$, and $S'd$ successively as radii, describe arcs cutting XX in F , E , and D . Join these points to S by lines cutting $O''G''$ in F' , E' , and D' .

Next draw SG (Fig. 54) equal to SG (Fig. 53), and with S as centre and SF , SE , and SD (Fig. 53) successively as radii describe arcs ff , ee , and dd . With G as centre and radius equal to Gf (Fig. 53) describe arcs cutting arc ff right and left of SG in F and F . With each of these points, F and F as centre and same radius describe arcs cutting arc ee right and left of SG in E and E . With same radius and each of the last-obtained points as centre describe arcs cutting dd right and left of SG in D and D . Join all the points right and left of SG to S . With S as centre and SG'' (Fig. 53) as radius, describe an arc cutting SG in G' . With same centre and SF' (Fig. 53) as radius, describe an arc $f'f'$ cutting the lines SF right and left of SG in F' and F' . With same centre and SE' (Fig. 53) as radius, describe an

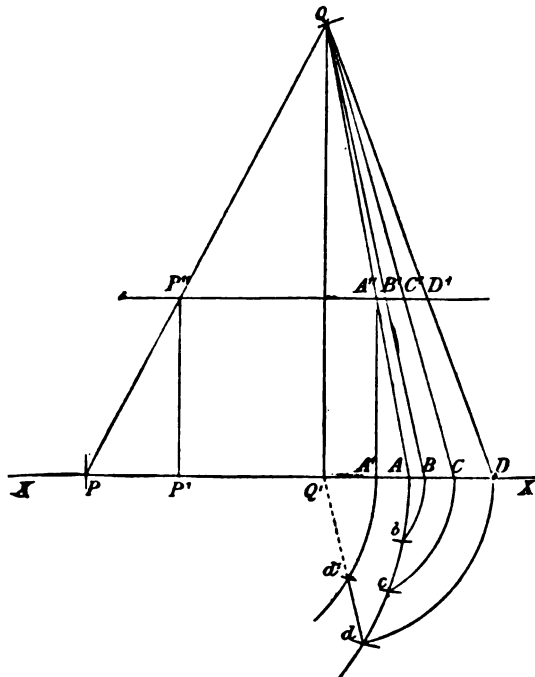


arc $e' e'$ cutting the lines $S E$ right and left of $S G$ in E' and E' . Similarly by arc $d' d'$ obtain points D' and D' . Through the points D, E, F, G, F, E, D , draw an unbroken curved line. Also through points $D', E', F', G', F', E', D'$, draw an unbroken curved line. Then $D G D' G' D'$ will be the required pattern for the ends of the body. It is in fact the development of the B portion of cone Z of Plate I.

To draw the pattern for the sides.

Draw separately (Fig. 55) the $A' A b d d'$ portion of Fig. 52, thus. Draw any line XX , Fig. 55. and with any point P

FIG. 55.

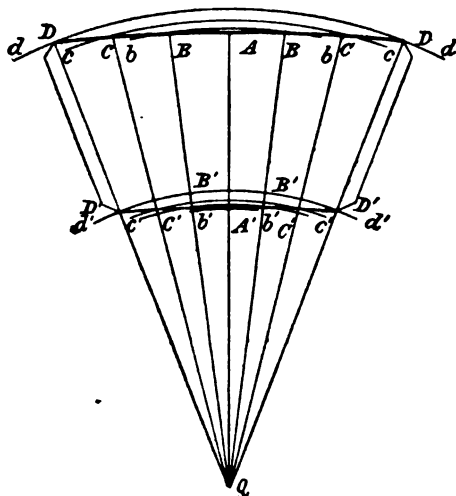


(corresponding to P , Fig. 52) in it as centre and PA (Fig. 52) as radius, describe an arc $A d$ equal to $A d$ of Fig. 52. Make

$A'P'$ equal to $A'P'$ (Fig. 52) and with P' as centre and $P'A'$ (Fig. 52) as radius describe an arc $A'd'$ equal to $A'd'$ of Fig. 52. Joining dd' completes the portion of Fig. 52 required. Now divide the arc $A'd'$ into any number of equal parts, here three, in the points b and c . At A' and P' draw $A'A''$, $P'P''$ perpendicular to XX , and each equal to the given height of the body. Join $P'P''$, $A'A''$, produce them to their intersection in Q (§ 80, p. 158); and from Q let fall $Q'Q'$ perpendicular to XX . Draw a line through $P''A''$. With Q' as centre and $Q'b$, $Q'c$, and $Q'd$ successively as radii describe arcs cutting XX in B , C , and D . Join these points to Q by lines cutting that through $P''A''$, in B' , C' , and D' .

Next draw QA (Fig. 56) equal to QA (Fig. 55), and with Q as centre and QB , QC , and QD (Fig. 55) successively as

FIG. 56.



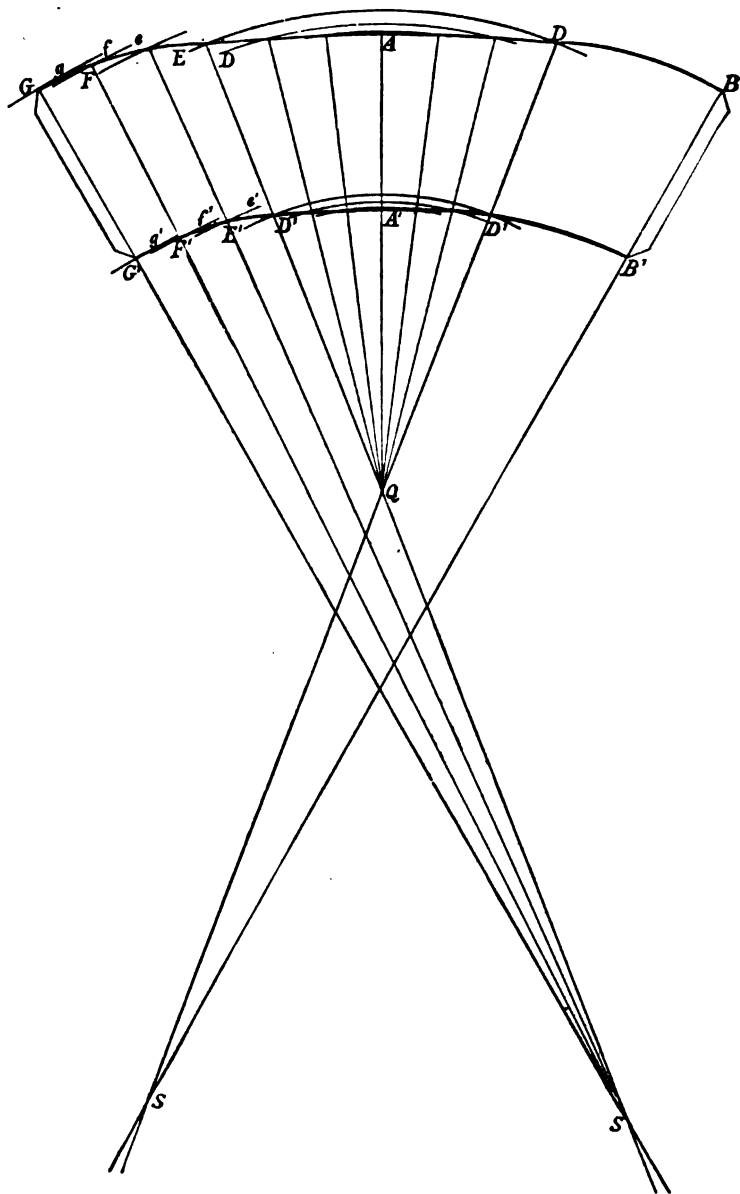
radii describe arcs bb , cc , and dd . With A as centre and radius equal to $A'b$ (Fig. 55) describe arcs cutting arc bb right and left of QA in B and B . With each of these points

B and B as centre and same radius describe arcs cutting arc *c c* right and left of Q A in C and C. With same radius and each of the last-named points as centre describe arcs cutting arc *d d* right and left of Q A in D and D. Join all the points right and left of Q A to Q. With Q as centre and Q A" (Fig. 55) as radius, describe an arc cutting Q A in A'. With same centre, and Q B' (Fig. 55) as radius, describe an arc *b' b'* cutting the lines Q B right and left of Q A in B' and B'. With same centre and Q C' (Fig. 55) as radius, describe arc *c' c'* cutting the lines Q C right and left of Q A in C' and C'. Similarly by arc *d' d'* obtain points D' and D'. Through the points D, C, B, A, B, C, D, draw an unbroken curved line. Also through the points D', C', B', A', B', C', D', draw an unbroken curved line. Then D A D D' A' D' will be the required pattern for the sides of the body, and is in fact the development of the A portion of cone *x* of Plate I.

CASE II.—Pattern when the body is to be made up of two pieces.

In this case the seams are usually made to correspond with B B' and G G' (Fig. 52). It is evident that only one pattern is now required, made up of a pattern for the side A' of the body (Plate I.) with right and left a half-end (B', B', Plate I.) pattern attached.

Draw (Fig. 57) a side pattern D A D D' A' D' as described in Case I. Produce D Q and make D S equal to D S (Fig. 53). With S as centre and S E, S F, and S G (Fig. 53) successively as radii describe arcs *e*, *f*, and *g*, and with D as centre and *d e* (Fig. 53) as radius describe an arc cutting arc *e* in E. With same radius and E as centre describe an arc cutting arc *f* in F, and similarly with F as centre and same radius find G. Join E S, F S, G S. Now with S E' (Fig. 53) as radius and S as centre describe an arc *e'* cutting S E in E', and with S F' and S G" (Fig. 53) successively as radii describe arcs *f'* and *g'* cutting S F and S G in F' and G'. From points D to G draw an unbroken curved line. Also from points D' to G' draw an unbroken curved line. Draw the other half-end





pattern $DBB'D'$ in the same manner; then $GABB'A'G'$ will be the pattern required.

CASE III.—Pattern when the body is to be made up of one piece.

We will put the seams at the middle of one end of the body, say, to correspond with BB' (Fig. 52). We now need an end pattern (the end $dGd'd'G'd'$ in plan), with side pattern attached right and left ($dEd'd'E'd'$, $dAd'd'A'd'$ in plan), and attached to each of these a half-end pattern ($dBB'd'$, $dBB'd'$ in plan). For want of space we do not give the pattern, but it is evident from what has just been stated, that the pattern will be double that shown in Fig. 57. It will be a useful exercise and should present no difficulties to the student, to himself draw the complete pattern, first drawing an end pattern (see Case I. and Fig. 54) and attaching right and left, a side pattern and a half-end pattern.

PROBLEM XXI.

To draw, without long radii, the pattern for an oval unequal-tapering body, with top and base parallel (an oval bath, for instance). The height and the dimensions of the top and bottom of the body being given.

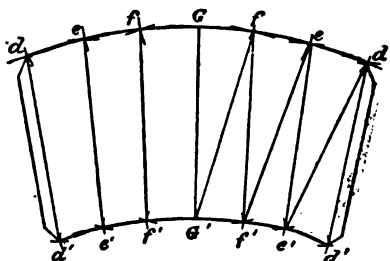
This problem is a fourth case of the preceding, and will be found very useful for both the end and side patterns, the radii of which are often of a most inconvenient length.

To draw the end pattern.

First draw (Fig. 58) the plan of the end of the body; that is the $dGd'd'G'd'$ portion of Fig. 52. Divide the arcs Gd , $G'd'$ each into the same number of equal parts, here three, in the points f , e , f' , e' ; join ff' , ee' . Through G draw GH perpendicular to GG' and equal to the given height of the body. From G along GG' mark off GF , GE , and GD respectively equal to ff' , ee' , and dd' ; join G' , F , E and D to H ; then $G'H$, $F'H$, $E'H$, and $D'H$ will be the true lengths of

arcs intersecting in e' and e' ; and with successively e' and e' right and left of $G'G'$ as centres, and radii respectively equal to $d'e''$ and $e'd$ describe arcs intersecting in d' and d' . Also with successively d' and e' as centres and DH and $e'd'$ respectively as radii describe arcs intersecting in d' and d' .

FIG. 59.



END PATTERN.

Through d, e, f, G, f, e, d , draw an unbroken curved line. Also through $d', e', f', G', f', e', d'$, draw an unbroken curved line. Join $d'd'$, right and left of $G'G'$, then $d'G'd'G'd'$ will be the end pattern required.

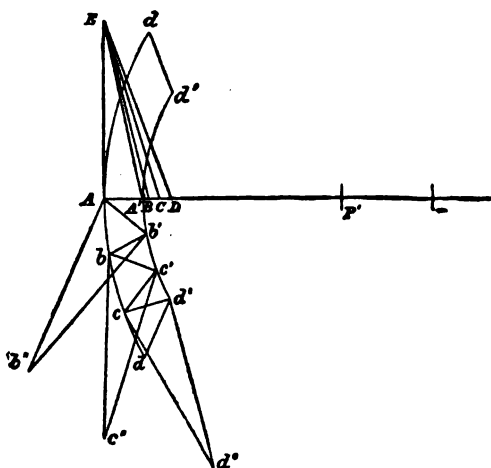
The lines $ff', ee', G'f, f'e$, &c., are not needed in the working, they are drawn for the reason stated in § 65, p. 121.

To draw the side pattern.

First draw (Fig. 60) the plan of the side of the body; that is the $dAd'd'A'd'$ portion of Fig. 52. Divide the arcs $Ad, A'd'$ each into the same number of equal parts, here three, in the points b, c, b', c' ; join bb', cc' . Through A draw AE perpendicular to AA' and equal to the given height of the body. From A along AA' mark off AB, AC , and AD respectively equal to bb', cc' , and dd' . Join A', B, C and D to E ; then $A'E, BE, CE$, and DE will be the true lengths of AA', bb', cc' , and dd' respectively. Next join $b'A$ and draw $b'b''$ perpendicular to $b'A$ and equal to the given height.

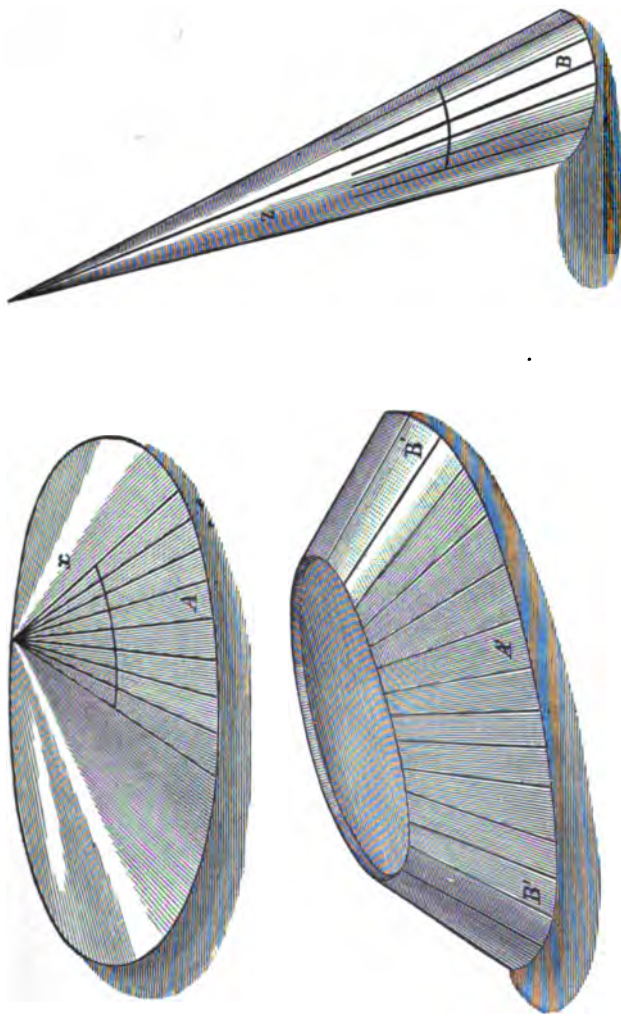
Join $A b''$; then $A b''$ may be taken as the true length $A b'$. Similarly join $c' b$ and $d' c$; through c' and d' draw $c' c''$, and $d' d''$ perpendicular to $c' b$ and $d' c$ respectively, and each equal to the given height; join $b c''$, $c d''$, then $b c''$ and $c d''$ may be taken as the true lengths of $b c'$ and $c d'$ respectively.

FIG. 60.



Next draw (Fig. 61) a line $A A'$ equal to $A'E$ (Fig. 60) and with A and A' as centres and radii respectively equal to $A b''$ and $A' b'$ (Fig. 60) describe arcs right and left of $A A'$, intersecting in b' and b'' . With b' , right of A' , and A as centres, and radii respectively equal to $B E$ and $A b$ (Fig. 60) describe arcs intersecting in b , right of A . With b' , left of A' , and A as centres, and radii respectively as before, describe arcs intersecting in b left of A . With successively b and b' right and left of $A A'$ as centres, and radii respectively equal to $b c''$ and $b' c'$ (Fig. 60) describe arcs intersecting in c' and c'' . With successively c' and b right and left of $A A'$ as centres, and radii respectively equal to $C E$ and $b c$ (Fig. 60) describe arcs intersecting in c and c' ; and with successively c and c' right

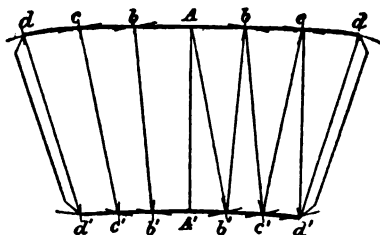
PLATE L (see p. 169).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (Part of Exhibit by the Author at the *Inventions Exhibition*, 1885.)

and left of $A A'$ as centres, and radii respectively equal to $c d''$ and $c' d'$ (Fig. 60) describe arcs intersecting in d' and d'' . Also with successively d' and c right and left of $A A'$ as centres, and $D E$ and $c d$ respectively as radii describe arcs

FIG. 61.



SIDE PATTERN.

intersecting in d and d' . Through d, c, b, A, b, c, d , draw an unbroken curved line. Also through $d', c', b', A', b', c', d'$, draw an unbroken curved line. Join $d d'$, right and left of $A A'$; then $d A d' A' d'$ will be the side pattern required.

The remark about lines $f f', e e', \&c.$, in (end pattern) Fig. 59, applies to lines $c c', b b', b c', \&c.$, in the present pattern.

PROBLEM XXII.

To draw the pattern for a tapering body with top and base parallel, and having circular top and oblong bottom with semicircular ends (tea-bottle top, for instance), the dimensions of the top and bottom of the body and its height being given.

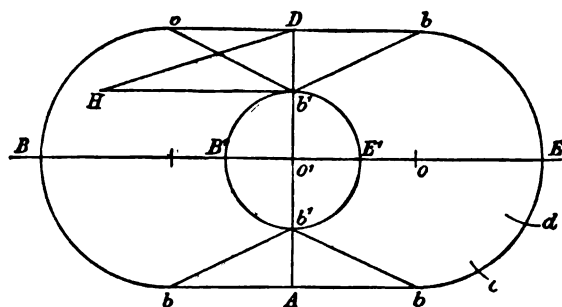
Four cases will be treated of; three in this problem and one in the problem following (see also § 79, p. 157).

CASE I.—Pattern when the body is to be made up of four pieces.

Draw (Fig. 62) the plan of the body (see Problem VIII., p. 132) preserving of its construction the centres O, O' ; the points b in plan of bottom where the extremities of the plan

lines of the sides meet the extremities of the plan semicircles of the ends; and the points b' in plan of top where the sides and ends meet in plan. Join $b'b$ at the four corners. The ends $bBbb'B'b'$, and $bEbb'E'b'$ of the body (see *g*, p. 129), are portions of frusta of oblique cones. Making the body in four pieces it will be best that the seams shall correspond with the lines Ab' , Bb' , Db' , and Eb' , then one pattern only, consisting of a half-end with a half-side pattern attached, will be required.

FIG. 62.

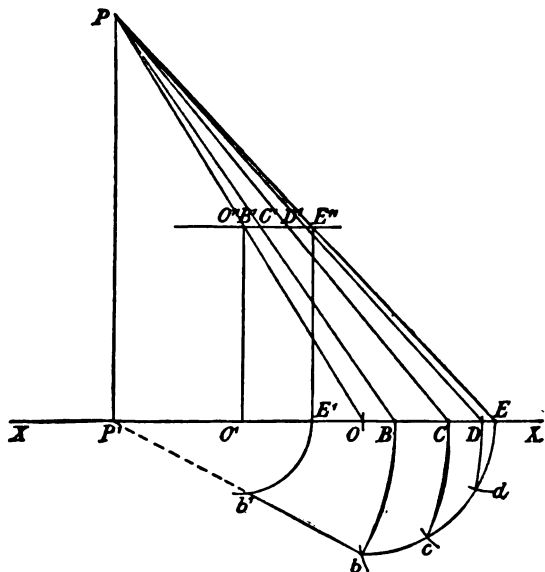


To draw the pattern.

Draw separately $E'Edbb'$ (Fig. 63), the $E'Edbb'$ portion of Fig. 62, thus. Draw any line XX and with any point O (to correspond with O , Fig. 62) in it as centre and OE (Fig. 62) as radius describe an arc (here a quadrant) Edb , equal to Edb of Fig. 62. Make EO' equal to EO' (Fig. 62), and with O' as centre and $O'E'$ (Fig. 62) as radius describe an arc (here a quadrant) $E'b'$ equal to $E'b'$ of Fig. 62. Joining bb' completes the portion of Fig. 62 required. Now divide Eb into any number of equal parts, here three, in the points d and c . From E' and O' draw $E'E''$, $O'O''$ perpendicular to XX and each equal to the given height of the body. Join $E'E''$, $O'O''$; produce them to intersect in P (§ 80, p. 158); from P let fall $P'P''$ perpendicular to XX , and join $E''O''$. With P'

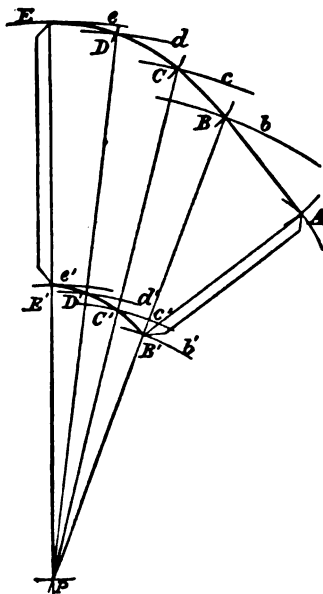
as centre and $P'd$, $P'c$, and $P'b$ successively as radii describe arcs cutting XX in D , C , and B . Join these points to P by lines cutting $O''E''$ in D' , C' , and B'

FIG. 63.



with same centre and $P C'$ and $P B'$ (Fig. 63) successively as radii find points C' and B' . Through b' (Fig. 62) draw $b' H$ perpendicular to $b' D$ and equal to the given height, and join $D H$, then $D H$ will be the true length of the line of which $b' D$ is the plan, that is, will be the length of a slant of the body at the middle of the side, where one of the seams

FIG. 64.



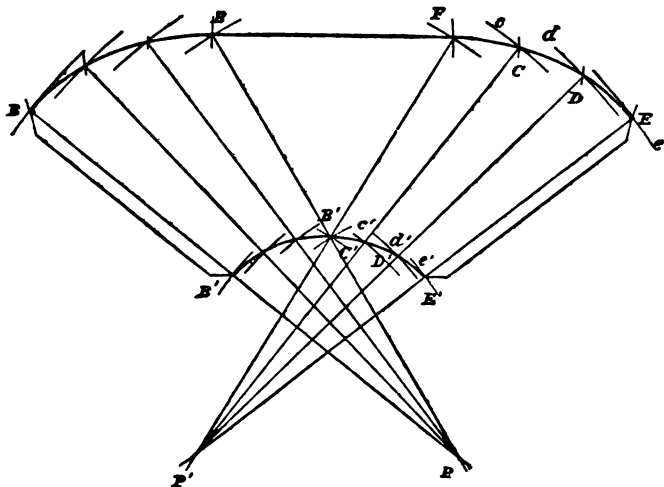
will come. With B' and B (Fig. 64) as centres and radii respectively equal to $D H$ and $b A$ (Fig. 62) describe arcs intersecting in A . Through the points E, D, C , and B draw an unbroken curved line. Also through the points E', D', C' , and B' draw an unbroken curved line. Join $B A, A B'$; then $E C A B' E'$ will be the pattern required.

CASE II.—Pattern when the body is to be made up of two pieces.

We will suppose the seams are to correspond with the lines BB' and EE' . It is evident that here we need but one pattern only, which will combine a side of the body and two half-ends, in fact will be double that of Fig. 64.

First draw $BB'B'B$ (Fig. 65) a half-end pattern exactly as the half-end pattern $EE'B'B$ in Fig. 64 is drawn, and with B' as centre and $B'B$ as radius and B as centre and bb

FIG. 65.



(Fig. 62) as radius, describe arcs intersecting in F . Join BF , FB' ; and produce FB' indefinitely. Make FP' equal to PB (Fig. 63), and with P' as centre and PC , PD and PE (Fig. 63) successively as radii describe arcs c , d , and e . With F as centre and bc (Fig. 63) as radius describe an arc cutting arc c in C , and with C as centre and same radius describe an arc cutting arc d in D . With same radius and D as centre describe an arc cutting arc e in E . Join CP' , DP' , EP' . Now with P' as centre and PC' (Fig. 63)

as radius describe an arc c' cutting $P'C$ in C' , and with same centre and PD' , PE'' (Fig. 63) successively as radii describe arcs d' and e' respectively cutting $P'D$ and $P'E$ in D' and E' . Through F , C , D , and E draw an unbroken curved line. Also through B' , C' , D' , and E' draw an unbroken curved line. Then $BBFFE'E'B'B'$ will be the complete pattern required.

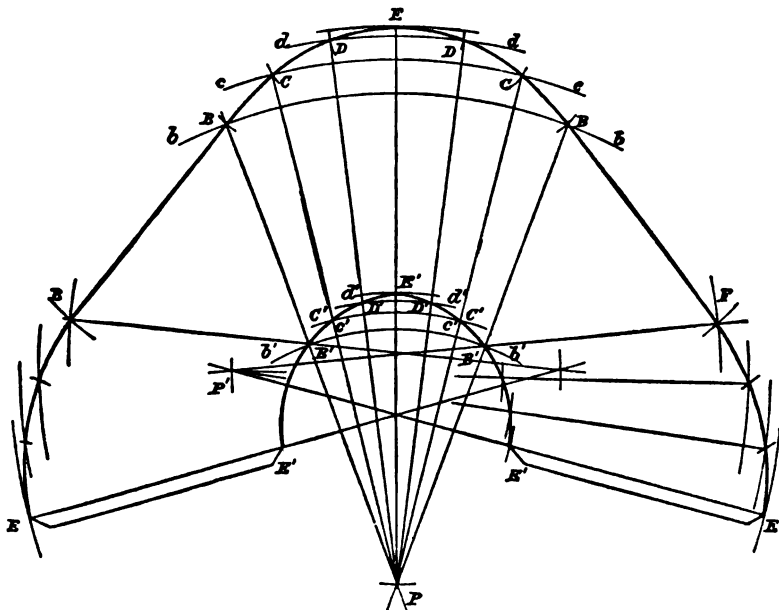
CASE III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with BB' (Fig. 62). We now need an end pattern (the end $bEb'b'E'b'$ in plan), with right and left a side pattern attached ($bAbb'$, $bDbb'$ in plan), and joined to each of these, a half-end pattern ($b'b'B'B$, $b'b'B'B$ in plan).

First draw Fig. 63; then draw (Fig. 66) PE equal to PE (Fig. 63) and with P as centre and PD , PC and PB (Fig. 63) successively as radii describe arcs dd , cc , and bb . With E as centre and radius equal to Ed (Fig. 63) describe arcs cutting arc dd right and left of PE in D and D . With points, D , D , successively as centres and same radius describe arcs cutting arc cc right and left of PE in C and C ; and with same radius and the last found points as centres describe arcs cutting arc bb right and left of PE in B and B . Join all the points found to P . With P as centre and PE'' (Fig. 63) as radius describe an arc cutting PE in E' . With same centre and PD' (Fig. 63) as radius describe an arc $d'd'$ cutting lines PD right and left of PE in D' and D' . With same centre and PC' (Fig. 63) as radius describe an arc $c'c'$ cutting lines PC right and left of PE in C' and C' . Similarly by arc $b'b'$ find points B' and B' . Through B , C , D , E , D , C , B , draw an unbroken curved line. Also through B' , C' , D' , E' , D' , C' , B' , draw an unbroken curved line. This gives us $BEBB'E'B'$ a complete end pattern. Now with B' on the right-hand side of the end pattern as centre and $B'B$ as radius, and B as centre and bb (Fig. 62) as radius describe arcs intersecting in F . Join BF , $F'B'$; produce $F'B'$ indefinitely, and to $F'B'$ attach the half-end pattern $FEE'B'$ in precisely the same manner that

$F E E' B'$ the half-end pattern in Fig. 65 is attached to the side pattern $B F B'$. By a repetition of the foregoing

FIG. 66.



construction on the left of the end pattern $B E B' E' B'$ we can attach $B B E E' B'$ and complete $E E E' E' E' E'$ the pattern required.

PROBLEM XXIII.

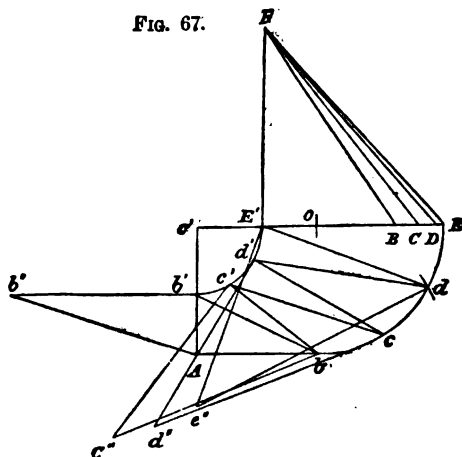
To draw, without long radii, the pattern for a tapering body with top and base parallel, and having circular top and oblong bottom with semicircular ends. The dimensions of the top and bottom of the body and its height being given.

This problem is a fourth case of the preceding, and is exceedingly useful where the work is so large that it is

inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces, as in Case I. of preceding problem).

(81.) Draw (Fig. 67) $E'E b A b'$ one quarter of the plan of the body. Divide the quadrants $E b$, $E' b'$, each into the same number of equal parts, here three, in the points d, c, d', c' ;

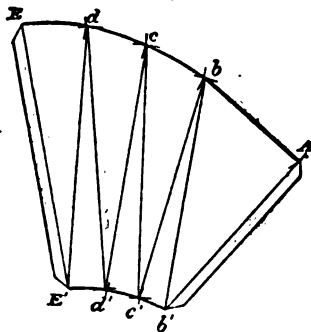


join $d d'$, $c c'$. Through E' draw $E'F'$ perpendicular to $E'E$ and equal to the given height of the body. From E' along $E'E$ mark off $E'D$, $E'C$, and $E'B$ respectively equal to $d d'$, $c c'$, and $b b'$; and join $E'F'$, $D'F'$, $C'F'$, and $B'F'$; then $E'F'$, $D'F'$, $C'F'$, and $B'F'$ will be the true lengths of $E'E'$, $d d'$, $c c'$, and $b b'$ respectively. Next join $d E'$, and draw $E'e''$ perpendicular to it and equal to the given height, and join $d e''$; then $d e''$ may be taken as the true length of $d E'$. Also join $c d'$ and $b c'$; through d' and c' draw $d' d''$ and $c' c''$ perpendicular to $c d'$ and $b c'$ respectively, and each equal to the given height, and join $c d''$ and $b c''$; then $c d''$ and $b c''$ may be taken as the true lengths of $d' c$ and $c' b$ respectively. Through b' draw

$b'b''$ perpendicular to $b'A$ and equal to the given height, and join $A b''$, then $A b''$ will be the true length of $b'A$.

Next draw (Fig. 68) EE' equal to EF (Fig. 67), and with E' and E as centres and radii respectively equal to $d e''$ and $E d$ (Fig. 67) describe arcs intersecting in d , and with d and E' as centre and radii respectively equal to DF and $E'd'$ (Fig. 67) describe arcs intersecting in d' . With d' and d as centres and radii respectively equal to $c d''$ and $d c$ (Fig. 67) describe arcs intersecting in c , and with c and d' as centres

FIG. 68.



and radii respectively equal to CF and $d'c'$ (Fig. 67) describe arcs intersecting in c' . With c' and c as centres and radii respectively equal to bc'' and cb (Fig. 67) describe arcs intersecting in b . Similarly with b and c' as centres and radii respectively equal to BF and $c'b'$ (Fig. 67) describe arcs intersecting in b' . With b' and b as centres and radii respectively equal to $b''A$ and bA (Fig. 67) describe arcs intersecting in A . Through E, d, c, b , draw an unbroken curved line. Also through E', d', c', b' , draw an unbroken curved line. Join $b'A, bA$; then $E c A b' c' E'$ is the pattern required.

(82.) The lines dd', cc', dE' , &c., are drawn in Fig. 68 simply to show the position that the lines which correspond to them in Fig. 67 (dd', cc', dE' , &c.) take upon the developed surface of the tapering body.

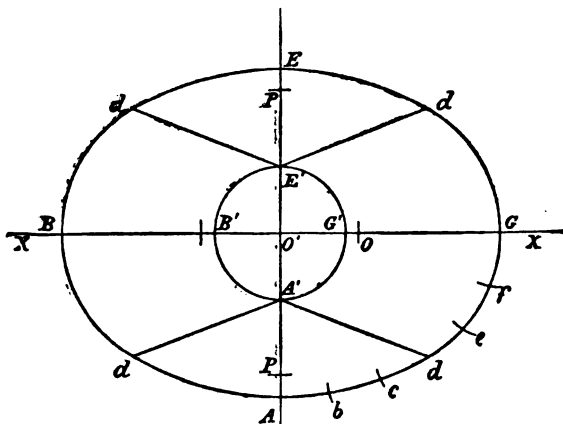
PROBLEM XXIV.

To draw the pattern for a tapering body with top and base parallel, and having an oval bottom and circular top (oval canister top, for instance). The height and dimensions of the top and bottom of the body being given.

Again four cases will be treated of; three in this problem and one in the problem following (see also § 79, p. 157).

Draw (Fig. 69) the plan of the body (see Problem IX., p. 132), preserving of its construction the centres O , O' , P , P' , and the four points (d) where the end and side curves of the

FIG 69.



plan of the bottom meet one another, also the points $A'E'$ where the axis AE cuts the circular top. Join dA' (two places) and dE' (two places). From the plan we know (see *g*, p. 129) that $dGdA'G'E'$, $dBdA'B'E'$, the ends of the body, are like portions of the frustum of an oblique cone; we also know that $dAdA'$, $dEdE'$, the sides of the body, are like portions of the frustum of an oblique cone.

(83.) It is evident that in this problem the arcs dGd , $E'G'A'$ and dBd , $E'B'A'$ are, neither pair, proportional (§ 67, p. 124). We have hitherto in Problems XVIII., XX., and XXII. been dealing with proportional arcs. The working will therefore differ, though but slightly, from that of problems mentioned.

In Plate II. (p. 203) is a representation of the tapering body for which patterns are required, also of two oblique cones (x and Z). The oblique cones show to what portions of their surfaces the several portions of the tapering body correspond. Thus the sides, B' , of the body correspond to the B portion of cone Z , and the ends, A' , correspond to the portion A of cone x . The correspondence will be more fully recognised as we proceed with the problem.

CASE I.—Pattern when the body is to be made up of four pieces.

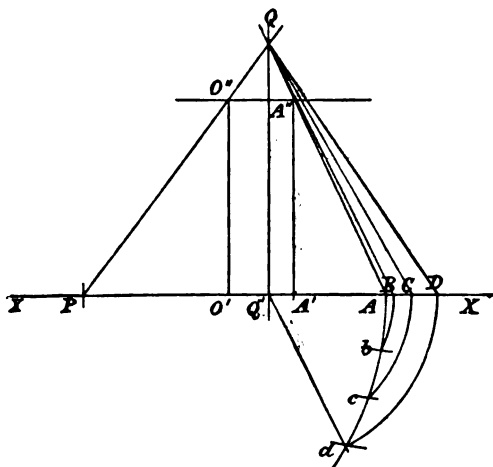
We will suppose the seams are to correspond with the plan lines $G'G'$, BB' , AA' , EE' , of ends and sides, as in Problem XXII. Then one pattern only, consisting of a half-end pattern, with, attached, a half-side pattern will be required.

To draw the pattern.

Draw (Fig. 70) separately $G'GfdA'$ the $G'GfdA'$ portion of Fig. 69, thus. Draw any line XX and with any point O in it (corresponding to O , Fig. 69) as centre and OG (Fig. 69) as radius describe an arc Gd , equal to Gd of Fig. 69. Make GO' equal to GO' (Fig. 69), and with O' as centre and $O'G'$ (Fig. 69) as radius describe an arc $G'A'$ equal to $G'A'$ of Fig. 69. Joining dA' completes the portion of Fig. 69 required. Now divide Gd into any number of equal parts, here three, in the points f and e . From G' and O' draw $G'G''$, $O'O''$ perpendicular to XX and each equal to the given height of the body. Join $G'G''$, $O'O''$; produce them to intersect in S (§ 80, p. 158); from S let fall SS' perpendicular to XX , and join $O''G''$. Now join d to S' , by a line cutting arc $G'A'$ in d' , then (§ 68, p. 124, and d , p. 126)

AA' and AO' (Fig. 69). Divide Ad into any number of equal parts, here three, in the points b and c . From A and O' draw AA'' , $O'O''$ perpendicular to XX and each equal to the given height of the body. Join AA'' , PO'' ; produce

FIG. 71.

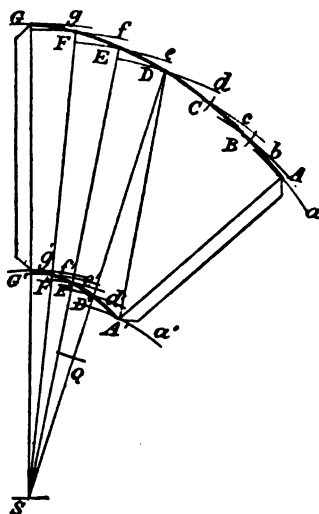


them to intersect in Q (§ 80, p. 158); from Q let fall QQ' perpendicular to XX , and join $O''A''$. With Q' as centre and $Q'b$, $Q'c$, and $Q'd$ successively as radii describe arcs cutting XX in B , C , and D , and join these points to Q .

Next draw (Fig. 72) a line SG equal to SG (Fig. 70) and with S as centre and SF , SE , and SD (Fig. 70) successively as radii describe arcs f , e , and d . With G as centre and radius equal to Gf (Fig. 70) describe an arc cutting arc f in F . With same radius and F as centre describe an arc cutting arc e in E , and with E as centre and same radius describe an arc cutting arc d in D . Join the points F , E , and D to S . With S as centre and SG'' (Fig. 70) as radius describe an arc g' cutting SG in G' . With same centre and SF' (Fig. 70)

as radius describe arc f' cutting $S F$ in F' . With same centre and $S E'$ (Fig. 70) as radius describe an arc e' cutting $S E$ in E' , and with same centre and $S D'$ (Fig. 70) as radius describe arc d' cutting $S D$ in D' . With same centre and $S A''$ (Fig. 70) as radius describe arc a' , and with D' as centre and radius $d' A'$ (Fig. 70) describe an arc intersecting arc a' in A' . Make $D Q$ equal to $D Q$ (Fig. 71) and with Q as centre and $Q C$,

FIG. 72.



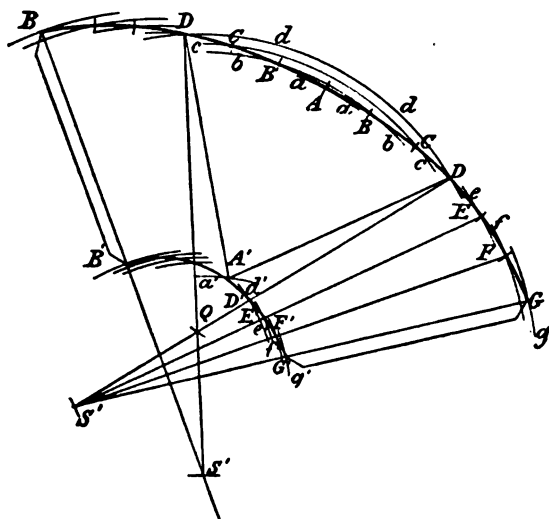
$Q B$, and $Q A$ (Fig. 71) successively as radii describe arcs c , b , and a . With D as centre and $d c$ (Fig. 71) as radii describe an arc cutting arc c in C , and with C as centre and same radius describe an arc cutting arc b in B . Similarly with same radius and B as centre find point A . Join $A A'$. Through the points G, F, E, D, C, B, A draw an unbroken curved line. Also through the points G', F', E', D', A' draw an unbroken curved line. Then $G D A A' E' G'$ is the pattern required.

CASE II.—Pattern when the body is to be made up of two pieces.

We will suppose the seams are to correspond with the lines GG' and BB' . It is evident that here we need but one pattern only, which will combine a side of the body and two half-ends, in fact will be double that of Fig. 72.

First draw $BDA'B'$ (Fig. 73) a half-end pattern exactly as the half-end pattern $GDA'G'$ in Fig. 72 is drawn, and make DQ equal to DQ (Fig. 71). With Q as centre and

FIG. 73.



QC , QB , and QA (Fig. 71) successively as radii describe arcs dd , cc , bb , and aa . With D as centre and radius equal to dc (Fig. 71) describe an arc cutting arc cc in C , and with same radius and C as centre describe arc cutting arc bb in B . With B as centre and same radius describe an arc cutting arc aa in A , and with same radius and A as centre describe an arc cutting arc bb in B . Similarly with same radius and

points B and C, to the right of A, successively as centres find points C and D. Join D Q and produce it indefinitely; make D S' equal to S D (Fig. 70). With S' as centre and S E, S F, and S G (Fig. 70) successively as radii describe arcs *e*, *f*, and *g*, and with D as centre and *d e* (Fig. 70) as radius describe an arc cutting arc *e* in E. With same radius and E as centre describe an arc cutting arc *f* in F. Similarly with F as centre and same radius find point G. Join the points E, F, and G to S'. With S' as centre and S D' (Fig. 70) describe arc *d'* cutting S' D in D'. With same centre and S E' (Fig. 70) as radius describe arc *e'* cutting S' E in E'. Similarly with same centre and S F' and S G' (Fig. 70) successively as radii find points F' and G'. Through the points D, C, B, A, B, C, D, E, F, G, draw an unbroken curved line. Also through points A', D', E', F', G' draw an unbroken curved line. Then B A G G' A B' is the complete pattern required.

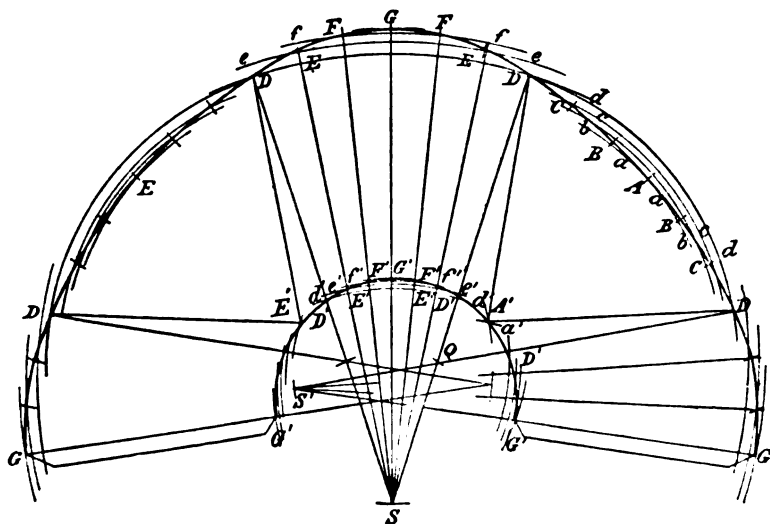
CASE III.—Pattern when the body is to be made up of one piece.

In this case we will put the seam to correspond with G G' (Fig. 69). We now need an end pattern (the end *d B d A' B' E'* in plan), with right and left a side pattern attached (*d A d A'*, *d E d E'* in plan), and joined to each of these a half-end pattern (*d A' G' G*, *d E' G' G* in plan).

First draw Figs. 70 and 71; then draw (Fig. 74) S G equal to S G (Fig. 70), and with S as centre and S F, S E, and S D (Fig. 70) successively as radii describe arcs *f f*, *e e*, and *d d*. With G as centre and radius G *f* describe arcs cutting arc *f f* right and left of S G in F and F. With points F, F, successively as centres and same radius describe arcs cutting arc *e e* right and left of S G in E and E, and with same radius, and the last found points as centres describe arcs cutting arc *d d* right and left of S G in D and D. Join all the points found to S. With S as centre and S G' (Fig. 70) as radius describe an arc cutting S G in G'. With same centre and S F' (Fig. 70) as radius describe an arc *f' f'* cutting lines S F right and left of S G in F' and F'. With same centre

and $S E'$ (Fig. 70) describe an arc $e' e'$ cutting lines $S E$ right and left of $S G$ in E' and E' , and with same centre and $S D'$ (Fig. 70) as radius describe arc $d' d'$ cutting lines $S D$ right and left of $S G$ in D' and D' . With S as centre and D' right and left of $S G$ as centres and radii respectively equal to $S A''$ and $d' A'$ (Fig. 70) describe arcs intersecting in A' and E' . Through D, E, F, G, F, E, D , draw an unbroken curved line. Also through $E', D', E', F', G', F', E', D', A'$, draw

FIG. 74.



an unbroken curved line, and join $D E'$, $D A'$. This gives us $D G D A' G' E'$ a complete end pattern. Now attach the side pattern $D A D A'$ and the half-end pattern $D G G' A'$ to the right and left of the complete end pattern we started with, in precisely the same manner that the side pattern $D A D A'$ and half-end pattern $D G G' A'$ in Fig. 73 is attached to $D A'$, which corresponds to $D A'$ in Fig. 74. This will complete $G E G A G' G' G'$ the pattern required.

PROBLEM XXV.

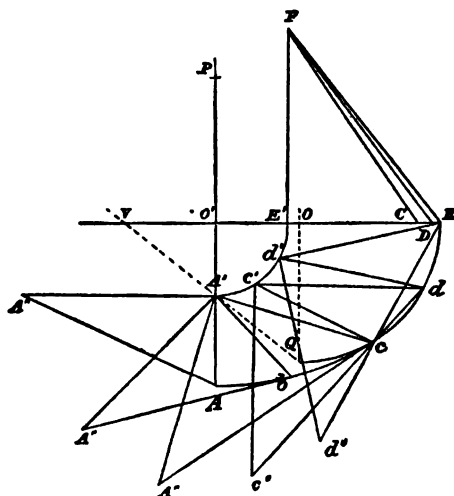
To draw, without long radii, the pattern for a tapering body with top and base parallel, and having an oval bottom and circular top. The height and dimensions of the top and base of the body being given.

This problem is a fourth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces, as in Case I. of preceding problem).

(84.) Draw (Fig. 75) $E d b A A' d' E'$ one quarter of the plan of the body. Join c (the point where the end and side curves

FIG. 75.



of the plan of the bottom meet) to A' , the extremity of the quadrant EA' . We must now get at corresponding points in

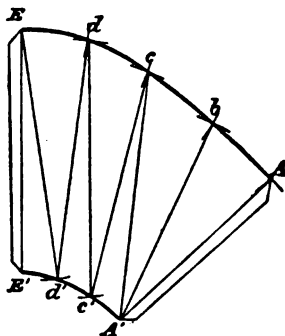
the arcs $E c$, $E' A'$. To do this, as the arcs are not proportional (§ 67), we must find the plan of the apex of the oblique cone of a portion of which $E c A' E'$ is the plan. It is in the finding of these points that our present working differs from the working of Problems XIX., XXI., XXII., and XXVII., where the corresponding points $d d'$, $c c'$ (Figs. 50, 58, 60, and 82) are found. With radius $O E$ produce arc $E c$ indefinitely, and through O draw $O Q$ perpendicular to $E' E$ and cutting $E c$ produced in Q . Then $E Q$ is a quadrant, and $E Q$, $E' A'$ (each a quadrant) are proportional. In Q and A' therefore we have corresponding points (§ 68), as well as in E and E' , which are points on the longest generating line (*b*, p. 126); and the intersection of $O O'$ produced, of which $E E'$ is part, and $Q A'$ joined and produced will give us V , the required plan of the apex. Next divide $E c$ into any number of equal parts, here two, in point d . Join $d V$, $c V$ cutting $E' A'$ in d' and c' , respectively (the lines from d and c are not carried to V in the fig.), then d and d' , c and c' are corresponding points.

Next through E' draw $E' F$ perpendicular to $E' E$ and equal to the given height; from E' along $E' E$ mark off $E' D$, $E' C$, respectively equal to $d d'$ and $c c'$, and join $E F$, $D F$, and $C F$; then $E F$, $D F$ and $C F$ will be the true lengths of $E E'$, $d d'$, and $c c'$. Join $E d'$ and $d c'$; through d' and c' draw $d' d''$ and $c' c''$ perpendicular to $d' E$ and $c' d$ respectively, and each equal to the given height, and join $E d''$, $d c''$; then $E d''$ and $d c''$ may be taken respectively as the true lengths of $d' E$ and $c' d$. Now divide arc $A c$ into any number of equal parts, here two, in the point b , and join $b A'$; through A' draw three lines $A' A''$; one perpendicular to $A' c$, the second perpendicular to $A' b$, and the third perpendicular to $A' A$, and each equal to the given height, and join $c A''$, $b A''$, and $A A''$; then $c A''$, $b A''$, and $A A''$ may be taken as the true lengths of $A' c$, $A' b$, and $A' A$ respectively.

Next draw (Fig. 76) $E E'$ equal to $E F$ (Fig. 75), and with E and E' as centres and radii respectively equal to $E d''$ and $E' d'$ (Fig. 75) describe arcs intersecting in d' , and with d' and E as centres and radii respectively equal to $D F$ and $E d$

(Fig. 75) describe arcs intersecting in d . With d and d' as centres and radii respectively equal to $d c''$ and $d' c'$ (Fig. 75) describe arcs intersecting in c' , and with c' and d as centres and radii respectively equal to $C F$ and $d c$ (Fig. 75) describe arcs intersecting in c . With c and c' as centres and radii respectively equal to $c A''$ and $c' A'$ (Fig. 75) describe

FIG. 76.

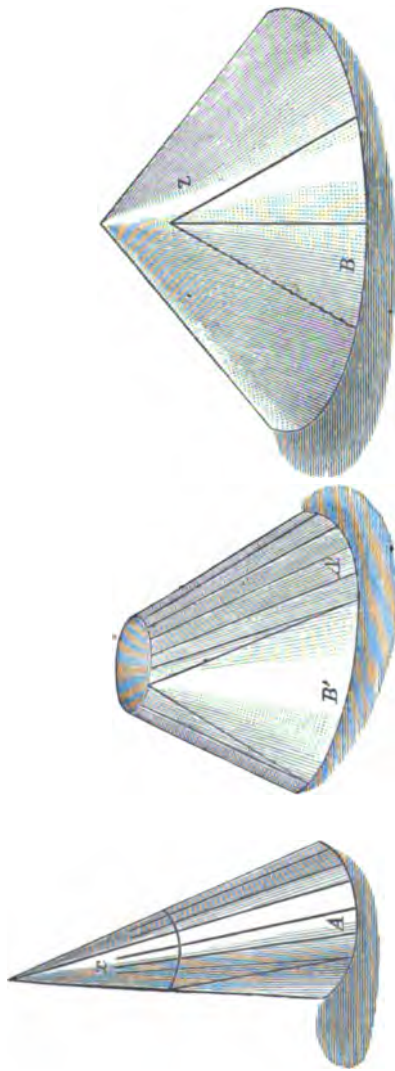


arcs intersecting in A' , and with A' and c as centres and radii $b A''$ and $c b$ (Fig. 75) describe arcs intersecting in b . Similarly with A' and b as centres and radii respectively equal to $A A''$ and $b A$ (Fig. 75) describe arcs intersecting in A . Join $A A'$. Through E, d, c, b, A draw an unbroken curved line. Also through E', d', c', A' draw an unbroken curved line. Then $E c A A' E'$ is the pattern required.

The lines $d d', c c', E d', d c', \&c.$, are not needed for the working, they are drawn for the reason stated in § 82, end of Problem XXIII.

(85.) If V is inaccessible, corresponding points c, c', d, d' can thus be found. From the point E' along the arc $E' A'$ set off an arc proportional to the arc $E c$ in the following manner. Join $O c$ (line not shown in fig.) and through O' draw $O' c'$ (also not shown in fig.) parallel to $O c$ and cutting arc $E' A'$ in c' ; then arcs $E' c'$ and $E c$ will be proportional. (The student must particularly notice this

PLATE II. (see p. 193).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (*Part of Exhibit by the Author at the Inventions Exhibition, 1885.*)

method of drawing proportional arcs. It is outside the scope of the book to prove the method.) Now divide arcs $E'c'$, Ec each into the same number of equal parts, here two, in the points d and d' ; then d, d' and c, c' are corresponding points.

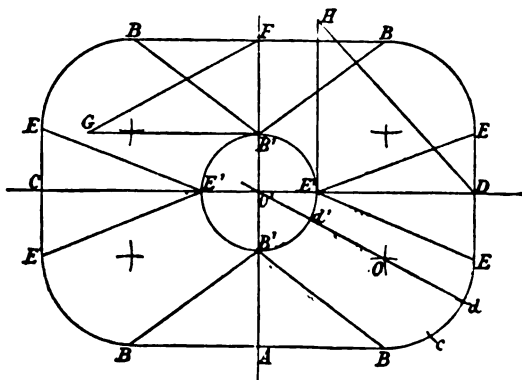
PROBLEM XXVI.

To draw the pattern for a tapering body with top and base parallel, and having oblong bottom with round (quadrant) corners, and circular top. The dimensions of the top and base of the body and its height being given.

Again four cases will be treated of, three in this problem, and one in the problem following so § 79, p. 157).

Draw (Fig. 77) the plan of the body (see Problem X., p. 133) preserving of its construction the centres OO' and the

FIG. 77.



points $E'B'$ where the flat sides and flat ends meet the circle of the top. Join BB' , EE' each in four places. From the plan we know (see *g*, p. 129) that the round corners of the body are portions of frusta of oblique cones.

(86.) Looking at the plan, we can at once see that what we have to deal with differs somewhat from what has as yet been before us. Hitherto a line passing through the centres of the plan arcs bisected the arcs, and the cone development was consequently identical each side of a central line. In Fig. 77, however, the line drawn through OO' does not bisect the plan arcs EB , $E'B'$. This affects the working but little, as will be seen.

In Plate III. (p. 213) the tapering body is represented; also an oblique cone Z , the A portion of which corresponds to the A' portion of the body, and the development of the former is the development of the latter.

CASE I.—Pattern when the body is to be made up of four pieces.

We will suppose the seams to correspond with the plan lines CE' , DE' , FB' , AB' , of ends and sides, as in Problems XXII. and XXIV. just preceding. Then one pattern, comprising a half-end, a complete corner, and a half-side, will be the pattern required.

To draw the pattern.

Draw separately (Fig. 78) an EE' , $B'B$ portion of Fig. 77, thus. Draw an indefinite line $S'd$ (Fig. 78), and with any point O (corresponding to O , Fig. 77) in it as centre and OB (Fig. 77) as radius describe an arc BE . Join OO' (Fig. 77) and produce it cutting arc BE in d ; make dB and dE (Fig. 78) equal respectively to dB and dE (Fig. 77). Now (Fig. 78) make dO' equal to dO' (Fig. 77), and with O' as centre and $O'B'$ (Fig. 77) as radius describe an arc $B'E'$. Make $d'B'$ and $d'E'$ equal respectively to $d'B'$ and $d'E'$ (Fig. 77). Joining EE' , BB' completes the portion of Fig. 77 required. Now divide (Fig. 77) BE into any number of parts. It is convenient to take d as one of the division points, and to make dc equal to dE ; leaving cB without further division, thus making the division of BE into three portions not all equal. In actual practice the dimensions of the work will suggest the number of parts expedient. Now (Fig. 78) make dc equal

cutting arc b in B . Join E , C , and B to S . Make SD' equal to SD' (Fig. 78) and with S as centre and SC' (Fig. 78) as radius describe an arc $c'c'$ cutting SE and SC in E' and C' respectively. With same centre and SF' (Fig. 78) as radius describe an arc b' cutting SB in B' . Through E , D , C , and B draw an unbroken curved line. Also through E' , D' , C' , and B' draw an unbroken curved line; this will complete the pattern of the round corner. To attach the half-end and half-side patterns to EE' and BB' respectively, the true lengths of $E'D$ and $B'A$ (Fig. 77) must be found. Draw (Fig. 77) $E'H$ perpendicular to $E'D$ and equal to the given height of the body; join DH , then DH is the true length of $E'D$. The lines $B'E$ and $B'A$ being equal, their true lengths are equal, we will therefore for convenience find the true length of $B'A$ in that of $B'F$. Draw $B'G$ perpendicular to $B'F$ and equal to the given height, join FG , then FG is the true length required. Now with E' (Fig. 79) as centre and DH (Fig. 77) as radius, and E as centre and radius ED (Fig. 77) describe arcs intersecting in G . Join EG , GE' ; this attaches to $E'E$ the half-end pattern. With B' (Fig. 79) as centre and FG (Fig. 77) as radius, and B as centre and radius BA (Fig. 77) describe arcs intersecting in A . Join BA , $A'B'$; this attaches to $B'B$ the half-side pattern. Then $A'B'E'G$ is the complete pattern required.

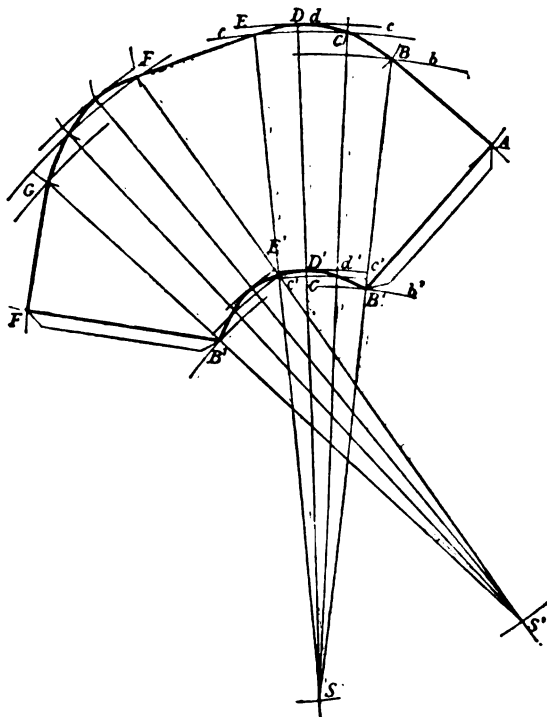
CASE II.—Pattern when the body is to be made up of two pieces.

Here it will be best that the seams shall correspond with the lines AB' , FB' , that is with the middle of each side. The required pattern will then be double that of Case I.

Draw (Fig. 80) $EBB'E'$, the corner pattern, in exactly the same manner that $EBB'E'$ (Fig. 79) is drawn. With E' as centre and $E'E$ as radius, and E as centre and EE (Fig. 77) as radius describe arcs intersecting in F . Join EF , FE' . Produce FE' indefinitely and make $F'S'$ equal to ES . Using S' as centre, the round corner $FG B'E'$ can be drawn as was $EBB'E$. With B' as centre and FG (Fig. 77)

as radius, and B as centre and BA (Fig. 77) as radius describe arcs intersecting in A. Similarly with B' and G as

FIG. 80.



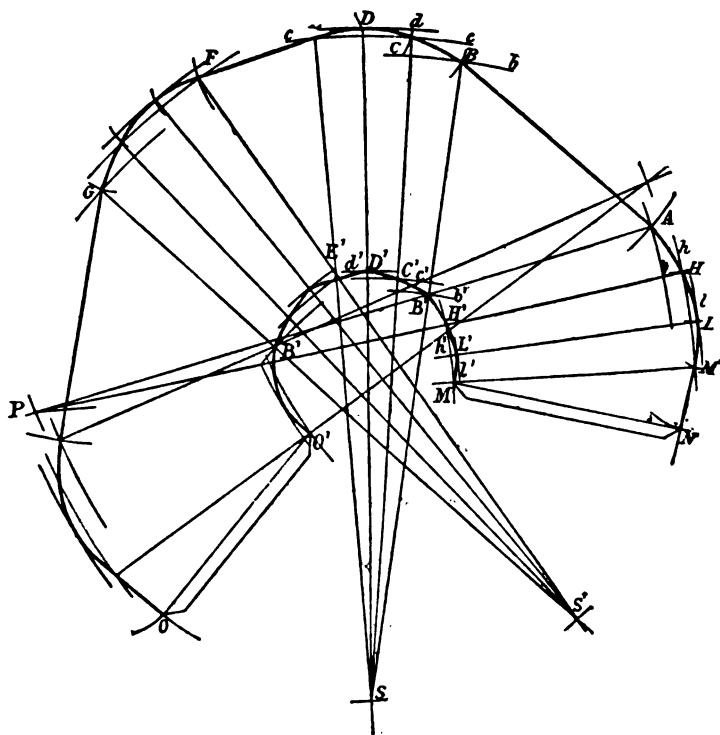
centres and same radii respectively describe arcs intersecting in F. Join BA, AB', GF, FB', then A D F F' B' E' B' will be the pattern required.

CASE III.--Pattern when the body is to be made up of one piece.

We will suppose the seam to correspond with CE' the middle of one end. Draw GFDBB'E'B' (Fig. 81) in the

same manner that $G F D B B' E' B'$ (Fig. 80) is drawn. With B' as centre and $B' B$ as radius, and B as centre and $B B'$ (Fig. 77) as radius describe arcs intersecting in A . Join $B A$, $A B'$; produce $A B'$ indefinitely. Make $A P$ equal to $S F$ (Fig. 78). With P as centre and $S C$ and $S D$ succes-

FIG. 81.



sively as radii describe arcs h and l , and with A as centre and $B c$ (Fig. 78) as radius describe an arc cutting arc h in H . With H as centre and $c d$ (Fig. 78) as radius describe an arc cutting arc l in L ; and with L as centre and same radius

describe an arc cutting arc h in M . Join H , L , and M to P . With P as centre and SC' (Fig. 78) as radius describe arc h' cutting PH and PM in H' and M' respectively. With same centre and SD' (Fig. 78) as radius describe arc l' cutting PL in L' . Through the points A , H , L , M draw an unbroken curved line. Also through the points B' , H' , L' , M' draw an unbroken curved line. With M' as centre and DH (Fig. 77) as radius, and M as centre and EC (Fig. 77) as radius describe arcs intersecting in N . Join MN , NM' . Repeating the working to the left of $B'G$, the $GOO'B'$ portion of the pattern can be drawn which completes the pattern required.

PROBLEM XXVII.

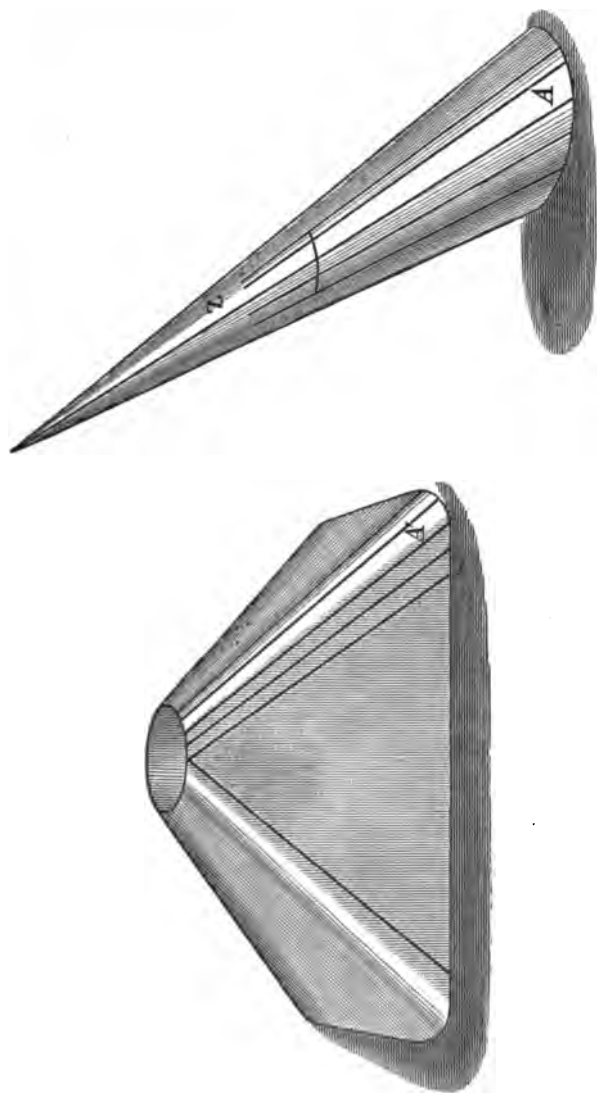
To draw, without long radii, the pattern for a tapering body with top and base parallel, and having oblong bottom with round quadrant corners, and circular top. The dimensions of the top and base of the body and its height being given.

This problem is a fourth case of the preceding, and is exceedingly useful where the work is so large that it is inconvenient to draw the whole of the plan, and to use long radii.

To draw the pattern (with the body in four pieces as in Case I. of preceding problem).

Draw (Fig. 82) $EcbAb'c'd'$, one quarter of the plan of the body. Divide the arc (quadrant) db into any number of equal parts, here two, in the point c , and the arc $d'b'$ into the same number of equal parts in the point c' ; and join cc' . Through d' draw $d'F$ perpendicular to $d'E$ and equal to the given height of the body. From d' along $d'E$ mark off $d'B$, equal to bb' , and $d'D$ equal to cc' and dd' (which two lines have happened to come in this particular fig. so nearly equal that we may take them as equal), and join BF , EF , and DF ; then BF and EF will be the true lengths of bb' and Ed' respectively, and DF may be taken as the true

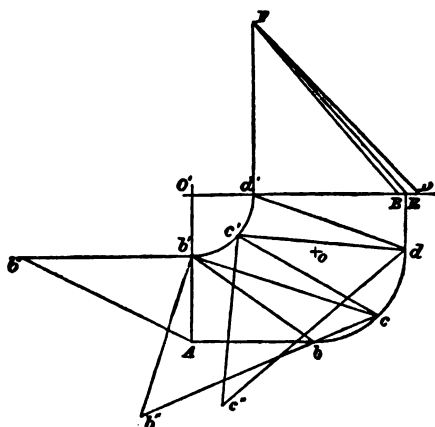
PLATE III. (see p. 208).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (Part of Exhibit by the Author at the Inventions Exhibition, 1885.)

length of both $c'c$ and $d'd$. Next join $c'd$, $b'c$; draw $c'c''$, $b'b''$ perpendicular to $c'd$, $b'c$ respectively, and each equal to the given height; and join $d'c''$ and $c'b''$; then $d'c''$, $c'b''$ may

FIG. 82.

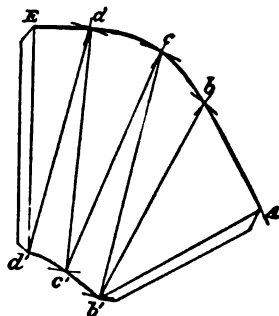


be taken as the true lengths of $c'd$ and $b'c$ respectively. Draw $b'b''$ perpendicular to $b'A$ and equal to the given height, and join $A b''$, then $A b''$ is the true length of $b'A$.

Next draw (Fig. 83) $d'd'$ equal to $D F$ (Fig. 82), and with d and d' as centres and radii respectively equal to $d'c'$ and $d'c$ (Fig. 82), describe arcs intersecting in c' . With c' and d as centres, and radii respectively equal to $D F$ and $d c$ (Fig. 82) describe arcs intersecting in c ; and with c and c' as centres and radii respectively equal $c'b''$ and $c'b'$ (Fig. 82) describe arcs intersecting in b' ; also with b' and c as centres and radii respectively equal to $B F$ and $c b$ (Fig. 82) describe arcs intersecting in b . With d' and d as centres and radii respectively equal to $E F$ and $d E$ (Fig. 82) describe arcs intersecting in E ; and with b' and b as centres and radii respectively equal to $b'' A$ and $b A$ (Fig. 82) describe arcs

intersecting in A. Through d, c, b , draw an unbroken curved line. Also through d', c', b' draw an unbroken curved line.

FIG. 83.



Join dE , $d'E$, bA , and $b'A$; then $Ecb'd'$ is the pattern required.

The lines cc' , bb' , &c., are not needed for the working, they are drawn for the reason stated in § 82, end of Problem XXIII.

PROBLEM XXVIII.

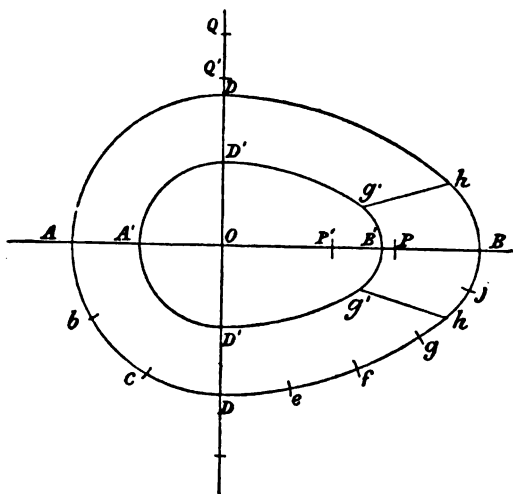
To draw the pattern for an Oxford hip-bath, the like dimensions to those for Problem XI. being given.

It is only necessary to treat of two cases, one in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 84) the plan of the body (see Problem XI., p. 134), preserving of its construction the centres O, P', P, Q', Q ; the points D, D' (two sets) in which the arcs, in plan, of the back and sides meet each other; and the points h, g' (two sets) in which the plan arcs of the sides and front meet each other. Join hg' (two places) as shown in the fig. Examining the plan of the bath we see (*d*, p. 55) that the back of it,

$DA D'D' A'D'$, is a portion of a right cone; that the sides $DD'g'h$ are (*g*, p. 123) each of them a portion of an oblique cone; and that the portion $hg'g'h$ is also a portion of an

FIG. 84.



oblique cone. Similarly as in Problem XXIV. (§ 83, p. 193) the arcs Dh , $D'g'$ and Bh , $B'g'$ are, neither pair, proportional.

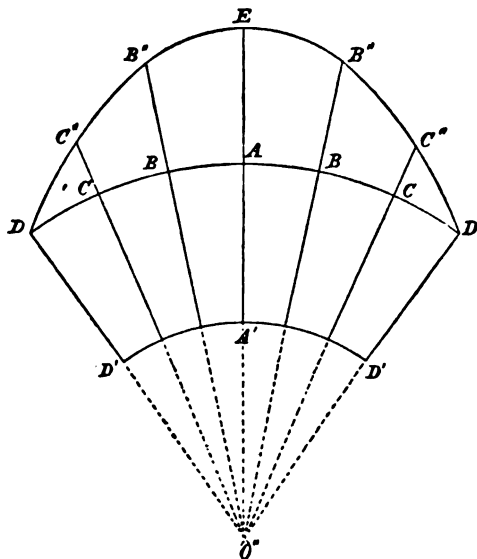
In Plate IV. (p. 227) is a representation of an Oxford hip-bath, also of a right cone x , and two oblique cones Y and Z . The cones show to what portions of their surfaces the several portions of the bath correspond. Thus the back, A' , of the bath corresponds to the A portion of right cone x ; the half-fronts, C' , of the bath correspond each of them to the C portion of oblique cone Y , and the sides, B' , of the bath correspond each of them to the B portion of oblique cone Z .

Patterns when the body is to be made up of three pieces.

We will put the seams to correspond with the lines DD'

Next draw (Fig. 86) $O''A$ equal to $O''A$ (Fig. 85), and with O'' as centre and radius $O''A$ describe an arc DAD , and with same centre and $O''A'$ (Fig. 85) as radius describe an

FIG. 86.



arc $D'A'D'$. Mark off, right and left of A , AB , BC , and CD , each equal to Ab (Fig. 85), one of the equal parts into which the quadrant AD is divided. Join DO'' , CO'' , BO'' right and left of AO'' , and produce $O''A$, $O''B$, $O''C$ indefinitely. Make AE equal to AE (Fig. 85); BB'' equal to AB'' (Fig. 85); and CC'' equal to AC'' (Fig. 85); and through D , C'' , B'' , E , B'' , C'' , D , draw an unbroken curved line. Then $DEDD'A'D'$ is the required pattern for back of bath.

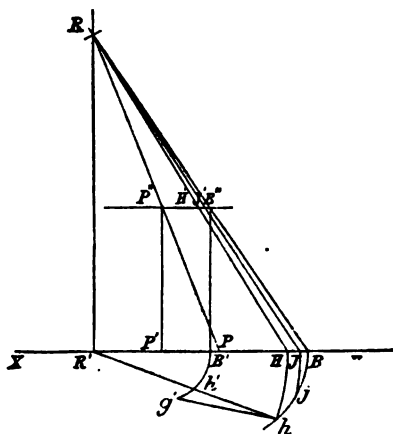
To draw the pattern for a side and a half-front.

METAL-PLATE WORK.

Joining $h g'$ completes the portion of Fig. 84 required. Now from D' and Q' draw $D'D''$, $Q'Q''$ perpendicular to XX and each equal to the given height of the DBD portion of the bath. Join DD'' , QQ'' ; produce them to intersect in S (§ 80, p. 158); and from S let fall SS' perpendicular to XX . Join $S'g'$, and produce it to cut arc Dh in g , then g and g' will be corresponding points. Divide Dg into any number of equal parts, here three, in the points e and f . Join $Q''D''$. With S' as centre and $S'e$, $S'f$, and $S'g$ successively as radii describe arcs cutting XX in E , F , and G ; join these points to S by lines cutting $Q''D''$, produced, in E' , F' , and G' . With S' as centre and $S'h$ as radius describe an arc cutting XX in H , and join HS .

Next draw separately $B'Bhg'$ (Fig. 88) one of the $B'Bhg'$ portions of Fig. 84, thus. Draw any line XX and

FIG. 88.

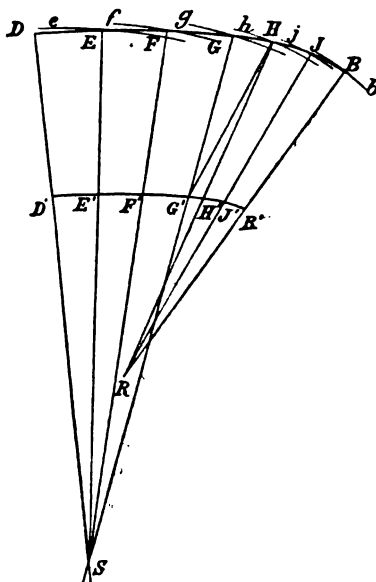


with any point P' (to correspond with P, Fig. 84) in it as centre and PB (Fig. 84) as radius describe an arc B*h* equal to B*h* of Fig. 84. Make BP' equal to B P' (Fig. 84), and with P' as centre and P' B' (Fig. 84) as radius describe an arc

$B'g'$ equal to $B'g'$ of Fig. 84. Joining hg' completes the portion of Fig. 84 required. Now from B' and P' draw $B'B''$, $P'P''$ perpendicular to XX and each equal to the given height of the DBD portion of the bath. Join BB'' , PP'' ; produce them to intersect in R (§ 80, p. 158); and from R let fall RR' perpendicular to XX . Join $R'h$, cutting arc $B'g'$ in h' , then h and h' will be corresponding points. Divide Bh into any number of equal parts, here two, in the point j . Join $P''B''$. With R' as centre and $R'j$ and $R'h$ successively as radii describe arcs cutting XX in J and H ; join these points to R by lines cutting $P''B''$ in J' and H' .

Next draw (Fig. 89) a line DS equal to DS (Fig. 87).

FIG. 89.



With S as centre and SE , SF , SG , and SH (Fig. 87) successively as radii describe arcs e , f , g , and h . With D as centre

and radius equal to $D e$ (Fig. 87) describe an arc cutting arc e in E , and with same radius and E as centre describe an arc cutting arc f in F . With F as centre and same radius describe an arc cutting arc g in G , and with G as centre and radius $g h$ (Fig. 87) describe an arc cutting arc h in H . Join the points E, F , and G (not H) to S . Make SD' equal to SD'' (Fig. 87); and make SE', SF' , and SG' respectively equal to SE', SF' , and SG' (Fig. 87). Through the points D, E, F, G, H draw an unbroken curved line. Also through points D', E', F', G' draw an unbroken curved line, and join $H G'$. This completes the side pattern, to which we have now to attach, at $H G'$, a half-front pattern. With H and G' as centres and radii respectively equal to $H H'$ and $g' h'$ (Fig. 88) describe arcs intersecting in H' . Join $H H'$; produce it indefinitely and make $H R$ equal to $H R$ (Fig. 88). With R as centre and $R J, R B$ successively as radii describe arcs j and b . With H as centre and $h j$ (Fig. 88) as radius describe an arc cutting arc j in J , and with same radius and J as centre describe an arc cutting arc b in B . Join the points J and B to R , and make $R J'$ and $R B'$ respectively equal to $R J'$ and $R B''$ (Fig. 88). Through H, J , and B draw an unbroken curved line. Also through H', J' , and B' draw an unbroken curved line. Then $D F H B B' G' D'$ is the complete pattern required.

PROBLEM XXIX.

To draw, without long radii, the pattern for an Oxford hip-bath; given dimensions as before.

This problem is a second case of the preceding.

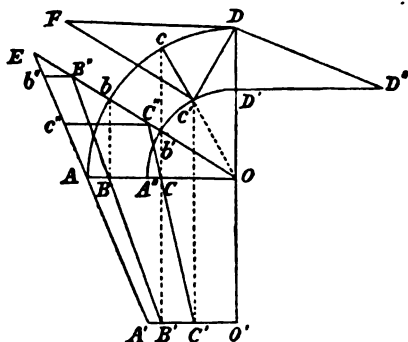
Patterns when the body is to be made up of three pieces with seams as in the preceding problem.

To draw the pattern for the back.

Draw $E A A' O' O$ (Fig. 90) the elevation of the back as in Fig. 85, and produce $O' O$. With O as centre and $O A$ as

radius describe a quadrant AD (corresponding with AD , Fig. 84), and divide it into any number of equal parts, here three, in the points b and c . Join bO , cO , and with O as centre and radius $O'A'$ describe a quadrant $D'A'$ (corresponding with $D'A'$, Fig. 84), and cutting lines Oc , Ob in c' and b' respectively. Then $DD'A'A$ will be the plan of that portion of the back of the bath of which $O'A'O A$ is the elevation. Through b and c draw bB and cC perpendicular to OA ; and through b' and c' draw $b'B'$ and $c'C'$ perpendicular to $O'A'$. (Here part of $b'B'$ happens to coincide with

FIG. 90.

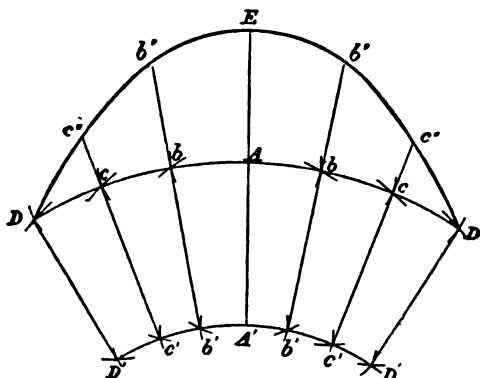


part of $c'C$). Join $B'B$, $C'C$, and produce them to meet OE in B'' and C'' respectively. Through C'' draw $C''c''$ parallel to OA , and through B'' draw $B''b''$ parallel to OA . Through D' draw $D'D''$ perpendicular to $D'D$ and equal to the given height of that portion of the bath, and join DD'' , then DD'' will be the true length of $D'D$. Join Dc' ; through c' draw $c'F$ perpendicular to $c'D$ and equal to the given height, and join DF , then DF may be taken as the true length of Dc' .

In drawing the pattern, we will first set out that for the $OA'O'A'$ portion of the back, and then attach to it the pattern for the $OE A$ portion. It is evident that the $OA'O'A'$

portion is half a right cone frustum, and therefore its pattern can be drawn (see Problem VIII., p. 41), thus. Draw (Fig. 91) a line DD' (the line DD' left of EA') equal to DD'' (Fig. 90). With D and D' as centres and radii respectively equal to DF and $D'c'$ (Fig. 90) describe arcs intersecting in c' . With D' and D as centres and radii respectively equal to DF and Dc (Fig. 90) describe arcs intersecting in c . To find points b and b' proceed as just explained and with the same radii, but c and c' as centres instead of D and D' .

FIG. 91.



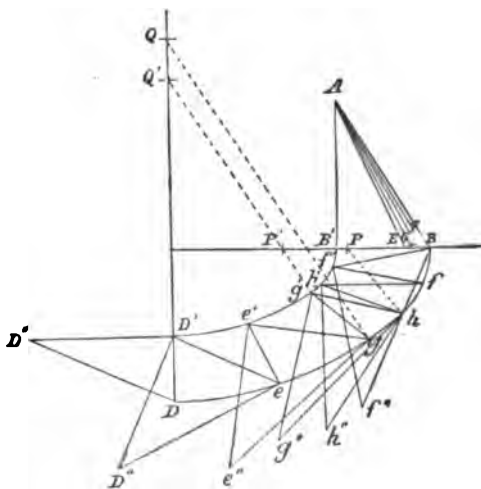
Similarly to find A and A' , and the points b and b' , &c., on the right-hand side of AA' . Through the points D' , c' , b' , A' , b' , c' , D' draw an unbroken curved line. Also through D , c , b , A , b , c , D draw an unbroken curved line, and join DD' . This completes the pattern for the $OA O'A'$ portion of back of bath, to which we have now to attach, at $DA D'$, the pattern for the OEA portion. Join AA' ; produce it indefinitely and make AE equal to AE (Fig. 90). Next join, right and left of AA' , $b'b$, $c'c$, and produce them indefinitely; make $b'b''$, right and left of AA' , equal to Ab'' (Fig. 90). Also make $c'c''$, right and left of AA' , equal to Ac'' (Fig. 90), and

through the points $D, c'', b'', E, b'', c'', D$ draw an unbroken curved line; then $D E D D' A' D'$ is the complete pattern required.

To draw the side and half-front pattern.

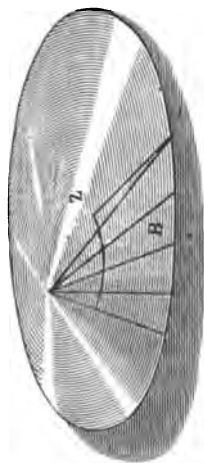
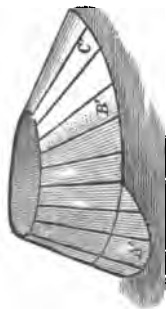
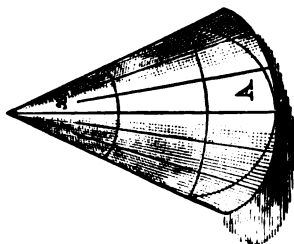
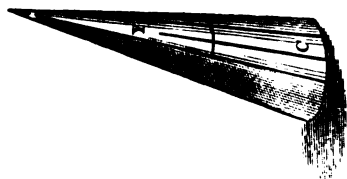
Draw separately (Fig. 92) the $B h D D' g' B'$ portion of Fig. 84, that is, the plan, of a side and half-front of the bath. Join $h g'$ (as was done in Fig. 84). We now have to get at corresponding points in the arcs $D h, D' g'$, and also in the arcs $h B, g' B$. We do this by the method given in § 85, p. 202,

FIG. 92.



thus. Join $Q' g'$ and through Q draw $Q g$ parallel to $Q' g'$ and cutting arc $D h$ in g ; then the arcs $D g$ and $D' g'$ will be the proportional. Now divide arc $D g$ into any number of equal parts, here two only to avoid confusion, in the point e , and divide the arc $D' g'$ into two equal parts in the point e' ; then e, e' and g, g' are corresponding points. Next join $P h$ and through P' draw $P' h'$ parallel to $P h$ and cutting arc $B' g'$ in h' . Divide arcs $B h$ and $B' h'$ each into the same number of equal

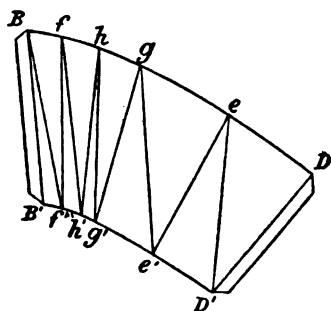
PLATE IV. (see p. 217).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (Part of Exhibit by the Author at the *Inventions Exhibition*, 1885.)

parts, here two, in the points f and f' respectively; then f, f' and h, h' are corresponding points. Join ee', gg', ff' , and hh' . Through B' draw $B'A$ perpendicular to $B'B$ and equal to the given height of that portion of the bath; and from B' along $B'B$ mark off $B'F, B'H, B'G$, and $B'E$ respectively equal to ff', hh', gg' , and ee' . Join BA, FA, HA, GA and EA ; then BA, FA, HA, GA , and EA will be respectively the true lengths of BB', ff', hh', gg' , and ee' . Join $f'B, h'f', e'g$, and $D'e$; through f' draw $f'f''$ perpendicular to $f'B$, and equal to the given height, and join Bf'' , then Bf'' may be taken as the true length of $f'B$. Through h', g', e' , and D' draw $h'h'', g'g'', e'e''$, and $D'D''$, perpendicular to $h'f', g'h', e'g$, and $D'e$ respectively, and each equal to the given height; also draw $D'D''$ perpendicular to DD' and equal to the given height; and join $fh'', hg'' g'e'', eD''$, and DD'' ; then $fh'', hg'', g'e'', eD''$, and DD'' may be taken as the true lengths of $h'f', g'h', e'g, D'e$, and $D'D$ respectively.

FIG. 93.



Next draw (Fig. 93) BB' equal to BA (Fig. 92), and with B and B' as centres and radii respectively equal to Bf'' and $B'f'$ (Fig. 92) describe arcs intersecting in f' , and with f' and B as centres and radii respectively equal to FA and Bf (Fig. 92) describe arcs intersecting in f . With f and f' as centres and radii respectively equal to fh'' and $f'h'$ (Fig. 92)

describe arcs intersecting in h' , and with h' and f as centres and radii respectively equal to HA and fh (Fig. 92) describe arcs intersecting in h . With h and h' as centres and radii respectively equal to hg'' and $h'g'$ (Fig. 92) describe arcs intersecting in g' , and with g' and h as centres and radii respectively equal to GA and hg (Fig. 92) describe arcs intersecting in g . With g and g' as centres and radii respectively equal to ge'' and $g'e'$ (Fig. 92) describe arcs intersecting in e' , and with e' and g as centres and radii respectively to EA and ge (Fig. 92) describe arcs intersecting in e . With e and e' as centres and radii respectively equal to eD'' and $e'D'$ (Fig. 92) describe arcs intersecting in D' , and similarly with D' and e as centres and radii respectively equal to DD'' and eD (Fig. 92) describe arcs intersecting in D . Join DD' . Through B, f, h, g, e, D draw an unbroken curved line. Also through B', f', h', g', e', D' draw an unbroken curved line. Then $BgD'D'g'B'$ is the pattern required.

PROBLEM XXX.

To draw the pattern for an oblong taper bath, the like dimensions to those for Problem XIII. being given.

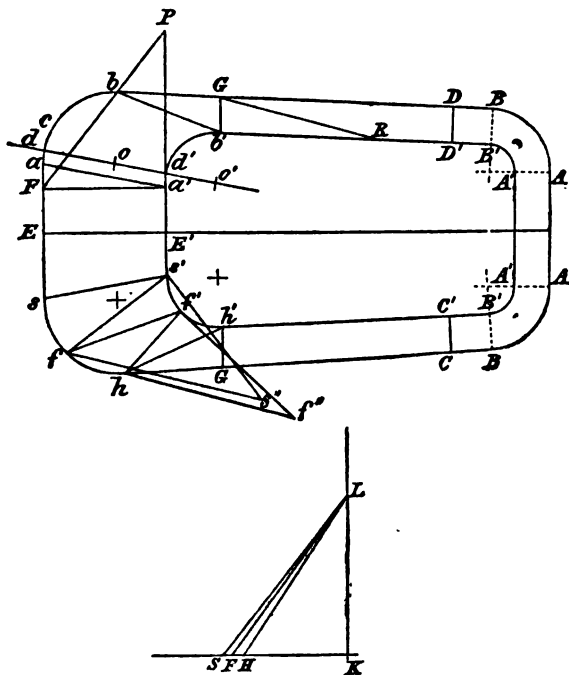
Again, it is only necessary to treat of two cases—one in this problem, and one in the problem following (see also § 79, p. 157).

Draw (Fig. 94) the plan of the body (see Problem XIII., p. 140), preserving of its construction the centres O, O' ; and the points $b, b', a, a', s, s', h, h', A, A', B, B'$, in which the straight lines and arcs meet each other. Join $b, b', a, a', s, s', h, h', AA'$ (two places), and BB' (two places) as shown in the fig. Examining the plan we see (*d*, p. 55) that each round corner $AA'BB'$ of the toe is the same portion of a right cone frustum; and each of the round corners $aa'b'b$, $ss'h'h$, of the head are the same portion (*g*, p. 129) of an oblique cone frustum. As we proceed it will be seen that

the construction of the pattern for the round corners of the head of bath is exactly the same as that for the round corners of the body in Problem XXVI. (see also § 86, p. 206).

In Plate V. (p. 237) is a representation of an oblong

FIG. 94.



taper bath, also of an oblique cone Z, the A portion of which corresponds to the A' portion of the body, and the development of the former is the development of the latter.

Patterns when the body is to be made up of four pieces.

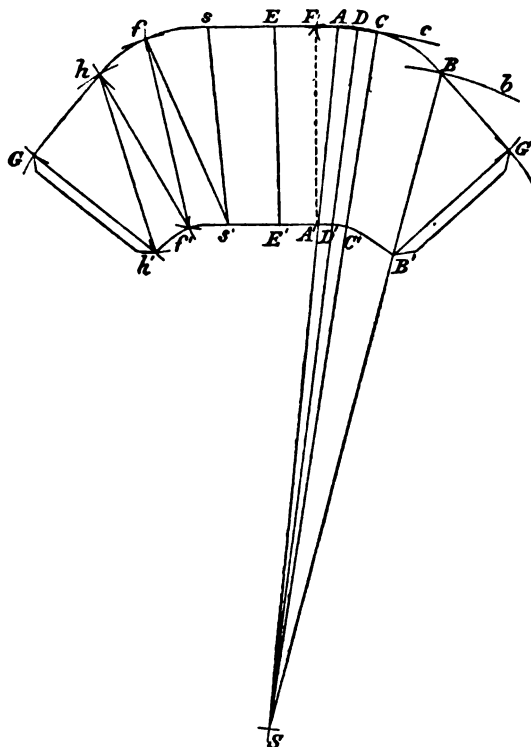
We will put the seams to correspond with the lines GA' , GB' , DD' , and CC' . The patterns required will be three,

ba in d ; make db and da (Fig. 95) equal respectively to db and da (Fig. 94). Now (Fig. 95) make dO' equal to dO' (Fig. 94), and with O' as centre and $O'a'$ (Fig. 94) as radius describe an arc $b'a'$. Make $d'b'$ and $d'a'$ equal respectively to $d'b'$ and $d'a'$ (Fig. 94). Joining bb' , aa' completes the portion of Fig. 94 required. Now divide (Fig. 94) ba into any number of parts. It is convenient to take d as one of the division points, and to make dc equal to da ; leaving cb without further division, thus making the division of ba into three portions, not all equal. In actual practice the dimensions of the work will suggest the number of parts necessary. Here bc is left without further division in order to make clear the correspondence of this problem to Problem XXVI., p. 205. Now (Fig. 95) make dc equal to dc (Fig. 94), and then ba will be divided correspondingly to ba (Fig. 94). Draw XX parallel to $S'd$; and at d and O draw dD , OQ perpendicular to $S'd$; and meeting XX in D and Q ; also through d' and O' draw $d'D'$, $O'O''$ perpendicular to $S'd$; the line $O'O''$ cutting XX in Q' . Make $Q'O''$ equal to the given height of the bath, and draw $O'D'$ parallel to XX , and cutting $d'D'$ in D' . Join DD' , QO'' ; produce them to intersect in S (§ 80, p. 158); and from S let fall a perpendicular to $S'd$, cutting $S'd$ in S' . With S' as centre and $S'a$, or $S'c$ (which is equal to $S'a$) and $S'b$ successively as radii describe arcs cutting $S'd$ in g and f . Draw gC , fB perpendicular to XX and cutting it in C and B ; and join CS , BS , cutting $O'D'$ in C' and B' .

Next draw SD (Fig. 96) equal to SD (Fig. 95) and with S as centre and SC , SB (Fig. 95) successively as radii describe arcs c and b . With D as centre and radius equal to da (Fig. 95) describe arcs cutting arc c in A and C . With C as centre and radius cb (Fig. 95) describe an arc cutting arc b in B . Join A , C , and B to S . Make SD' equal SD' (Fig. 95) and with S as centre and SC' (Fig. 95) describe an arc (not shown in the fig.) cutting SA and SC in A and C respectively; make SB' equal to SB' (Fig. 95).

Through A, D, C, and B draw an unbroken curved line. Also through A', D', C', and B' draw an unbroken curved line; this will complete the pattern for a head-corner. To attach the patterns for the flat portions of the head to A A'

FIG. 96.



and B B' respectively; draw through a' (Fig. 94) a line $a'F$ perpendicular to $a's'$, and through b' draw $b'G$ perpendicular to $b'D'$.

Now draw (Fig. 94) $a'P$ perpendicular to $a'F$ and equal

to the given height of the bath, and join $F P$, then $F P$ is the true length of $F a'$.

Next draw $b' R$ perpendicular to $b' G$ ($b' R$ will of course coincide with the line $b' B'$) and equal to the given height; join $G R$, then $G R$ is the true length of $G b'$. Now with B' (Fig. 96) as centre and $G R$ (Fig. 94) as radius, and B as centre and radius $b G$ (Fig. 94), describe arcs intersecting in G . Join $B G$, $B' G$. With A' (Fig. 96) as centre and $F P$ (Fig. 94) as radius, and A as centre and radius $a F$ (Fig. 94) describe arcs intersecting in F . Join $A F$ and produce it indefinitely, and make $A s$ equal to $a s$ (Fig. 94); through A' draw $A' s'$ parallel to $A s$ and equal to $a' s'$ (Fig. 94); and join $s s'$. The pattern for the portion, seen in plan in Fig. 94, $G b a s s' a' b'$ of the head of the bath is now completed. It is needless to work out in detail the addition of the portion (Fig. 96) $s h G h' f' s'$ of the pattern, by which we complete the head pattern $G E G h' E' B'$. The extra lines in this latter portion of the pattern appertain to the next problem.

PROBLEM XXXI.

To draw, without long radii, the pattern for an oblong taper bath; given dimensions as in Problem XXX.

This problem is a second case of the preceding.

Patterns when the body is to be made up of four pieces with seams as in preceding problem.

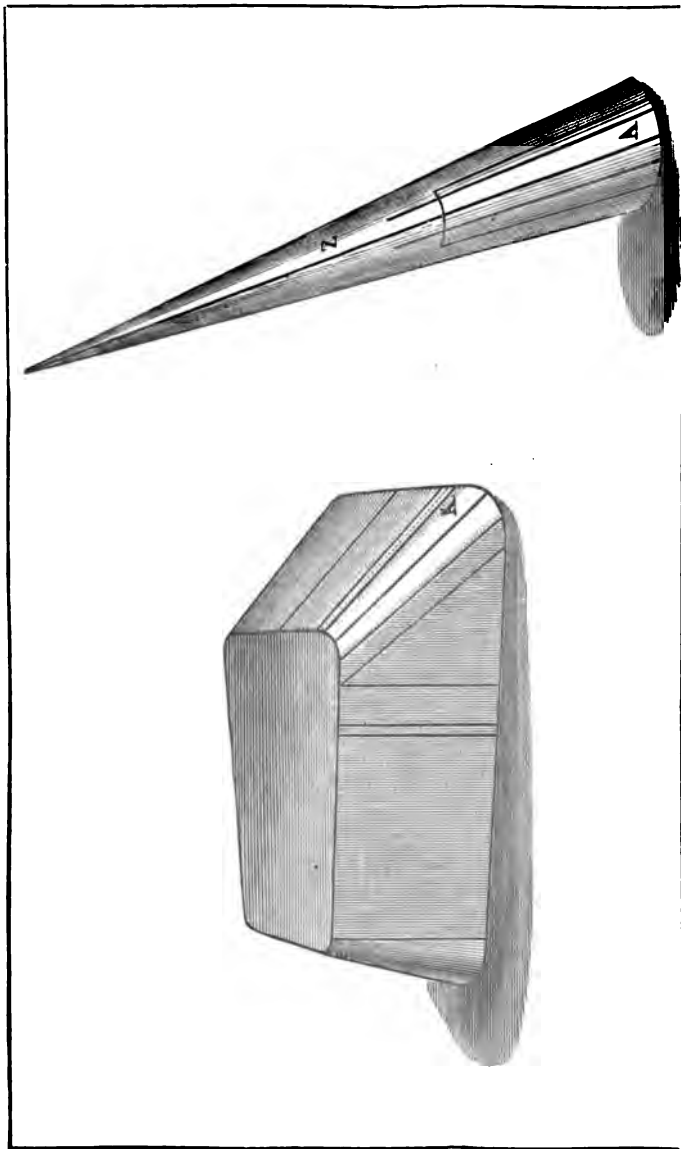
Again, the patterns required will be three; one for the head of the bath, one for the toe, and one for the sides. The latter pattern needs no description. The pattern for the toe can be readily drawn by Problem XXVIII., p. 94. The pattern for the head can be drawn as follows.

Draw half the plan of the bath, as the lower half of Fig. 94, and divide the arcs $s h$, $s' h'$, each into any number of equal parts, here two, in respectively the points f and f' , and join $f f'$. Draw (Fig. 94a) any two lines $K S$, $K L$ perpendicular to each other, and make $K L$ equal to the given

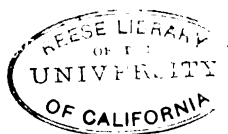
height of the bath. From K along KS mark off KH equal to $h'h'$, KF equal to $f'f'$, and KS equal to $s's'$; and join LH, LF, LS; then LH, LF, and LS are the true lengths of $h'h'$, $f'f'$, and $s's'$ respectively. Next join (Fig. 94) $f'h$, $s'f$; draw $f'f''$, $s's''$, perpendicular to $f'h$, $s'f$ respectively, and each equal to the given height; and join hf'' , fs'' ; then hf'' , fs'' may be taken respectively as the true lengths of $f'h$ and $s'f$. The true length of $h'G$ may be found along $b'B'$ as was that of bG in Problem XXX. along $b'B'$; it will of course be equal to GR, and we shall speak of it as GR.

Next draw (see Fig. 96, left-hand portion) ss' equal to LS (Fig. 94a), and with s' and s as centres and radii respectively equal to fs'' and sf (Fig. 94) describe arcs intersecting in f . With f and s' as centres and radii respectively equal to LF (Fig. 94a) and $s'f'$ (Fig. 94) describe arcs intersecting in f' ; and with f' and f as centres and radii respectively equal to hf'' and fh (Fig. 94) describe arcs intersecting in h ; also with h and f' as centres and radii respectively equal to LH (Fig. 94a) and $f'h'$ (Fig. 94) describe arcs intersecting in h' . With h' and h as centres and radii respectively equal to GR and hG (Fig. 94) describe arcs intersecting in G. Through s, f, h , draw an unbroken curved line. Also through s', f', h' draw an unbroken curved line; and join hG , Gh' . Then $s h G h' f' s'$ is the pattern for the portion of the head of the bath represented in plan in Fig. 94 by the same lettering. It is unnecessary to pursue the pattern further.

PLATE V. (see p. 231).



DRAWINGS FROM MODELS IN SOUTH KENSINGTON MUSEUM. (Part of Exhibit by the Author at the *Inventions Exhibition*, 1885.)



BOOK III.

CLASS III.

PATTERNS FOR MISCELLANEOUS ARTICLES.

THE book we now reach is made up of miscellaneous problems, all of which are of considerable practical importance, as well as are also typical cases.

PROBLEM I.

To draw a pattern for the elbow formed by two equal circular pipes (cylinders of equal diameters) which meet at any angle.

First draw (Fig. 1) a side elevation and a part-plan of the elbow, as follows. Draw any two lines $G'G'$, $O'G'$ at an angle to each other equal to the given angle of the elbow; through any point G in $G'G'$ draw GA perpendicular to $G'G'$ and equal to the diameter of the pipes; and through A draw an indefinite line AA' parallel to $G'G'$. Then from any point O in $G'O$ draw ON perpendicular to $G'O$ and equal to the diameter of the pipe; and through N draw an indefinite line NA parallel to $G'O$ and meeting AA' in A' . Join $A'G'$ (the line $A'G'$ produced always bisects the angle $G'G'O$); then $ONAG$ is a side elevation of the elbow, and the line $A'G'$ represents the joint or 'mitre' of the pipes. On GA describe a semicircle GdA (this will be the part-plan of the elbow); which divide into any number, here six, of equal parts in the points b, c, d, e, f . From each of these points draw lines parallel to AA' , namely, the lines bB' , cC' , dD' , eE' , and fF' , cutting GA in the points B, C, D, E , and F respectively.

FIG. 1.

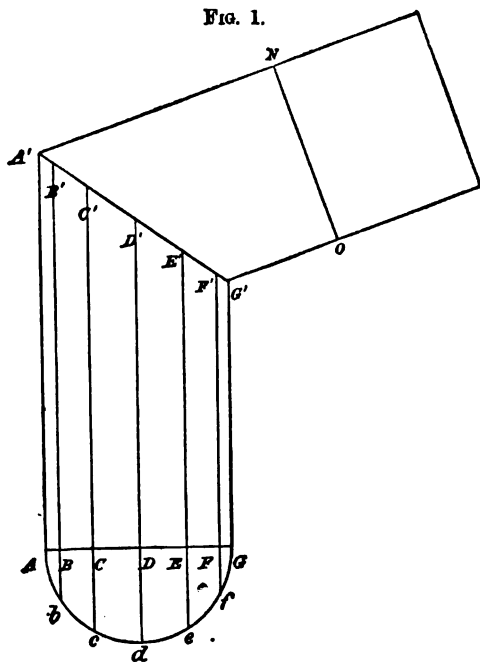
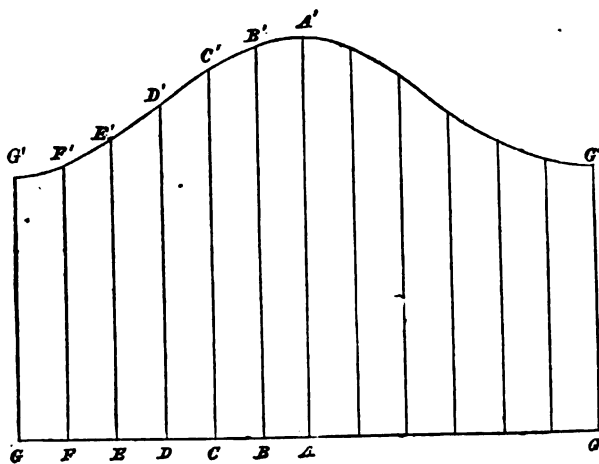


FIG. 2.



To draw the pattern, with the longitudinal seams of the pipes to correspond with the lines $G'G$, $G'O$. Draw (Fig. 2) an indefinite line GG , and, from a point G in it set off along it distances GF , FE , ED , DC , CB , and BA , equal respectively to Gf , fe , ed , dc , cb , and bA of Fig. 1. For small work the chord distances round the semicircle give satisfactory results; of large work we shall speak immediately. Through G , F , E , D , C , B , and A draw indefinite lines GG' , FF' , EE' , DD' , CC' , BB' , and AA' perpendicular to GG ; and make GG' equal to GG' (Fig. 1), FF' equal to FF' (Fig. 1), and EE' , DD' , CC' , BB' , and AA' equal respectively to EE' , DD' , CC' , BB' , and AA' (Fig. 1). From G' through F' , E' , D' , C' , B' , and A' draw an unbroken curved line; this completes one-half the pattern required. The half to the right of AA' can be drawn in similar manner, setting off along AG the same above distances, but in reverse order and starting from A .

(87.) The line GG (Fig. 2) must of course be equal to the circumference of the pipe. Therefore if a somewhat close accuracy is required, as in large work, it will be found best to make GG equal to the circumference of the pipe (Problem XI., p. 12); then to divide GG into twice as many equal parts as the semicircle AdG (Fig. 1) is divided into; then through each division point to draw lines perpendicular to GG and proceed as above explained.

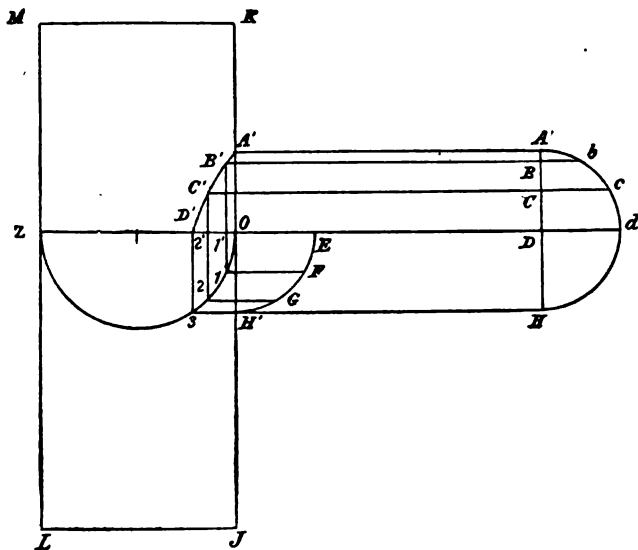
(88.) By making a start in Fig. 2 with GG' for an outer line of the pattern, we ensure that the longitudinal seams of the pipes shall, as we desired, correspond with GG' and $G'O$ of Fig. 1. If the seams are to correspond with the lines $A'A$ and $A'N$ of this fig., we should letter our commencing indefinite line, not GG , but AA , and should start with, for an outer line of the pattern, a perpendicular AA' equal to AA' of Fig. 1. The pattern we should then get would be like to that obtained by dividing Fig. 2 into halves along AA' , turning each portion half-way round, and making the lines GG' coincide.

PROBLEM II.

To draw the pattern for the T-piece formed by two equal or unequal circular pipes (cylinders of equal or unequal diameter), which meet at right angles.

First draw (Fig. 3) a side elevation and a part-plan of the two circular pieces of pipe, which we will suppose unequal, thus. Draw two indefinite lines Zd , and KJ , intersecting each other at right angles in O . Make OZ equal to the

FIG. 3.



diameter of the larger pipe, and through Z draw an indefinite ML parallel to KJ . Make OA' and OH' each equal to half the diameter of the smaller pipe, and through A' and H' draw indefinite lines $A'A$ and $H'H$ each perpendicular to KJ . In $A'A$ take any point A , on $H'H$ set off $H'H$ equal

to $A'A$, and join AH cutting Od in D ; then $A'AHH'$ will represent, in elevation, a piece of the smaller pipe. Next in $A'K$ take any point K , and through K draw KM perpendicular to KJ and meeting ML in M ; also in $H'J$ take any point J , and through J draw JL perpendicular to KJ and meeting ML in L ; then $MKJL$ will represent, in elevation, a piece of the larger pipe, and $MKA'AHH'JL$ a side elevation (except the curve of junction) of the T-piece. With D as centre and radius DA , that is, half the diameter of the smaller pipe, describe a semicircle $A d H$; divide the quadrant $A d$ of it into any number of equal parts, here three, in the points b and c ; and through b and c draw indefinite lines bB' and cC' parallel to $A'A$. Now on ZO describe a semicircle $Z3O$ (this will be a part-plan of the large pipe), and with O as centre and radius DA describe a quadrant $H'E$ (this may be regarded as a part-plan of the smaller pipe) which divide, exactly as quadrant $A d$ was divided in the points F and G ; through F , G , and H' draw lines $F1$, $G2$, and $H'3$ parallel to Zd and cutting the semicircle $Z3O$ in points 1, 2, and 3. Through point 1 draw a line $1B'$ parallel to KJ and meeting bB' in B' ; through 2 draw $2C'$ parallel to KJ and meeting cC' in C' ; and through 3 draw $3D'$ parallel to KJ and meeting dD' in D' . From D' to A' through the points C' and B' draw an unbroken curved line, then $A'C'D'$ is the elevation of one-half of the curve of junction of the two pipes. In practice it is only necessary to draw the $A'ODA$ (Fig. 3) portion of the elevation of the smaller pipe. The other half elevation $H'ODH$ of it is drawn here simply to make the full side-elevation of the T-piece clearer.

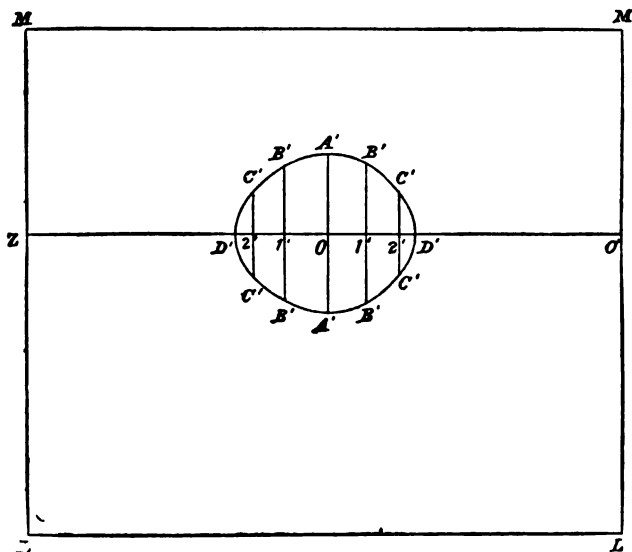
To get at the whole T-piece, it is evident that we require two patterns, one for the smaller piece of pipe, up to its junction with the larger, and one for the larger with the hole in it that the smaller pipe fits to.

To draw the pattern for the larger pipe, with the longitudinal seam to correspond with the line ML .

First set out apart from the pipe itself the shape for the

hole in it. Draw (Fig. 4) two indefinite lines ZO' and $A'A'$ intersecting at right angles in O ; from O , on ZO' , right and left of $A'A'$ set off distances $O1'$, $1'2'$, and $2'D'$ equal respectively to $O1$, 12 , and 23 (Fig. 3), that is, to the actual distances on the round curve of the pipe at ZO that the lines $1B'$, $2C'$, and $3D'$ are apart. Through points $1'$ and $2'$

FIG. 4.



right and left of $A'A'$ draw $B'B'$ and $C'C'$ perpendicular to ZO' . Make $1'B'$, above and below ZO' , and right and left of $A'A'$, equal to $1'B'$ (Fig. 3); and make $2'C'$, above and below ZO' , and right and left of $A'A'$, equal to $2'C'$ (Fig. 3) and through all the points as above found, namely D' , C' , B' , A' , B' , C' , D' , C' , B' , &c., draw an unbroken curved line; then $D'A'D'A'D'$ will be the shape of the hole required.

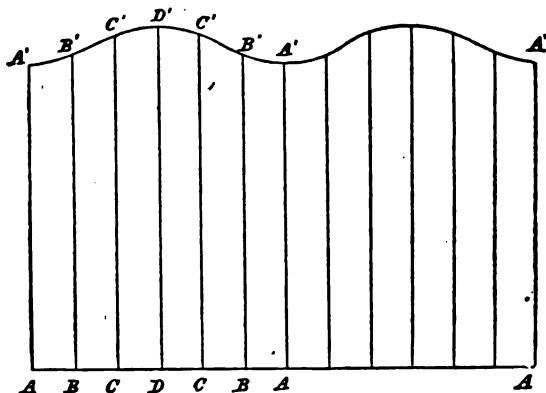
To complete the pattern for the $MKJL$ (Fig. 3) piece of the larger pipe, make OZ and OO' each equal to half

its circumference (Problem XI., p. 12); and through Z and O' draw indefinite lines ML perpendicular to ZO'. Make ZM of left-hand line ML, and O'M of right-hand line ML each equal to ZM (Fig. 3); similarly make ZL and O'L each equal to ZL (Fig. 3). Then MLLM will complete the pattern required.

(89.) We have shown how to mark out by itself the hole the larger pipe, because in cases where the pipe is already made up, it is convenient to be able to mark out the shape of the hole apart from the pipe, on, say, a thin piece of sheet metal, which shape can then be cut out and used as a template; being applied to the pipe and bent to it, and the shape of the hole marked on it from the template. Even when the pipe is not made up, it is useful when the pipe is large to be able to mark out the hole quite apart from the pipe itself.

To draw the pattern for the smaller piece of pipe, the longitudinal seam to correspond with the line A'A (Fig. 3).

FIG. 5.



Draw (Fig. 5) an indefinite line A A. In it take any point D, and from D, right and left, set off distances DC,

CB , BA , equal respectively to dc , cb , and ba (Fig. 3), that is, equal to one another; and from the point D and each of the points C , B , and A draw lines perpendicular to AA . Make DD' equal to DD' (Fig. 3), and the lines CC' , BB' , AA' , right and left of DD' , equal respectively to CC' , BB' , and AA' (Fig. 3). From either point A' to A' on the other side of DD' draw through B' , C' , D' , C' , and B' an unbroken curved line; then $AA'A'A$ will be half the required pattern. The other half can be similarly drawn. If a somewhat close accuracy in the length of AA (Fig. 5) is required proceed as in Problem I. (§ 87, p. 241). See also § 88, p. 241.

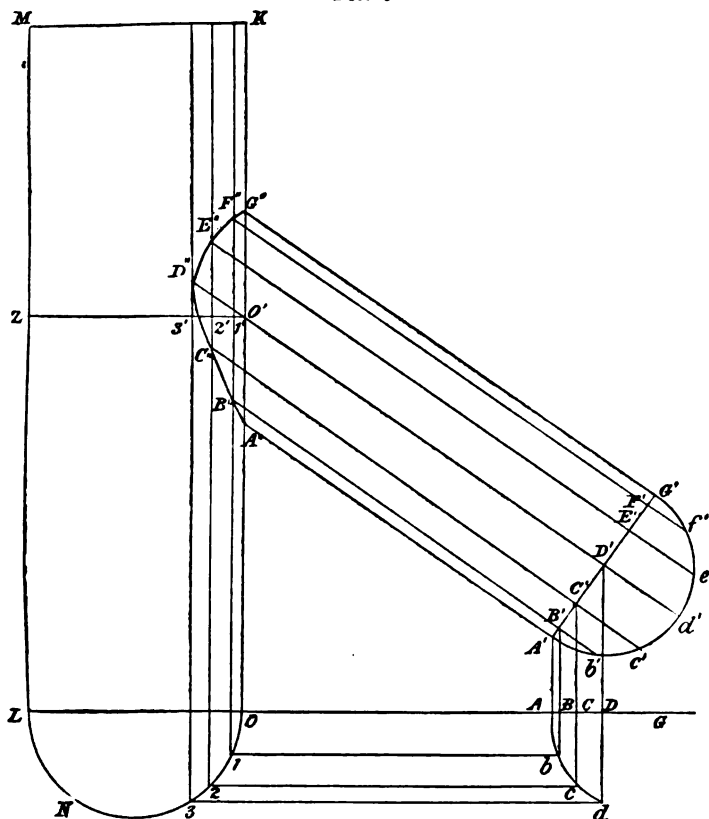
PROBLEM III.

To draw the pattern for the 'slanting T' formed by two equal or unequal circular pipes (cylinders of equal or unequal diameter) which meet otherwise than at right angles.

We will suppose the pipes unequal. First draw (Fig. 6) a side elevation and a part-plan of the pipes, thus. Draw two indefinite lines KO , and $A''A'$ at that angle to one another that the pipes are to be, the angle $OA''A'$, say. In $A''A'$ take any point A' : through it draw $A'G'$ perpendicular to $A''A'$ and equal to the diameter of the smaller pipe, and through G' draw a line parallel to $A''A'$ and meeting KO in G'' ; then $A''A'G'G''$ will represent, in elevation, a piece of the smaller pipe. Next in KO take any point O , and through it draw an indefinite line LG perpendicular to KO . Make OL equal to the diameter of the larger pipe; through L draw an indefinite line LM parallel to KO ; in LM take any point M , and through M draw MK perpendicular to ML . Then $MKOL$ will represent, in elevation, a piece of the larger pipe; and $MKG''G'A'A''OL$ a side elevation (except the curve of junction) of the slanting T . On $A'G'$ describe a semicircle $A'd'G'$; and divide it into any number of equal parts, here six, in the points

$b', c', d', e',$ and f' ; and through these division points draw indefinite lines $b' B'', c' C'', d' D'', e' E'',$ and $f' F''$ parallel to $G' G''$, and cutting $A' G'$ in the points $B', C', D', E',$ and F' .

FIG. 6



Now from A' let fall a perpendicular to LG meeting LG in A , and through B', C' and D' draw indefinite lines $B' b, C' c,$ and $D' d$ perpendicular to and cutting LG in $B, C,$ and D .

Make Bb equal to $B'b'$, Cc equal to $C'c'$, and Dd equal to $D'd'$; and from A through b and c to d draw an unbroken curved line. On LO describe the semicircle $L3O$, and through the points b, c, d , just found draw lines $b1, c2$, and $d3$ parallel to LG and cutting the semicircle $L3O$ in points $1, 2$, and 3 ; then $LN3dD$ is a part-plan of the slanting T . Through point 1 draw a line $1F''$ parallel to OK and meeting $B'B''$ and $F'F''$ in B'' and F'' respectively; through 2 draw $2E''$ parallel to OK and meeting $C'C''$ and $E'E''$ in C'' and E'' respectively; and through 3 draw $3D''$ parallel to OK and meeting $D'D''$ in D'' . From A'' through the points $B'', C'', D'', E'',$ and F'' to G'' draw an unbroken curved line, then this curve is the elevation of the curve of junction of the two pipes. In the straight line $A''G''$ take any point O' , about midway between A'' and G'' , and through it draw $O'Z$ perpendicular to KO , and cutting lines $1F'', 2E'',$ and $3D''$ in points $1', 2',$ and $3'$.

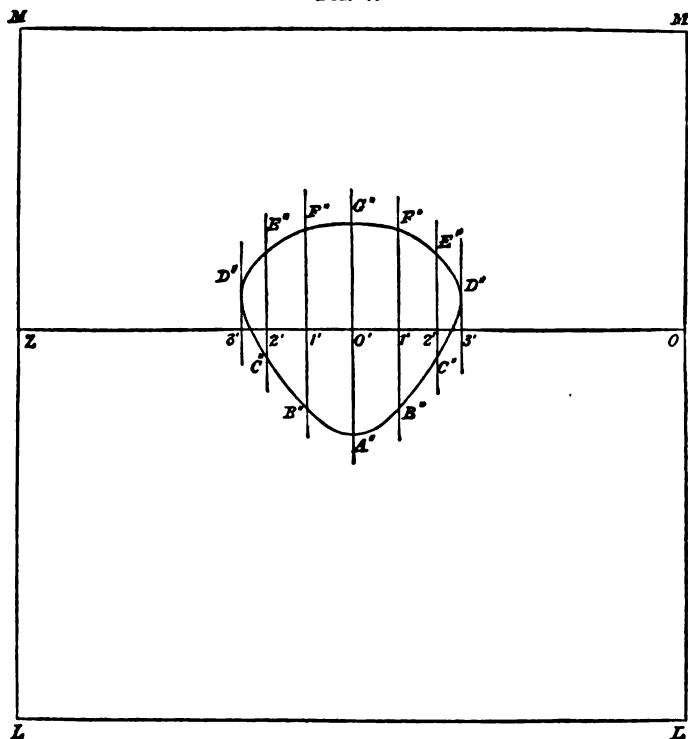
It is evident that for slanting T we require two patterns, one for the smaller piece of pipe up to its junction with the larger, and one for the larger with the hole in it that the smaller pipe fits to.

To draw the pattern for the larger pipe, the longitudinal seam to correspond with the line ML .

First to set out apart from the pipe itself the shape of the hole in it. Draw (Fig. 7) two indefinite lines ZO and $G''A''$ intersecting at right angles in O' ; and from O' , on ZO , right and left of $G''A''$ set off distances $O'1', 1'2',$ and $2'3'$ equal respectively to $O1, 12,$ and 23 (Fig. 6), that is, to the actual distances on the curve of the pipe at ZO that the lines $B''F'', C''E'',$ and $3'D''$ are apart. Through points $1', 2',$ and $3'$, right and left of $G''A''$ draw $F''B'', E''C'',$ and $D''3'$ perpendicular to ZO . Make $O'G''$ equal to $O'G''$ (Fig. 6), $O'A''$ equal to $O'A''$ (Fig. 6). Also make $1'F'', 2'E'', 3'D'',$ right and left of $G''A''$ and above ZO , equal respectively to $1'F'', 2'E'', 3'D''$ above ZO' (Fig. 6); and $1'B'', 2'C''$ right and left of $G''A''$ and below ZO , equal respectively to $1'B'', 2'C'',$ below ZO' (Fig. 6). Through all the points as above

found, namely D'' , E'' , F'' , G'' , F'' , E'' , D'' , C'' , B'' , &c., draw an unbroken curved line; then $D''G''D''A''$ will be the shape of the hole required.

FIG. 7.



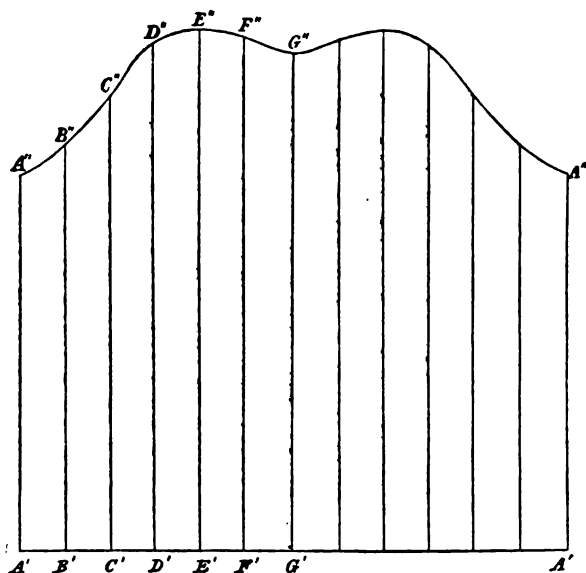
To complete the pattern for the $MKOL$ (Fig. 6) piece of the larger pipe, make $O'Z$ and $O'O$ each equal to half its circumference (Problem XI., p. 12); and through Z and O draw indefinite lines ML perpendicular to ZO . Make ZM of left-hand line ML , and OM of right-hand line ML each equal to ZM (Fig. 6); similarly make ZL and OL each

equal to ZL (Fig. 6). Then $MLLM$ will complete the pattern required. In connection with this see § 89, p. 245.

To draw the pattern for the smaller piece of pipe, the longitudinal seam to correspond with the line $A''A'$ (Fig. 6).

Draw (Fig. 8) an indefinite $A'A'$. In it take any point A' , and from A' set off distances $A'B'$, $B'C'$, $C'D'$, $D'E'$, $E'F'$, and $F'G'$ equal respectively to $A'b'$, $b'c'$, $c'd'$, $d'e'$, $e'f'$,

FIG. 8.



and $f'G'$ (Fig. 6), that is, equal to one another. Through A' , B' , C' , D' , E' , F' , and G' draw indefinite lines $A'A''$, $B'B''$, $C'C''$, $D'D''$, $E'E''$, $F'F''$, and $G'G''$ perpendicular to $A'A'$; and make $A'A''$ equal to $A'A'$ (Fig. 6), $B'B''$ equal to $B'B'$ (Fig. 6), $C'C''$ equal to $C'C'$ (Fig. 6), and $D'D''$, $E'E''$, $F'F''$, and $G'G''$ equal respectively to $D'D'$, $E'E'$, $F'F'$, and $G'G'$ (Fig. 6). From A'' through B'' , C'' , D'' , E'' ,

F'', and G'' draw an unbroken curved line; this completes one-half the pattern required. The half to the right of G' G'' can be drawn in similar manner, setting off along G' A' the same above distances, but in reverse order and starting from G'. With this see § 87, p. 241, and § 88, p. 241.

PROBLEM IV.

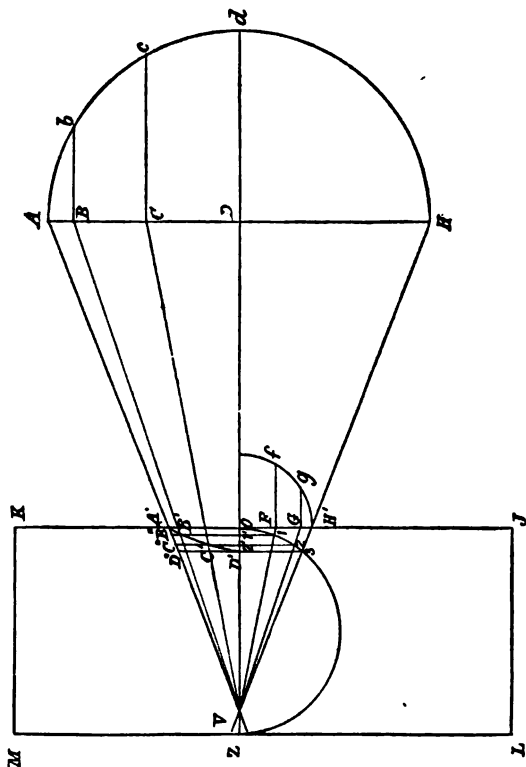
To draw the pattern for the T formed by a funnel-shape piece of pipe and a circular piece, the former being 'square' to the latter (part-cone joined 'square' to a cylinder); the diameter of the circular pipe, and the diameters of the ends of the funnel-shape pipe and its length being given.

By 'being square' we mean that the axes of the pieces of pipe intersect and are at right angles. The given diameter of the smaller end of the funnel-shape pipe is the diameter in the direction of the length of the circular pipe and that coincides with its surface.

First draw a side elevation and a part-plan of the T, thus. Draw (Fig. 9) any two indefinite lines Z d and K J, intersecting at right angles at O. Make O Z equal to the diameter of the circular pipe; and through Z draw M L parallel to K J; make O D equal to the length of the funnel-shape pipe and through D draw a line A H perpendicular to O d. Now make D A and D H each equal to half the given diameter of the larger end of the funnel, and O A' and O H' each equal to half the given diameter of its smaller end, which small end we will suppose is not let into, but fits against the circular pipe; join A A' and H H', then A' A, H H' will be, in elevation, the main portion of the funnel-shape pipe. Next in A' K take any point K, and through K draw K M perpendicular to K J and meeting M L in M; also in H' J take any point J, and through J draw J L perpendicular to K J and meeting M L in L; then M K J L will represent, in elevation, a piece of the circular pipe: and M K A' A H H' J L a side elevation (except the curve of

junction) of the Γ . Produce AA' and HH' to intersect ZO in V . With D as centre and radius DA describe a semi-circle $A d H$ and divide the quadrant $A d$ of it into any number of equal parts, here three in the points b and c ;

FIG. 9.



through b and c draw bB and cC each perpendicular to AH and cutting it in B and C , and join BV and CV . Now on ZO describe a semicircle $Z d O$ (this will be a part-plan of the circular pipe) cutting VH' in point 3. With O as centre and radius OH' describe a quadrant $H'E$ (this may be regarded as a part-plan of the funnel-shape pipe) which

divide into the same number of equal parts that the quadrant $A d$ is divided into, in the points f and g . Through f and g draw fF and gG each perpendicular to $A'H'$ and cutting it in F and G ; join FV , GV , cutting the semicircle ZO in points 1 and 2 respectively. Through point 1 draw a line $1B''$ parallel to KJ , meeting AV in B'' and cutting ZO and BV in $1'$ and B' respectively; through 2 draw $2C''$ parallel to KJ , meeting AV in C'' , and cutting ZO and CV in $2'$ and C' respectively; and through 3 draw $3D''$ parallel to KJ , cutting ZO in D' and meeting AV in D'' . From D' through C' and B' to A' draw an unbroken curved line, then $A'C'D'$ is the elevation of one-half the curve of junction of the two pipes. In practice it is only necessary to draw the $O A' A D$ (Fig. 9) portion of the elevation of the smaller pipe. The other half-elevation $O H' H D$ of it is drawn here simply to make the full side elevation of the T clearer.

It is evident that the T requires 2 patterns, one for the circular pipe with the hole in it that the funnel-shape pipe fits to, and one for the funnel-shape itself.

To draw the pattern for the circular pipe; the longitudinal seam to correspond with the line ML .

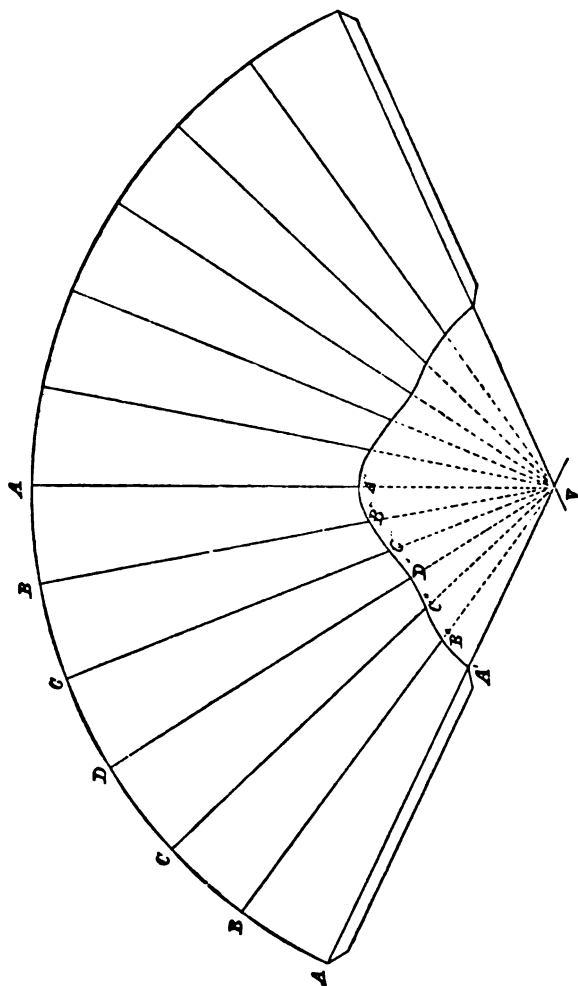
Proceed in exactly similar manner as explained for the pattern of the corresponding pipe in Problem II., p. 242.

To draw the pattern for the funnel-shape pipe; the longitudinal seam to correspond with the line AA' (Fig. 9).

With V (Fig. 10) as centre and VA (Fig. 9) as radius describe an arc AA , and from any point in it set off along the arc distances AB , BC , CD , DC , BC , and BA each equal to Ab (one of the equal parts in which quadrant dA (Fig. 9) is divided). Join AV , BV , CV , DV , CV , BV , and AV ; and make the extreme lines AA' right and left of DV equal to AA' (Fig. 9), also the lines BB'' , right and left of DV , equal to AB'' (Fig. 9), CC'' right and left of DV , equal to AC'' (Fig. 9) and DD'' equal to AD'' (Fig. 9). Through points A' , B'' , C'' , D'' , C'' , B'' , A' draw an unbroken curved line, then $AA'A'A$ will be one-half the pattern required. By continuing to the right, say, the arc AA , and setting off on

it the same above equal distances A B, B C, &c., and proceeding in exactly similar manner the other half pattern can be drawn to complete the pattern required.

FIG. 10.



PROBLEM V.

To draw the pattern for a tapering piece of pipe joining two circular pipes of unequal diameter and not in the same line with each other.

A representation of such tapering piece of pipe joining two circular pipes will be found in Plate VI. (p. 259); also (except apex portion) of an oblique cone (Z) the frustum B'D'DEB of which (the line B'D' should be continued to the outside of the cone) corresponds to the tapering piece of pipe.

This tapering piece of pipe being an oblique cone frustum, its pattern can be drawn by Problem II., p. 113, Book II. In connection here see also §§ 58, 59, p. 111, and Figs. 5 and 6, p. 111.

PROBLEM VI.

To draw the pattern for a piece of pipe joining two circular pipes of equal diameters, and not in the same line with each other.

This problem is a special case of the preceding, and the piece of pipe joining the two equal pipes is an oblique cylinder. Being so, we can draw its pattern by Problem V., p. 121, Book II. See also § 61, p. 112, and Figs. 7a and 7b, p. 112.

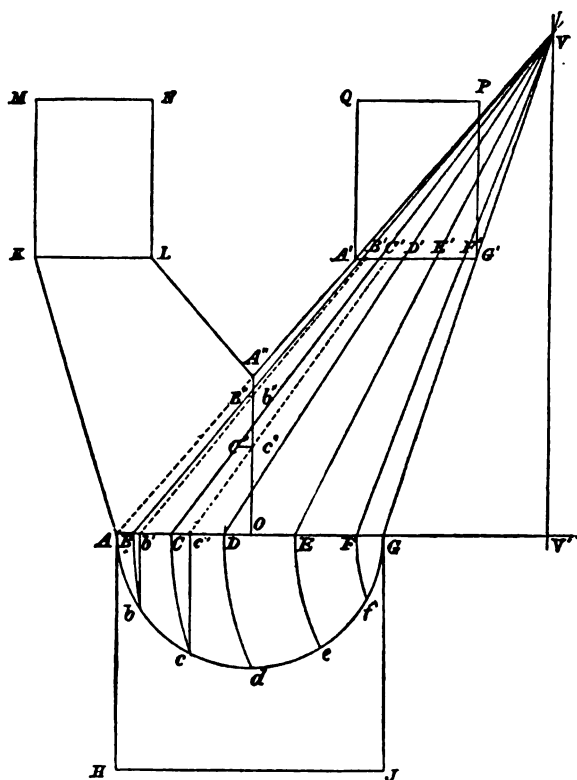
PROBLEM VII.

To draw the pattern for the 'Y' formed by two tapering pieces of pipe uniting two equal circular pipes at the arms of the Y to a third larger piece of circular pipe at its stem, the Y being both ways symmetrical.

In Plate VI. (p. 259) is a representation of such 'Y'. The frustum B'D'DEB of the oblique cone Z in same

Plate (the line $B'D'$ being continued to the outside lines of the cone) corresponds to either of the tapering pieces of pipe of the Y , except that at their junction $C'D'$ a piece ($B C D$ of

FIG. 11.



cone Z) of each frustum is cut off. It is manifest that the base of either frustum, supposing the frustum completed, would coincide with the top of the stem of the Y .

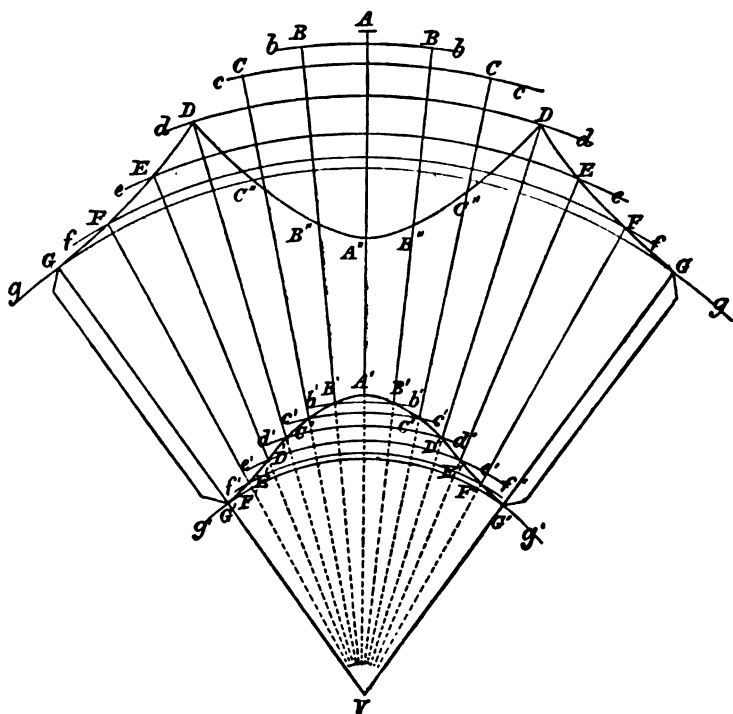
Let $M A H J G P Q A'' N$ (Fig. 11) represent the Y in

elevation. As it is both ways symmetrical, $G'G$ is the shortest generating line. Produce $A'A'$ to A ; then $A'A$ is the longest generating line. Also produce AA' , GG' , to their intersection in V . On AG describe a semicircle AdG , which divide into any number of equal parts, here six, in the points b, c, d, e, f . Produce AG indefinitely and from V let fall a perpendicular to the produced line, meeting it in V' . With V' as centre and $V'f, V'e, V'd, V'c$, and $V'b$ respectively as radii, describe arcs meeting AG in F, E, D, C , and B . Join each of these points to V by lines cutting $A'G'$ in F', E', D', C' , and B' . From c and b draw cc' and bb' perpendicular to AG and join c' and b' to V , by lines cutting $A''O$ in c'' and b'' . Through c'' draw $c''C''$ parallel to AG and cutting VC in C'' ; also through b'' draw $b''B''$ parallel to AG and cutting VB in B'' .

Now to draw the pattern (Fig. 12), so that the seam shall correspond with $G'G$ (Fig. 11) the shortest generating line. Draw VA (Fig. 12) equal to VA (Fig. 11) and with V (Fig. 12) as centre and radii successively equal to VB, VC, VD, VE, VF , and VG (Fig. 11), describe, respectively, arcs bb, cc, dd, ee, ff , and gg . With A as centre and radius equal to Ab (Fig. 11), describe arcs cutting the arc bb right and left of VA in B and B . With these points B and B as centres and radius as before, describe arcs cutting the arc cc right and left of VA in C and C . With same radius and the last-named points as centres, describe arcs cutting dd right and left of VA in D and D . With D and D as centres and same radius, describe arcs cutting ee right and left of VA in E and E , and with E and E as centres and same radius describe arcs cutting ff in F and F . Similarly, with same radius and F and F as centres find points G and G . Join the points B, C, D, E , &c., right and left of VA to V . With V as centre and VA' (Fig. 11) as radius describe an arc cutting VA in A' . With same centre and VB' (Fig. 11) as radius describe an arc $b'b'$ cutting VB right and left of VA in B' . With same centre and VC' (Fig. 11) as radius describe an

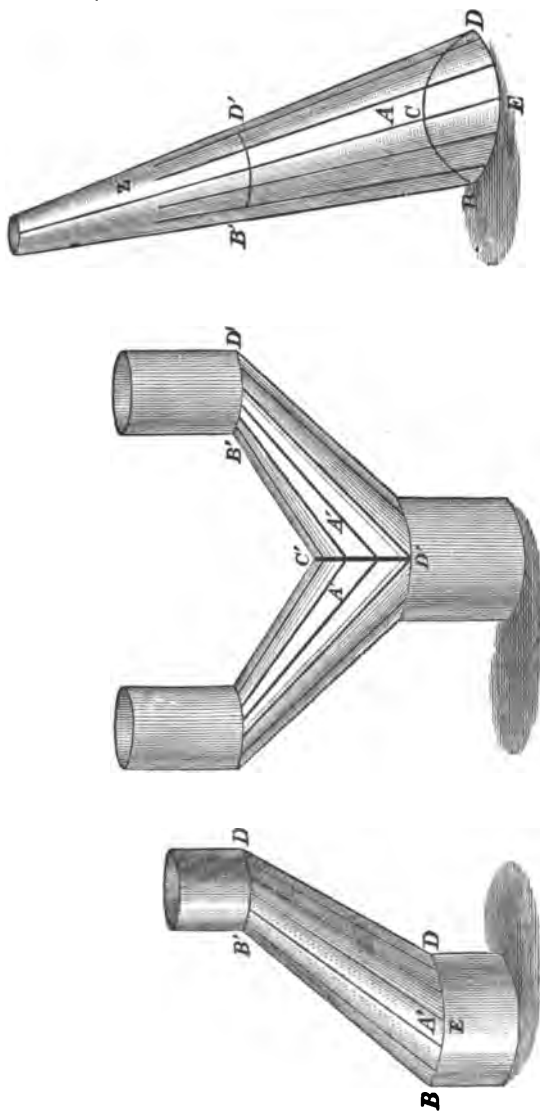
are $c'c'$ cutting VC right and left of VA in C' . Similarly, with same centre and VD' , VE' , VF' , and VG' (Fig. 11) successively as radii describe, respectively, arcs $d'd'$, $e'e'$, $f'f'$, and $g'g'$, cutting VD , VE , VF , and VG , right and left of VA respectively in D' , E' , F' , and G' .

FIG. 12.



Draw through A' , and the points B' , C' , D' , E' , F' , and G' , right and left of VA , an unbroken curved line. Also through the points G , F , E , D , right and left of VA draw unbroken lines. Now make AA'' equal to AA' (Fig. 11);

PLATE VI. (see p. 255).



DRAWINGS FROM MODELS IN SOUTH KENNINGTON MUSEUM. (Part of Exhibit by the Author at the Inventions Exhibition, 1885.)

B B" right and left of A V equal to B B" (Fig. 11); and C C" right and left of A V equal to C C" (Fig. 11). Through A", and the points B", C", and D, right and left of V A draw an unbroken curved line. Then G D A" D G G' A' G' will be the pattern for the pipe A" O G G' A' (Fig. 11) as well as also for the pipe A" O A K L. The patterns for the circular pipes need no explanation.

(90.) In this problem, the Y being symmetrical, the two tapering pieces of pipe are equal, and the mitre or joint line A" O, perpendicular to A G, bisects A G, and, produced, bisects also the angle L A" A'. If the Y were unsymmetrical, the line of junction of the tapering pipes should still be made perpendicular to A G; it will, however, neither bisect it nor the angle L A" A', and a distinct pattern for each tapering piece of pipe will be required.

PROBLEM VIII.

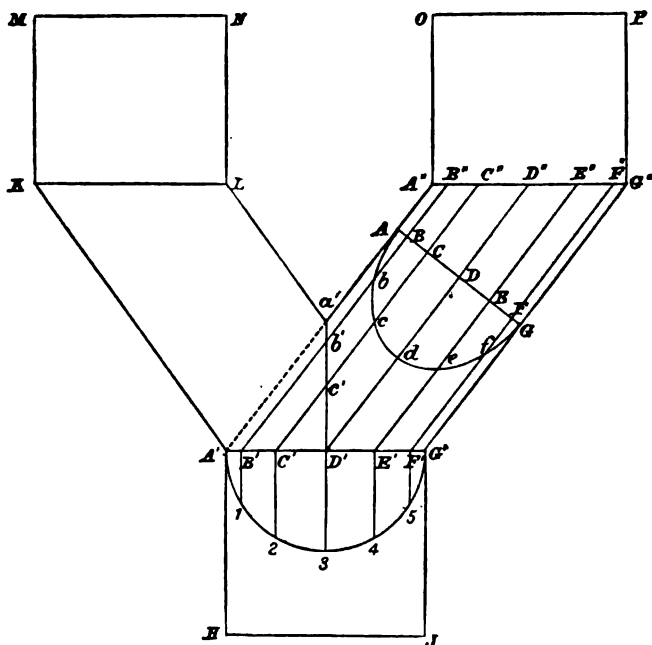
To draw the pattern for the Y formed by two cylindrical pieces of pipe of equal diameter uniting two further pieces at the arms of the Y to a third piece at its stem, the Y being both ways symmetrical.

This problem is a special case of the preceding, the oblique cone frusta of that problem now becoming oblique cylinders.

Let M A' H J G' P O a' N (Fig. 13) represent the Y in elevation. Produce A" a' to A'. On A' G' describe a semi-circle A' 3 G', which divide into any number of equal parts, here six, in the points 1, 2, 3, 4, and 5; through each point of division draw lines perpendicular to A' G', meeting it in the points B', C', D', E', and F', and through these points draw lines B' B'', C' C'', D' D'', &c., parallel to A' A". From any point A in A' A" draw A G perpendicular to A' A", meeting G' G" in G, and cutting the lines B' B'', C' C'', &c., in the points B, C, D, E, and F. Next make B b equal to

$B'1$, Cc equal to $C'2$, Dd equal to $D'3$, Ee equal to $E'4$, and Ff equal to $F'5$, and draw a curve from A through the points b, c, d, e , and f to G .

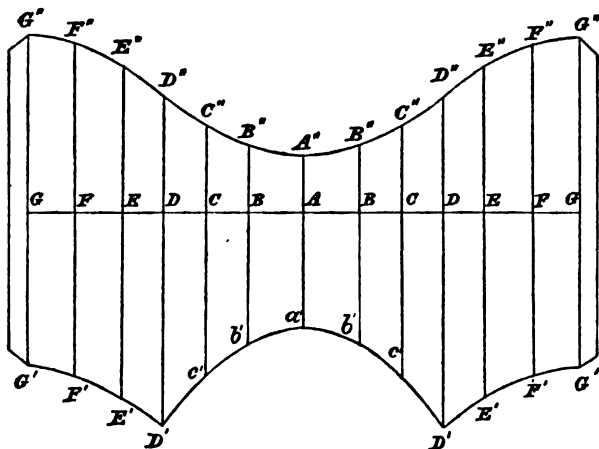
FIG. 13.



To draw the pattern (Fig. 14), so that the seam shall correspond with $G'G''$ (Fig. 13). Draw any line GG , and at or about its centre draw any line $A''a'$ perpendicular to it and cutting it in A . From A , right and left of it, on the line GG mark distances AB, BC, CD, DE, EF , and FG equal respectively to the distances Ab, bc, cd, de, ef , and fG (Fig. 13). Through the points B, C, D , &c., right and left of A , draw lines parallel to $A''a'$. Make Aa' , AA'' equal to Aa' , AA'' (Fig. 13) respectively. Similarly make

Bb' , BB'' , Cc' , CC'' , DD' , DD'' , &c., right and left of $A''a'$ equal respectively to Bb' , BB'' , Cc' , CC'' , DD' , DD'' , &c. (Fig. 13). Draw an unbroken curved line from A'' through B'' , C'' , &c., right and left of A'' , to G'' . Also draw unbroken curved lines from D' to G' , right and left of a' , and an

FIG. 14.



unbroken curved line from D' through c' , b' , a' , b' , and c' , to D' . Then $G''A''G''G'D'a'D'G'$ will be the pattern required for either of the portions of oblique cylinders $a'A''G''G'D'$, or $a'LK A'D'$. The patterns for the circular pipes need no explanation.

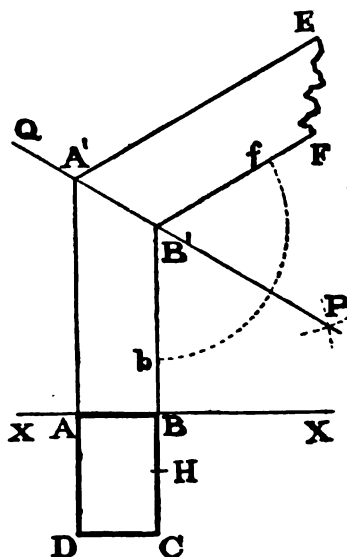
(91.) In this problem, as in the preceding, the Y being symmetrical the two obliquely cylindrical pipes are equal, and the mitre or joint line $a'D'$, perpendicular to $A'G'$, bisects $A'G'$, and, produced, bisects also the angle $L a' A''$. If the Y were unsymmetrical, the line of junction of the pipes should still be made perpendicular to $A'G'$; it will, however, neither bisect it nor the angle $L a' A''$, and a distinct pattern for each obliquely cylindrical piece of pipe will be required.

PROBLEM IX.

To draw the patterns for an elbow formed by two equal rectangular pipes (pipes of oblong section) meeting at any angle.

In the case under consideration, we assume that the throat of the elbow is on the broad face of the pipe, which is what happens when pipes of oblong section are fitted to a wall or between a floor and ceiling.

FIG. 15.

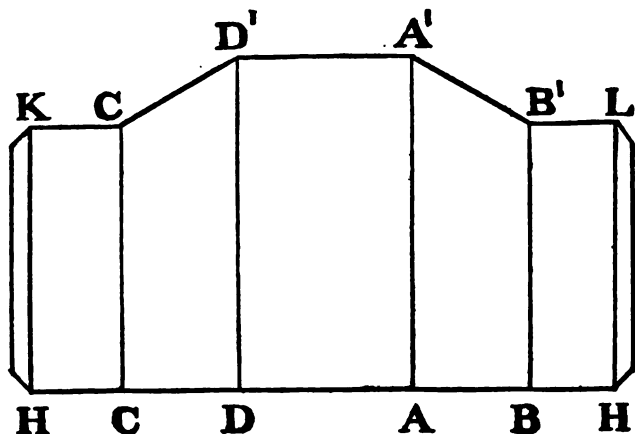


First draw a section of the pipe at right angles to it; also a side elevation of the elbow. Taking any point B (Fig. 15) to represent a definite point in one of the straight lengths of pipe, draw any two lines B'B' and F'B' at an angle equal to the given angle of the elbow, and through B draw an indefinite line XX perpendicular to B'B'. From B on XX

set off BA equal to the narrow face of the pipe, and through B and A draw BC and AD each perpendicular to XX and equal to the broad face of pipe; join DC ; then $ABCD$ will be the section of the pipe; it is also part plan (see Problem I. p. 239) of the elbow.

To get the side elevation of the elbow. Through A draw an indefinite line AA' and find the joint or mitre line $B'A'$ as follows:—With B' as centre and any convenient radius, describe an arc cutting $B'B$ and $B'F$ in b and f respectively,

FIG. 16.



and with b and f as centres and any radius greater than half the length of the arc $b f$, describe arcs intersecting in P ; join $P B'$ and produce it indefinitely to Q cutting the line AA' in A' ; then $B'A'$ is the mitre line. The line $A'E$ drawn through A' parallel to $B'F$ completes the elevation.

To draw the pattern for the $A A' B' B$ piece of pipe, the seam to be in the middle of the broad face, of which the line $B B'$ is the end elevation. Bisect BC in H , and draw (Fig. 16) an indefinite line HH , and from a point H in HH , set off distances HC , CD , DA , AB , and BH equal

respectively to H C, C D, D A, A B, and B H (Fig. 15), and through H, C, D, A, B, and H draw indefinite lines H K, C C', D D', A A', B B' and H L perpendicular to H H; make H K, C C', B B', and H L, each equal to B B' (Fig. 15), and D D', A A' each equal to A A' (Fig. 15). Join K C', C' D', D' A', A' B' and B' L by straight lines, then the figure H K C' D' A' B' L H will be the pattern required. The pattern will also be that of an equal length of the pipe E A' B' F; and the two patterns will be the pattern for the elbow.

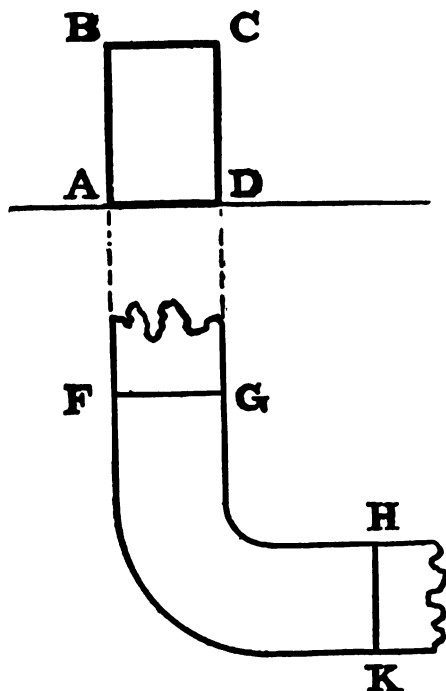
The procedure is exactly similar if the throat of the elbow has to be on the narrow face of the pipe.

PROBLEM X.

To draw the patterns for a curved elbow (bend) to join two equal rectangular pieces of pipe.

Let F G H K (Fig. 17) be a side elevation of the bend, and A B O D a section of the pipe at right angles to its length. The pattern for the F G H K faces of the bend will obviously be of the same size and shape as the figure F G H K. The pattern for the throat of the bend from G to H will be a rectangle, two sides of which are equal to D C, and the other two sides equal in length to the 'stretch out' of the curve from G to H. The pattern for the outside of the bend will also be a rectangle, having two of its sides equal to A B and the other two sides equal to the 'stretch out' of the curve from F to K. Thus the necessary patterns for the elbow consist of two rectangles and two pieces of shape F G H K; and the elbow is formed by seaming them at the angles corresponding with the lines F K and G H.

FIG. 17.



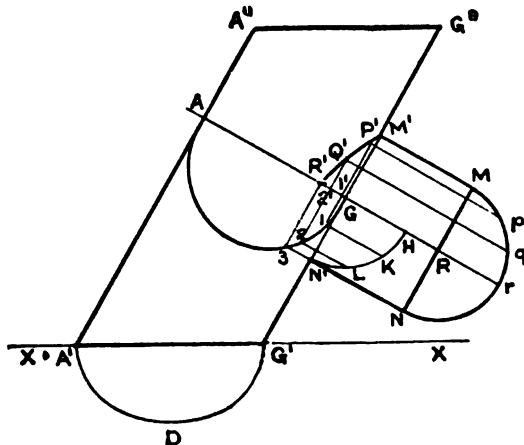
PROBLEM XI.

To draw the patterns for the T-piece formed by two circular pipes, one of which is an oblique cylinder, the axes of the pipes being at right angles and intersecting.

Let $A'G'G''A''$ (Fig. 18), drawn as in Problem V. p. 121 (Fig. 13), be the elevation of the oblique cylinder pipe on which the T is to be formed. On $A'G'$ describe a semi-circle $A'DG'$; this will be part plan of the pipe. In $A'A''$ take any point A, draw AG perpendicular to $A'A''$, and on

it describe, as in the problem referred to, the semi-ellipse $A 3 G$. Produce $A G$ indefinitely to r , and let $G r$ be the axis of the pipe which, of smaller diameter than the oblique cylinder, is to form a T with it. Make $G N'$ and $G M'$ each equal to half the given diameter of the smaller pipe, and through N' and M' draw indefinite lines $N' N$ and $M' M$ perpendicular to $G' G''$. In $N' N$ take any point N ; on $M' M$ set off $M' M$ equal to $N' N$ and join $N M$; then $N' M' M N$ will be the elevation of a piece of the smaller pipe. With point R where $N M$ outs $A r$ as centre and $R M$ as radius,

FIG. 18.



describe a semicircle $N r M$, which will be part plan of the smaller pipe. Divide the quadrant $M r$ into any number of equal parts, here three, in the points p and q , and through p and q draw indefinite lines $p P'$, $q Q'$ parallel to $M M'$. With G as centre and radius $G N'$ describe an arc terminating at H in the line $A r$, which arc, as the lines $G N'$ and $A r$ are perpendicular to each other, will be a quadrant, and may be regarded as part plan of the pipe $N' M' M N$. Divide this quadrant, exactly as the quadrant $M r$ was divided, in the

METAL-PLATE WORK.

points K and L, and through K, L, and N' draw lines K 1, L 2, and N' 3 parallel to A R and cutting the semi-ellipse A 3 G in the points 1, 2, and 3. Through point 1 draw 1 P' parallel to G' G'', cutting A r in 1' and meeting p P' in P'; through 2 draw 2 Q' parallel to G' G'', cutting A r in 2' and meeting q Q' in Q'; and through 3 draw 3 R' parallel to G' G'', and meeting A r in R'. From M' to R', through P' and Q' draw an unbroken curved line, then M' P' Q' R' is the elevation of one quarter of the curve of junction of the two pipes.

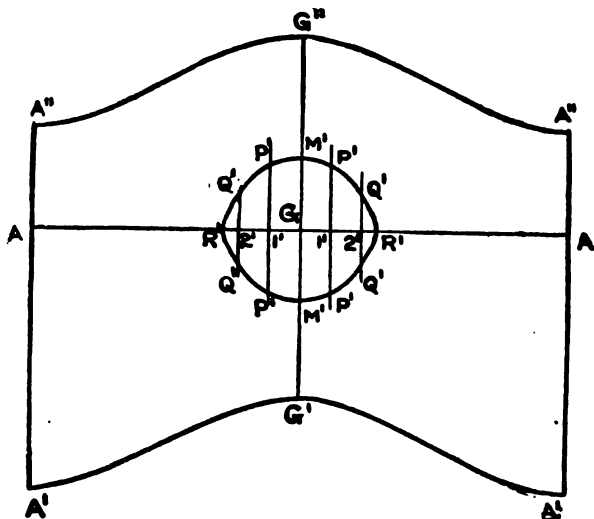
(92A.) Turning back now to Problem II. p. 242 (which problem the student should read with that in course of solution), and comparing its Fig. 3 with the Fig. just constructed, it will be seen that the *method* of obtaining the required part elevation of the curve of junction is the same in each problem; but that, while in the former problem the required part plan of the larger pipe (Z 3 O, p. 242; and see p. 243) is always a semicircle (sections of a right cylinder cut by planes perpendicular to its axis being circles), in the latter the required part plan of the larger pipe (A 3 G) is a semi-ellipse (sections of an oblique cylinder cut by planes perpendicular to its axis being ellipses, the ellipse varying according to the obliquity of the oblique cylinder). This characteristic difference between the larger pipes in the two instances should be carefully noted, as errors will thus be avoided, and the advantages be secured which arise from correct practice.

Just as with the T-piece of p. 242, so with the T now being dealt with, two patterns are required, one for the oblique cylinder pipe with the hole in it where it receives the smaller pipe, and one for the smaller pipe as it fits to the larger.

The pattern for the larger pipe is set out by Problem V., p. 121. To get out the shape of the hole in it draw (Fig. 19) two indefinite lines A A, G'' G' at right angles to one another, and intersecting in G. (These lines correspond respectively to the lines A A, G'' G' of Fig. 14, p. 122. From G on A A, right and left of G' G', set off distances

$G'1'$, $1'2'$, and $2'R'$ equal respectively to the curved distances $G'1$, 12 , and 23 (Fig. 18), that is to the actual distances, as shown on the plan curve $A3G$, that the lines $1'P'$, $2'Q'$, and $3'R'$ are apart from one another. Through points $1'$ and $2'$ right and left of $G'G''$, draw $P'P'$ and $Q'Q'$ perpendicular to AA . Make $G'M'$ above and below AA equal to $G'M'$ (Fig. 18); make $1'P'$ above and below AA and right and left of $G''G'$ equal to $1'P'$ (Fig. 18); and make $2'Q'$ above and below AA and right and left of $G''G'$ equal to $2'Q'$

FIG. 19.



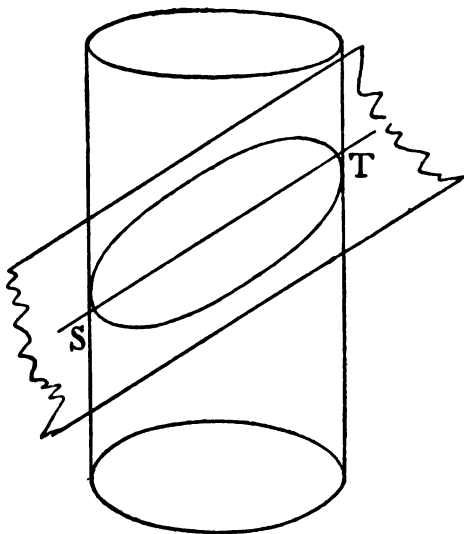
(Fig. 18). Through the points just found, namely R' , Q' , P' , M' , P' , Q' , R' , above and below the line AA , draw an unbroken curved line, the shape thus obtained will be the required pattern for the hole in the inclined pipe.

To draw the pattern for the smaller pipe at its junction with the larger, proceed in exactly the same manner as described for the corresponding pattern (shown in Fig. 5 p. 245) of Problem II. already referred to.

SECTIONS: PATTERNS OF SECTIONED CONES.

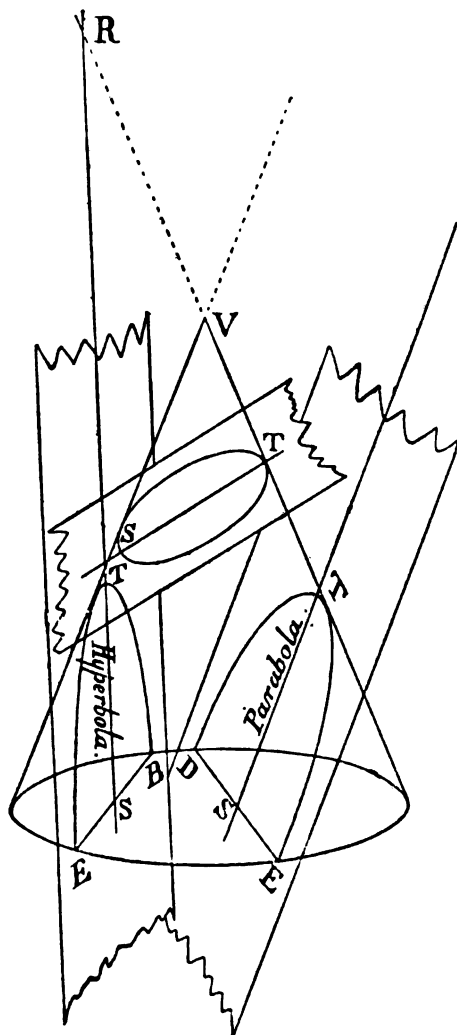
Workers in sheet metal come across in practice a large number of cases in which a knowledge is required of the sections of solids, especially of the cone and cylinder, and of short direct methods of finding the true shapes of the sections and of drawing the patterns for them. With such knowledge, many problems, said by some men to be very difficult, become extremely simple, and the serious errors arising in attempting their solution without the knowledge, are entirely avoided. We now briefly deal with the sections referred to.

FIG. 20.



DEFINITIONS.—(93A.) The section of a right cone or cylinder, or of an oblique cone or cylinder, formed by a plane which

Fig. 20a.



cuts all the generating lines of either solid. but is not parallel to the base of the solid, is an ellipse.

In connection with this definition the student must read §§ 44, 45, 46, and 47 (pp. 105 and 106), noting especially § 47. Further, he will find it convenient to regard a cylinder, whether or not oblique, as an extreme case of frustum of a cone, that is, as being a frustum of which the ends are so nearly equal that they may in practice be regarded as equal. The oblique cylinder has already (§ 61, p. 112) been spoken of as an instance of such frustum.

For examples of elliptic section see Figs. 20 and 20a. Also see Problem XI., p. 267, where the section of the cylinder, $A'A''G''G'$ is an ellipse; Problem XII., p. 274; and Problem XX., p. 284, where the section of the cone $V'A'G'$, by the plane (TS) of the end of the pipe B T S A, is an ellipse.

(94A.) The section of a right or oblique cone by a plane parallel to one of its generating lines is a parabola (see Fig. 20a and Problem XVI., p. 278).

(95A.) Every other section of a right or oblique cone by a plane not containing the axis or parallel to the base is an hyperbola.

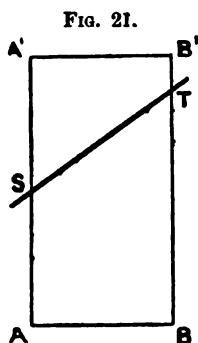
For instances of the hyperbola see Fig. 20a; also see Problem XVIII., p. 281, and Problem XXI., p. 286, where the conical cap is cut by the planes of the sides of the square pipes.

The foregoing definitions are general, and will be useful knowledge to the workman; in the few problems that follow, however, only sections of the right cone and of the cylinder will be dealt with, cases of other sections so seldom occurring in practice, that they cannot be treated here.

PROBLEM XII.

Given a cylinder cut obliquely (§ 93A), to find the dimensions of the cut and draw its ellipse.

Let Fig. 21 represent in elevation a cylinder $A A' B' B$ cut by a plane perpendicular to the plane of the paper in the



direction $S T$. The plane of the paper being the plane in which the axis (§ 61, p. 113) of the cylinder lies, the points S and T will be the extreme points of intersection, in the plane of the paper, of the cylinder with the plane of section. The ellipse of the section will then have $S T$ for its major axis or length, and $A B$, the diameter of the cylinder, for its minor axis or width (see B, p. 17). It may be drawn by either of the methods, as most convenient, given in Chap. II. of Book I., pp. 15–18, or by Problem XXXV., p. 330.

PROBLEM XIII.

To draw the pattern for a cylinder cut elliptically.

This problem has been already solved in Problem I., p. 239, where (Fig. 1, p. 240) $A A' G' G$ is a cylinder cut elliptically by a plane $A' G'$. The pattern required for the present problem is to be obtained exactly as was the pattern (Fig. 2, p. 240), of $A A' G' G$ (Fig. 1, p. 240).

PROBLEM XIV.

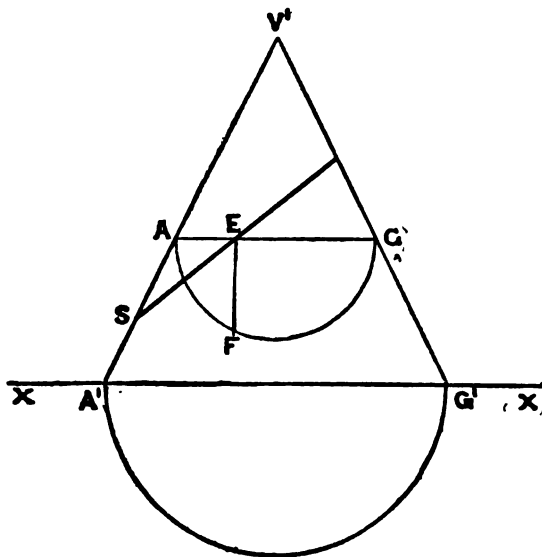
Given a cone cut elliptically (§ 93A), to find the dimensions of the cut and draw the ellipse.

Let Fig. 22 represent in elevation a cone $V' A' G'$ cut by a plane perpendicular to the plane of the paper in the direc-

tion ST . The plane of the paper being the plane in which the axis of the cone lies; the points S and T will be the extreme points of intersection, in the plane of the paper, of the cone with the plane of section. The ellipse of the section will then have ST for its length; and the width is found as follows:—

Bisect ST in E , and through E draw a line AG parallel

FIG. 22.



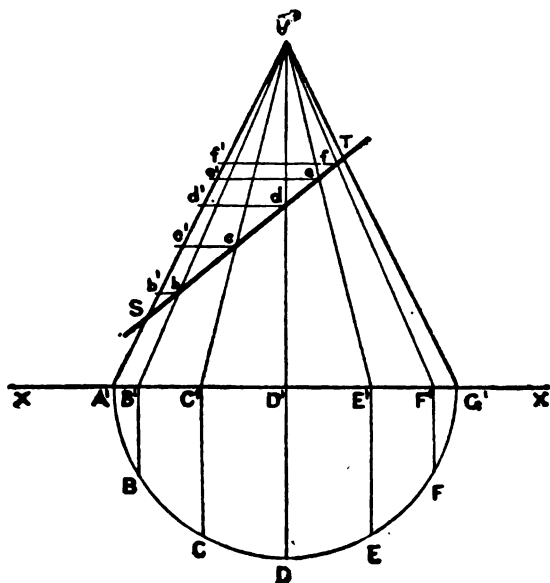
to $A'G'$ and cutting $V'A'$ and $V'G'$ in A and G respectively. On AG describe a semicircle AFG and from E let fall a perpendicular EF cutting the semicircle AFG in F ; then EF is half the minor axis or width required. An ellipse whose length is ST , and width twice EF , will be the ellipse required. For how to draw it see references just given in Problem XII., p. 274.

PROBLEM XV.

To draw the pattern for a cone cut elliptically.

Let $V'A'G'$ (Fig. 23) represent the cone in elevation, cut elliptically by a plane perpendicular to the plane of the paper in the direction ST . The points S and T will be the extremities (see preceding Problem) of the ellipse of section. The pattern is required of $A'STG'$ portion of the cone.

FIG. 23.



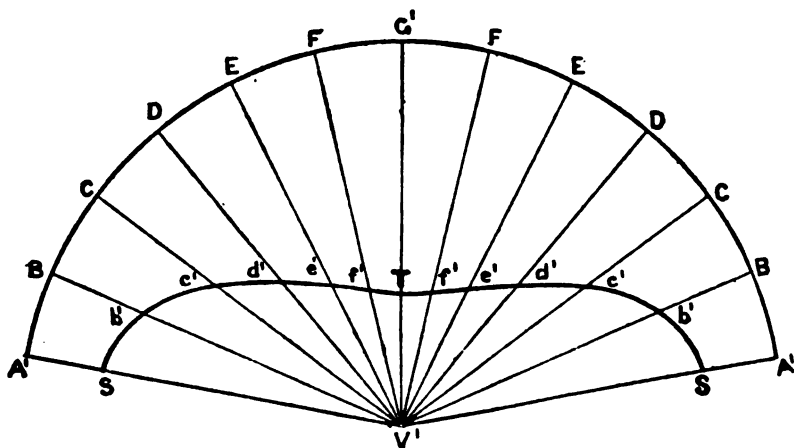
Draw the semicircle $A'D'G'$, half-plan of the cone and divide it into any number of equal parts, here six, in the points B, C, D, E , and F , and through these points draw lines perpendicular to $A'G'$ and intersecting it in B', C', D', E' and F' respectively. Join $B'V', C'V', D'V', E'V'$, and $F'V'$ by lines

cutting ST in the points b, c, d, e , and f respectively, and through b, c, d, e , and f , draw lines bb', cc', dd', ee' , and ff' parallel to $A'G'$, and cutting $A'V'$ in the points b', c', d', e' , and f' respectively. The lengths $A'b', A'c', A'd', A'e'$ and $A'f'$ will be the true lengths respectively of the lines $B'b, C'c, D'd, E'e$, and $F'f$, these latter lines being elevation lengths only, and not true lengths.

(96A.) In dealing with a cone true lengths on its surface must always be obtained, as shown, and the workman should particularly notice this, because even in books mistakes are made in this important matter.

To draw the pattern; the seam to correspond with the line $A'S$. With V' (Fig. 24) as centre and $V'A'$ (Fig. 23)

FIG. 24.



as radius, describe an arc $A'G'A'$, and take any point in it as G' . Join $G'V'$, and set off along the arc right and left of $G'V'$, distances $G'F, FE, ED, DC, CB$, and BA' equal each to $A'B$ (Fig. 23), one of the equal parts into which the semi-circle $A'DG'$ is divided. Join FV', EV', DV', CV', BV' , and AV' right and left of $G'V'$, and make $G'T$ equal to $G'T$

(Fig. 23); also make Ff' , Ee' , Dd' , Cc' , Bb' , and $A'S$ right and left of $G'V'$, equal respectively to $A'f'$, $A'e'$, $A'd'$, $A'c'$, $A'b'$, and $A'S$ (Fig. 23). Through the points $S, b', c', d', e', f', T, f', e', d', c', b', S$, draw an unbroken curved line; then $A'G'A'Sd'Td'S$ will be the pattern for that portion of the cone of which the pattern is required, the portion, namely, $A'STG'$ (Fig. 23).

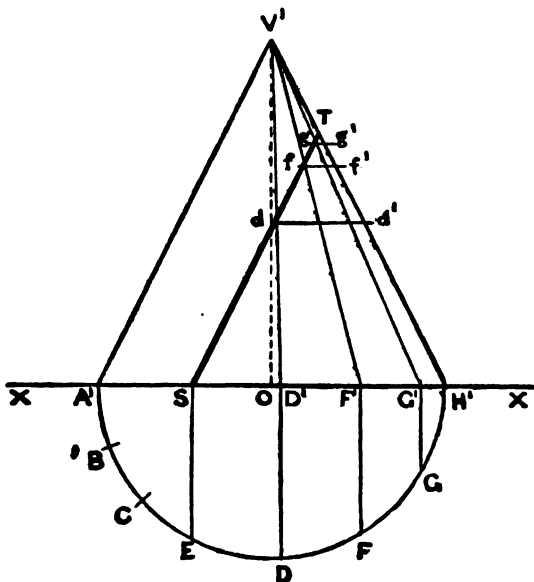
It should be noted that $V'Sd'Td'SV'$ (Fig. 24) is pattern for the $V'ST$ (Fig. 23) portion of the cone $V'A'G'$ should the pattern of that portion be needed.

PROBLEM XVI.

Given a cone cut in parabolic section (§ 94A), to find the dimensions of the cut and draw the parabola.

Let Fig. 25 represent in elevation a cone $V'A'H'$ cut by a plane perpendicular to the plane of the paper in the direction ST ; the semicircle $A'DH'$ being half-plan of the cone. The points S and T will be the extreme points of intersection, in the plane of the paper, of the cone with the plane of section, and the line ST will be the representation in end elevation of the parabola of the section. The line ST will also be the length of the parabola. And as dividing the curve into two equal parts (see Fig. 20a and Fig. 65, p. 331), ST is also its "axis." The width at S of the parabola can be found by drawing through S a line SE perpendicular to $A'H'$ cutting the semicircle $A'DH'$ in E ; then SE is half the width, at S , of the section of the cone. A parabola having ST for its length, and twice SE for its width, or "double ordinate" at S that is, will be the parabola required. It may be drawn by the method given in Problem XXXVI, p. 331.

FIG. 25.



PROBLEM XVII.

To draw the pattern for a cone cut in parabolic section.

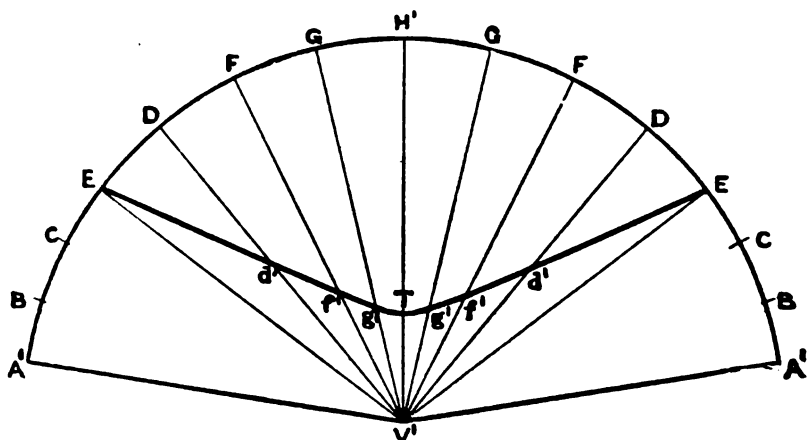
Before commencing this Problem the student should again read § 96A, p. 277.

Let $V'A'H'$ (Fig. 25) represent in elevation a cone (axis $V'O$), cut in parabolic section by a plane perpendicular to the plane of the paper in the direction ST , the semicircle $A'D'H'$ being half-plan of the cone. The points S and T will be the extremities of the length (see preceding Problem), of the parabola. The pattern is required of, say, either the STH' or $A'V'TS$ portion of the cone. Through S draw SE perpendicular to $A'H'$, cutting the half-plan $A'D'H'$ in E . Divide the EH' portion of $A'D'H'$ into any number of equal

parts, here four, in the points G, F and D, and through these points draw lines perpendicular to $A'H'$ and intersecting it in G' , F' , and D' respectively. Join $G'V'$, $F'V'$, and $D'V'$ by lines intersecting ST in the points g , f , and d , respectively, and through g , f , and d draw lines gg' , ff' and dd' parallel to $A'H'$, and cutting $H'V'$ in the points g' , f' , and d' respectively. The lines $H'g'$, $H'f'$, and $H'd'$ will be the true lengths respectively of the lines $G'g$, $F'f$, and $D'd$, these latter lines being only elevation lengths and not true lengths.

To draw the pattern of the STH' portion of the cone. With V' (Fig. 26) as centre and $V'A'$ (Fig. 25) as radius,

FIG. 26.



describe an arc $A'H'A'$ and take in it any point H' . Join $H'V'$, and set off along the arc right and left of $H'V'$ distances $H'G$, GF , FD , and DE , equal each to $H'G$ (Fig. 25), one of the equal parts into which the arc $H'E$ is divided. Join $G'V'$, $F'V'$, and $D'V'$, right and left of $H'V'$, and make $H'T$ equal to $H'T$ (Fig. 25); also make Gg' , Ff' , and Dd' , right and left of $H'V'$ equal respectively to $H'g'$, $H'f'$, and $H'd'$ (Fig. 25), and through the points E , d' , f' , g' , T , g' , f' , d' , E

draw an unbroken curved line. Then $E H' E d' T d'$ is the pattern required.

The pattern for the $A' V' T S$ portion of cone is obtained by dividing the arc $E A'$ (Fig. 25) into any number of equal parts, here three, in the points C and B , and setting off from E (Fig. 26) (a point obtained in getting the pattern for the $S T H'$ portion of the cone) along the arc, $A' H' A'$, right and left of $H' V'$, distances $E C$, $C B$, and $B A'$, equal each to $E C$ (Fig. 25), one of the equal parts into which the arc $E A'$ is divided. Then joining $A' V'$, right and left of $H' V'$, the figure $E d' T d' E A' V' A'$ will be the pattern sought. The seam will correspond with $A' V'$ (Fig. 25).

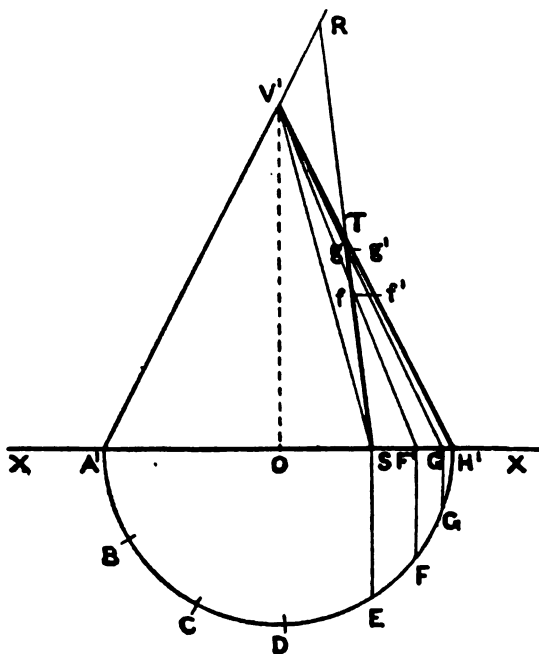
PROBLEM XVIII.

Given a cone cut in hyperbolic section (§ 95A), to find the dimensions of the cut and draw the hyperbola.

Let Fig. 27 represent in elevation a cone $V' A' H'$ cut by a plane perpendicular to the plane of the paper in the direction $S T$; the semicircle $A D H'$ being part-plan of the cone. The points S and T will be the extreme points of intersection, in the plane of the paper, of the cone with the plane of section, and the line $S T$ will be the representation in end elevation of the hyperbola of the section. The line $S T$ will also be the length of the hyperbola. And as dividing the curve into two equal parts (see Fig. 20a and Fig. 66, p. 332), $S T$ is also its axis. Another line is required, which is called the "major" axis of the hyperbola, and can be found as follows:—Produce $A' V'$ and $S T$ to intersect in R ; then $T R$ is the major axis required. The width at S of the hyperbola is ascertained by drawing a line $S E$ drawn through S perpendicular to $A' H'$, and cutting the semicircle $A' D H'$ in E ; then $S E$ will be half the width at S , of the section of the cone. A hyperbola having $T R$ for its major axis, twice $S E$ for its double ordinate at S , or its width, that is, and $T S$ for its

length, will be the hyperbola required. It may be drawn by the method given in Problem XXXVII., p. 332.

FIG. 27.



PROBLEM XIX.

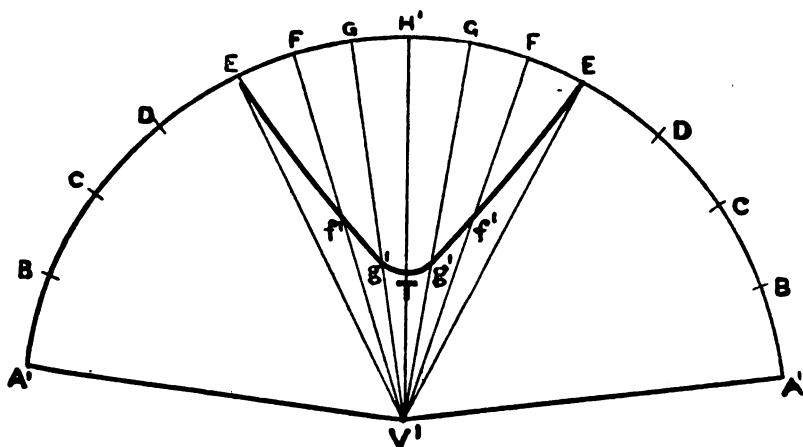
To draw the pattern for a cone cut in hyperbolic section.

The student is here again referred to the caution of § 96A, p. 277.

Let $V' A' H'$ (Fig. 27) represent in elevation a cone (axis $V'O$), cut in hyperbolic section by a plane perpendicular to the plane of the paper in the direction ST , the semicircle $A' D H'$ being half-plan of the cone. The points S and T will

be the extremities (see preceding Problem) of the length of the hyperbola of section, and the line ST the representation of the hyperbola in end elevation. The pattern is required of, say either the STH' , or $A'V'TS$ portion of the cone. Through S draw SE perpendicular to $A'H'$, cutting the half plan $A'DH'$ in point E . Divide the EH' portion of semicircle $A'DH'$ into any convenient number of equal parts, here three, in the points G and F , and through these points draw lines perpendicular to $A'H'$, intersecting it in G' and F' respectively. Join $G'V'$ and $F'V'$ by lines

FIG. 28.



cutting ST in the points g and f respectively; and through g and f draw lines gg' and ff' parallel to $A'H'$, and cutting $H'V'$ in the points g' and f' respectively. The lines $H'g'$ and $H'f'$ will be the true lengths respectively of the lines $G'g$ and $F'f$, these latter lines being only elevation lengths.

To draw the pattern for the STH' (Fig. 27) portion of the cone. With V' (Fig. 28) as centre and $V'A'$ (Fig. 27) as radius, describe an arc $A'H'A'$, and take in it any point H' . Join $H'V'$ and set off along the arc right and left of $H'V'$ dis-

tances $H'G$, GF , and FE , equal each to $H'G$ (Fig. 27) one of the equal parts into which the arc $H'E$ is divided. Join $G'V'$ and $F'V'$, right and left of $H'V'$, and make $H'T$ equal to $H'T$ (Fig. 27); also make $G'g'$ and $F'f'$ right and left of $H'V'$, equal respectively to $H'g'$ and $H'f'$ (Fig. 27). Through the points E, f', g', T, g', f', E draw an unbroken curved line, then $E H' E f' T' f'$ will be the pattern required.

The pattern for the $A'V'TS$ portion of cone is obtained by dividing the arc EA' (Fig. 27) into any number of equal parts, here four, in the points D, C , and B ; and setting off from E (Fig. 28) (a point obtained in getting the pattern of the STH' portion of the cone), along the arc $A'H'A'$, right and left of $H'V'$, distances ED, DC, CB , and BA' , equal each to $A'B$ (Fig. 27), one of the equal parts into which the arc EA' is divided. Then joining $A'V'$ right and left of $H'V'$ the figure $V'A'E f' T' f' E A'$ will be the pattern sought. The seam will correspond with $V'A'$ (Fig. 27).

PROBLEM XX.

To draw the patterns for the elbow formed by a circular pipe fitting on to an elliptic section of a conical pipe (part of right cone, § 93A, p. 271); also to determine the size of the circular pipe and the angle at which it must be cut to fill the cone ellipse.

The student should give attention to this Problem, because it is one, in the working of which, notwithstanding its simplicity, mistakes are made in several books. Elevation lengths of lines are taken as true lengths, nothing is said of determining the size of the circular pipe to fit the cone ellipse, and the elevation of the elbow is drawn more or less at random. It is plainly necessary for the formation of a correct elbow that the ellipse of section of the circular pipe must exactly equal the ellipse of the conical pipe, that is, that the diameter of the circular pipe (see Problem XII., p. 274), must be equal to the minor axis of the cone ellipse

(see Problem XIV., p. 274), and that the pipe must be cut at such an angle that the major axis of the cone ellipse of section shall equal the major axis of the cone ellipse.

Let $A'TSG'$ (Fig. 29) represent a conical pipe or cap, the line ST being an elliptical section of it in side elevation; it is required to fit on a circular pipe to the cone at ST .

First to find the size of the circular pipe, and the angle

FIG. 29.

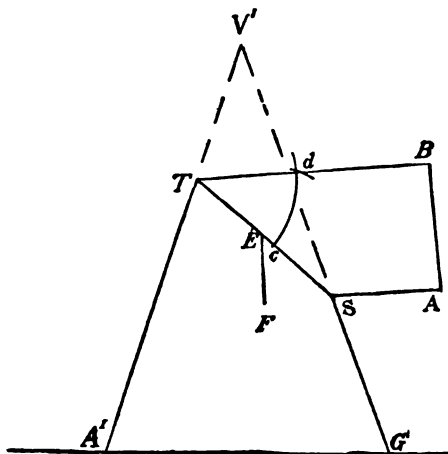
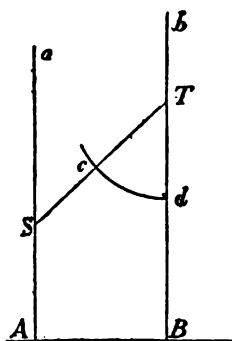


FIG. 30.



at which it must be cut. Produce $A'T$ and $G'S$, the sides of the cone to meet in V' and find EF the half of the minor axis of the ellipse ST by Problem XIV., p. 274, then twice EF will be the diameter of the required circular pipe. Now draw (Fig. 30) a line AB equal to this diameter, equal, that is, to twice EF , and through A and B draw Aa , Bb perpendicular to AB . With any point S on Aa as centre and radius equal to ST (Fig. 29) describe an arc cutting Bb in T and join ST ; then $ASTB$ will represent a cylinder cut at the necessary angle.

To complete the elevation of the conical cap with circular pipe attached (Fig. 29). With T (Fig. 30) as centre and

any convenient radius describe an arc cutting TS and TB in c and d respectively. With T (Fig. 29) as centre and radius equal to Tc (Fig. 30) describe an arc cutting TS in c , and make this arc from c equal to the arc cd (Fig. 30). The arc cd (Fig. 29) is the arc thus equal to cd (Fig. 30). Join Td and produce it as TB , and draw a line SA from S parallel to the line drawn from T . The required elevation is now completed.

To draw the patterns for the elbow. First, that of the conical pipe $A'TSG'$ (Fig. 29). This being a cone cut elliptically, its pattern can be drawn by Problem XV., p. 276. The pattern for the piece of circular pipe $ASTB$ is that of a cylinder cut obliquely, and can be found by Problem XIII., p. 274.

PROBLEM XXI.

To draw the patterns for a conical cap having a square pipe fitted to it concentrically.

First draw an elevation and a part plan of the arrangement, of which Fig. 31a is a perspective drawing. Draw an indefinite line XX (Fig. 31), and from any point V in it as centre, and radius equal to half the given diameter of the larger end of the cap, describe a semicircle $A'EK$. Also from V on XX set off distances $V4$, Vh , each equal to half the given side of pipe, through points 4 and h draw lines 41 , he , perpendicular to XX , also each equal to half the given side of pipe, and join $1e$. Then $A'EK$ with $4he1$ is the part plan required. Through V draw VO perpendicular to XX , and make it equal to the height from the base of the conical cap to the highest point of the cap, that is, the highest point where the square pipe meets it. Through O draw an indefinite line $4'K'$ perpendicular to VO , and set off distances $O4'$ and OK' , each equal to half the given side of the square pipe. Now join KK' , $A'4'$, and draw indefinite

lines $4'q$, $K'p$ perpendicular to $4'K'$. Produce the lines $A'4'$ and KK' to intersect each other in V' ; V' will be the apex of the cone, of which the cap is a part. Join $V1$, and produce it to intersect the semicircle $A'EK$ in D ; from D draw $D'D'$ perpendicular to XX , and intersecting it in D' , and join $D'V'$, intersecting $q4'$ produced in the point $1'$.

FIG. 31.

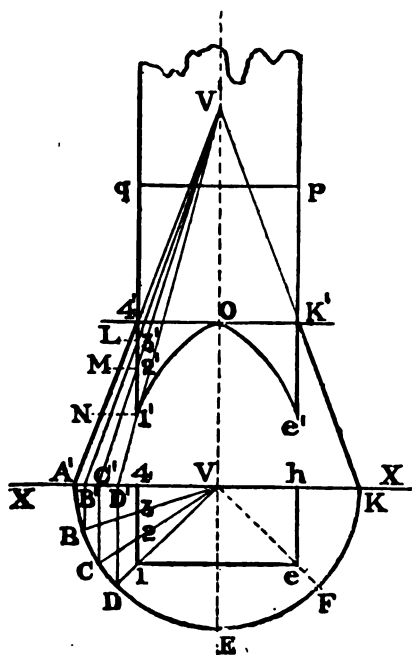
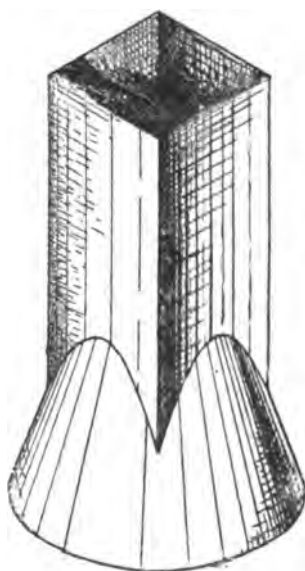


FIG. 31a.

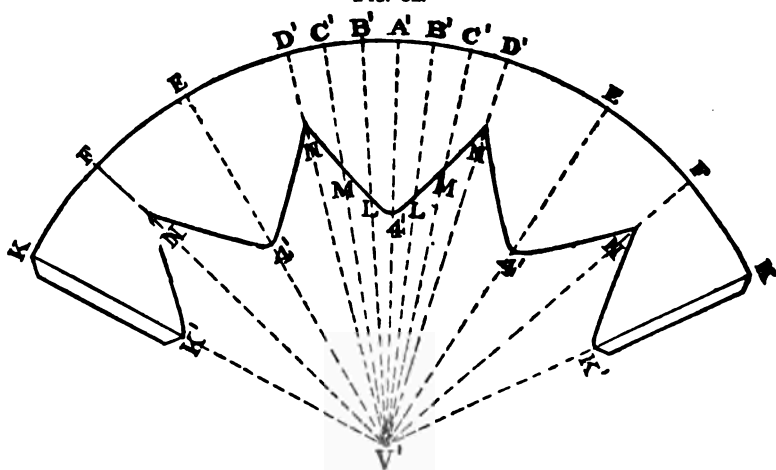


Then $A'4'qpK'K$ with the line $4'1'$, which is the end elevation of one of the lines (curves) of junction of the cap and pipe, will be the elevation required. The curve $1'Oe'$ is the front elevation of two other of the curves of junction, which are all alike, of the cap and pipe, and the line $e'K'$ is the end elevation of another of these junctions.

For these lines no construction is given; the lines indeed are not required for the problem, and are added to the Fig. merely to make clear the arrangement of cap and pipe under consideration. Next divide the arc $A'D$ into any number of equal parts, here three, in the division points B and C , and join CV , BV by lines cutting 41 in points 2 and 3 respectively; also through C and B draw lines CC' , BB' perpendicular to XX , and intersecting it in the points C' and B' respectively; join $C'V'$ and $B'V'$ by lines cutting the line $1'4'$ in points 2' and 3' respectively. The points 2' and 3' will be the elevations of points in the curve of which the line $1'4'$ is the elevation.

To obtain the true lengths of the lines $B'3'$, $C'2'$, and $D'1'$, draw lines through the points 3', 2', and 1' perpendicular to $4'1'$, and cutting $A'V'$ in the points L , M , and N ; then

FIG. 32.



the lengths $A'L$, $A'M$, and $A'N$ are respectively the true lengths required; the lines $B'3'$, $C'2'$, $D'1'$ being elevations only, and not true lengths (§ 96A, p. 277).

To draw the pattern for the conical cap $A'4'1'O'e'K'K$,

32, 21, 1 *e* and *e h* respectively, equal to the distances 43, 32, 21, 1 *e* and *e h* (Fig. 31), and through the points 4, 3, 2, 1, *e* and *h*, right and left of 4, draw lines perpendicular to *h h*. Make 44' equal to 1' 4' (Fig. 31), and 33' and 22', right and left, equal respectively to 1' 3' and 1' 2' (Fig. 31), and through the points 1, 2', 3', 4', 3', 2', 1 draw an unbroken curved line, this will be the curve for one face of the pipe pattern. The curves 1 4' *e* and half curves *e K'* can be drawn in exactly similar manner. Assuming the line *q p* (Fig. 31) to represent the top of the pipe, make 4' *q* equal to 4' *q* (Fig. 31), and through *q* draw *p p* parallel to *h h*, cutting lines *h p* in points *p* and *p*. Then *p p K' e 4' 1 4' 1 4' e K'* will be the complete pattern for the square pipe, and the lines to *h h*, from 1 and *e*, right and left of 4, will be the lines for bending up.

PROBLEM XXII.

To draw the pattern for a conical cap having a rectangular pipe fitted to it concentrically.

This problem differs from the preceding in that all four curves of junction of pipe and cap are not alike, but opposite curves only; that is to say, two pairs of like curves have to be dealt with, and points found by which to draw their patterns.

First draw an elevation and a part plan of the arrangement. Draw an indefinite line *X X* (Fig. 34), and from any point *V* in it as centre, describe a semicircle *A' G K*. Also from *V* on *X X* set off distances *V 4*, *V h*, each equal to half the given dimension of the narrow side of the pipe, and through the points 4 and *h* draw lines 4 1, *h e* perpendicular to *X X*, each equal to half the given dimension of the wider side of the pipe, and join 1 *e*. Then *A' G K* with 4 *h e* 1 is the part plan required. Through *V* draw *V O* perpendicular to *X X*, and equal in length to the height from the base of the conical cap to the highest point of the cap, that is, the highest point where the oblong pipe meets it,

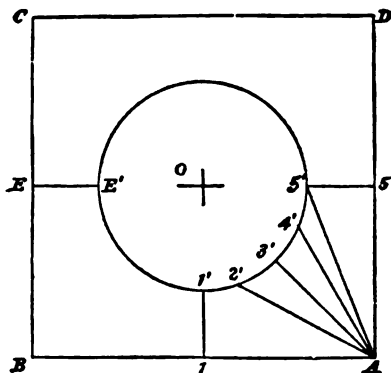
and produce it to intersect the semicircle $A'GK$ in D ; from D draw DD' perpendicular to XX and intersecting it in D' , and join $D'V'$ intersecting $q1'$ in the point $1'$; the line $1'4'$ will be the side elevation of that one of the curves of junction of cap and pipe of which 41 is the plan, and the point $1'$ will be necessarily a lowest or starting point of the part elevation of the curve of junction of which $1e$ is the plan. Next divide the arc $A'D$ into any number of equal parts, here three, in the division points B and C , and join CV , BV by lines cutting 41 in points 2 and 3 respectively; also through C and B , draw lines perpendicular to XX , and intersecting it in the points C' and B' respectively, and join $C'V'$ and $B'V'$ by lines cutting the line $1'4'$ in points $2'$ and $3'$ respectively. The points $2'$ and $3'$ will be the elevations of points in that curve of the cap and pipe of which $1'4'$ is the side elevation, of the points 2 and 3 in the part plan of the pipe. To find the elevation of the other pair of curves of junction; divide the arc DG into any number of equal parts, here three, in the division points E and F , and join EV , FV , and GV by lines cutting $1e$ in points 5 , 6 , and 7 respectively; also through E and F draw lines perpendicular to XX , and intersecting it in the points E' and F' respectively. Part of the line $E'E'$ in this particular figure happens to coincide with the line 14 , and the point E with the point 4 . Join $E'V'$, and $F'V'$, and through points 5 and 6 draw lines perpendicular to XX , and intersecting lines $E'V'$ and $F'V'$ in points $5'$ and $6'$ respectively; these points $5'$ and $6'$ will be the elevations of the points 5 and 6 in the part plan of the pipe. The elevation of point 7 will be found by setting off, on XX , VZ equal to $V7$, through Z drawing a line perpendicular to XX and intersecting KK' in Z' , and making $V7'$ equal to ZZ' . Through the points $1'$, $5'$, $6'$, and $7'$ draw an unbroken curve; this curve will be the front elevation of that part-curve of cap and pipe of which the line 17 is the plan, as well as also a representation for other like portions of the curves of junction. The necessary patterns can now be drawn by proceeding exactly as described for the patterns in the preceding Problem.

PROBLEM XXIII.

To draw the pattern for a body of which the bottom is rectangular and the top circular (base of a tall-boy).

First draw the plan of the body, thus. Draw (Fig. 35) ABCD the rectangle of the bottom of the body. The rectangle is here a square, but the working is the same of whatever nature the rectangle may be. Draw the diagonals of the rectangle, and with their intersection, which will be at O, as centre, and radius equal to half the given diameter of the circular top, describe a circle. Through O draw O 1 perpendicular to BA and cutting the circle in 1'.

FIG. 35.



Also through O draw a line perpendicular to AD, meeting CB and DA in points E and 5 respectively, and cutting the circle in E' and 5'. (Neither O 1 nor E 5 are fully drawn in the figure in order to keep this clear of confusing lines.) Divide the arc from 1' to 5' into any number of equal parts, here four, in the points 2', 3', and 4'. Join A 2', A 3', A 4', and A 5'; then 1' 1, 2' A, 3' A, 4' A, 5' A, and 5' 5 are the plans of lines on the body of which we need the true lengths, lengths which we find as follows :

Draw (Fig. 36) any two lines AF , $5'F$ at right angles to each other and intersecting in F . From F on FA set off FA equal to the height of the body, and from F on $F5'$ set off $F1'$ equal to $1'1$ or $5'5$ (Fig. 35), $F2'$ equal to $A2'$ (Fig. 35), $F3'$ equal to $A3'$ (Fig. 35), $F4'$ equal to $A4'$ (Fig. 35) (the point $4'$ here in Fig. 35 coincides with $2'$, because of $ABCD$ (Fig. 35) being a square), and $F5'$ equal to $A5'$ (Fig. 35). Join the points $5'$, $2'$, $3'$, and $1'$ to A ; then $A1'$ will be the true length of $1'1$ or $5'5$ (Fig. 35), $A2'$ will be the true length of $A2'$ or $A4$ (Fig. 35), and $A3'$ and $A5'$ will be respectively the true lengths of $A3$ and $A5'$ (Fig. 35).

FIG. 36.

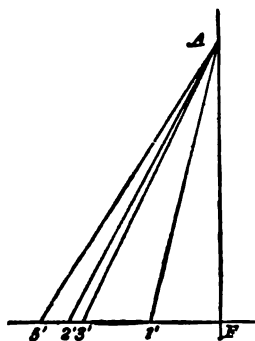
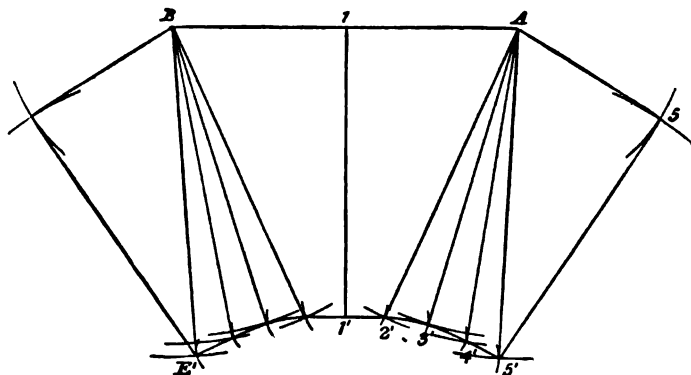


FIG. 37.



To draw the pattern (body in two pieces; seams to correspond with EE' and $5'5$, Fig. 35). Draw (Fig. 37) any line

A B and from point 1 at or about its centre draw a line 1 1' perpendicular to A B. Make the line 1 1' equal to A 1' (Fig. 36). Now make 1 A and 1 B equal each to 1 A or 1 B (Fig. 35), and with A and 1' as centres and radii respectively equal to A 2' (Fig. 36) and the arc 1' 2' (Fig. 35) describe arcs intersecting in 2'. With A and 2' as centres and radii respectively equal to A 3' (Fig. 36) and 2' 3' (Fig. 35) describe arcs intersecting in 3'. With A and 3' as centres and radii respectively equal to A 2' (Fig. 36) and 3' 4' (Fig. 35) describe arcs intersecting in 4'. And with A and 4' as centres and radii respectively equal to A 5' (Fig. 36) and 4' 5' (Fig. 35) describe arcs intersecting in 5'. Through the points 1', 2', 3', 4', and 5' draw an unbroken curved line. With A and 5' as centres and radii respectively equal to A 5 (Fig. 35) and A 1' (Fig. 36) describe arcs intersecting in point 5. Join A 5, 5 5'; then 1 A 5 5' 1' is half the required pattern. The other half can be similarly drawn, and E B A 5 5' 1' E' will be the complete pattern.

PROBLEM XXIV.

To draw the pattern for a dripping-pan with "well."

The pattern for the bottom can be drawn by the method given in the preceding problem, the rim and corners being added by means of Problem XXIII., p. 77.

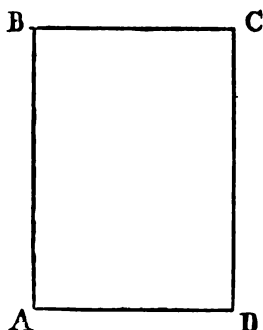
PROBLEM XXV.

To draw the pattern for a tall-boy base to fit a rectangular opening in the slant of a roof.

Let A B C D (Fig. 38) represent the opening in the roof, and let Q P X (Fig. 39) be the angle of its pitch or slant. On P Q from any point E' in it set off a distance equal to the

length of one of the sides of the rectangle that are in the direction of the slant of the roof, here a distance $E'G'$ equal

FIG. 38.



to AB (Fig. 38). The length of the tall-boy base has now to be marked. If this has been given from where the lowest point portion of it meets the roof, through E' draw an indefinite line perpendicular to PX , cutting it in A and on it set off a distance $E'a$ equal to the given height. If this height has been given from where the highest portion of the base meets the roof, then set off that height, $G'b$ say, on an indefinite line through G' perpendicular to PX

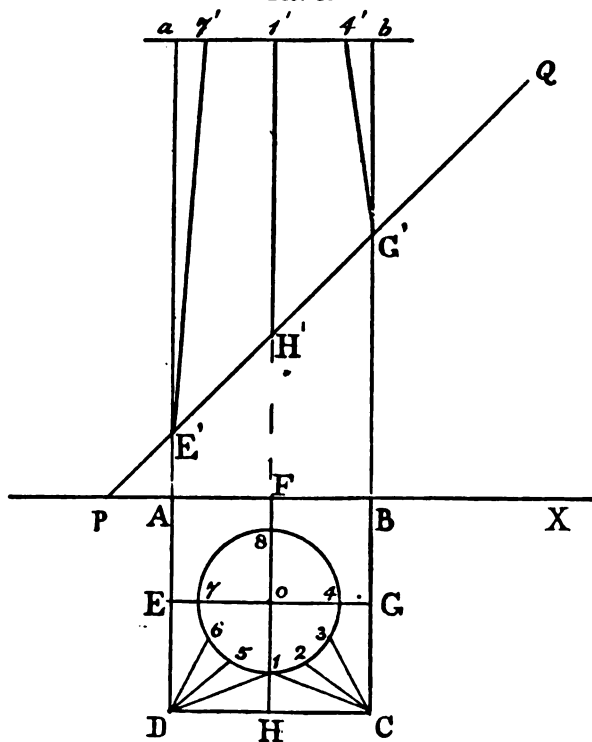
and cutting that line in B . Next through a (or b) draw an indefinite line parallel to PX ; bisect $E'G'$ in H' , and from H' draw an indefinite line perpendicular to PX cutting that line in F , and cutting ab in $1'$; from $1'$ set off $1'4'$ and $1'7'$ each equal to half the given diameter of the circular end of the tall-boy base; and join $E'7'$ and $G'4'$; then $E'7'4'G'$ will be the elevation of the tall-boy base.

To draw the complete plan of it, set off from B on the line through G' , a distance BC , and from A on the line through $E'a$ a distance AD , each equal to BC (Fig. 38), and join DC , cutting the line through H' in H . Then $ABCD$ will be the plan of the opening in the roof, and consequently of the bottom of the tall-boy base. The addition of its top will make the plan complete, and this is obtained by bisecting FH in O and describing a circle with O as centre and the distance $1'4$ as radius.

Let the points 1 and 8 be the points in which the circle cuts the line FH ; the line 18 is a diameter of the circle. Bisect AD in E and from E draw EG parallel to AB , cutting the circle in the points 7 and 4; the line 74 will be another diameter of the circle. Divide each of the quadrants 14 and 17 into any number of equal parts, here

three, in the points 3, 2, and 5, 6 respectively, and join C 3, C 2, C 1, D 1, D 5, and D 6. The lines H 1, E 7, F 8,

FIG. 39.



G 4, and the lines meeting the circle from C and D, are plans of lines on the tall-boy base. The true lengths of two of these lines, E 7 and G 4, we already have in E' 7' and G' 4'. To find the true lengths of C 3, C 2, and C 1, draw (Fig. 40) any two lines K 1, K I' at right angles to each other and intersecting in K. From K on K I' set off K C equal to G' b (Fig. 39) the height of the points 1, 2, and 3 above the point G', (G' being the elevation of C); and from K on K 1 set off K 3 equal to C 3 (Fig. 39), K 2 equal to C 2 (Fig. 39),

and K 1 equal to C 1 (Fig. 39), and join C 3, C 2, and C 1; these lines will be respectively the true lengths of C 3, C 2, and C 1 (Fig. 39). The true length of H 1 is found by making K H equal to H 1 (Fig. 39), and K I' equal to H' I' (Fig. 39), and joining H I'. This length H I' will be the true length of F 8 (Fig. 39) also. To find the true lengths

FIG. 40.

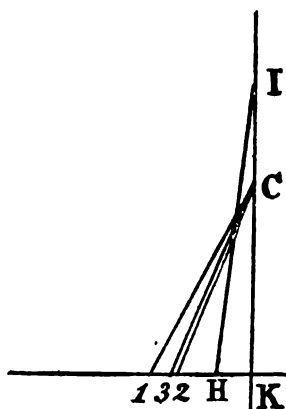
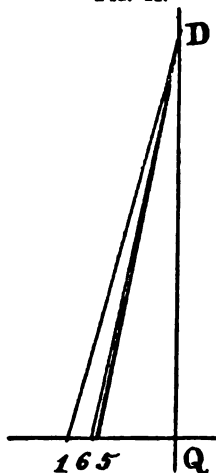


FIG. 41.



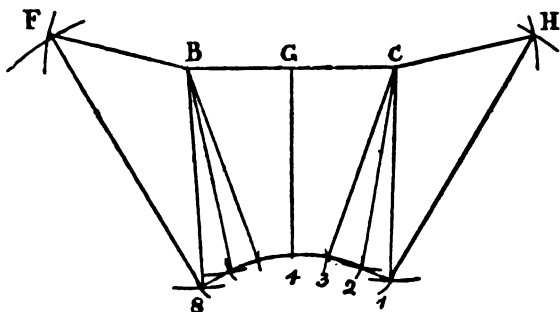
of D 1, D 5, and D 6, draw (Fig. 41) any two Q D, Q 1 at right angles to each other and intersecting in Q; from Q on Q D set off Q D equal to E' a (Fig. 39), the height of the points 1, 5, and 6 above the point E' (E' being the elevation of D); from Q on Q 1 set off Q 1 equal to D 1 (Fig. 39), Q 5 equal to D 5 (Fig. 39), and Q 6 equal to D 6 (Fig. 39), and join D 1, D 5, and D 6; these lines will be respectively the true lengths of D 1, D 5, and D 6 (Fig. 39).

CASE I.—To draw the pattern for the tall-boy base, to be made in two pieces; the seams to correspond with the lines F 8 and H 1 (Fig. 39). Two patterns will be required, one for the F B C H 1 4 8 portion and one for the F A D H 1 7 8 portion.

The pattern for the F B C H 1 4 8 portion. Draw (Fig. 42)

any line BC, and from a point G at or near its centre, draw a line G4 perpendicular to it, and make G4 equal to G'4' (Fig. 39). Make GB and GC each equal to G'B or G'C (Fig. 39), and with C and 4 as centres and radii respectively equal to C3 (Fig. 40) and the arc 4 to 3 (Fig. 39), describe arcs intersecting in point 3. With C and 3 as centres and

FIG. 42.

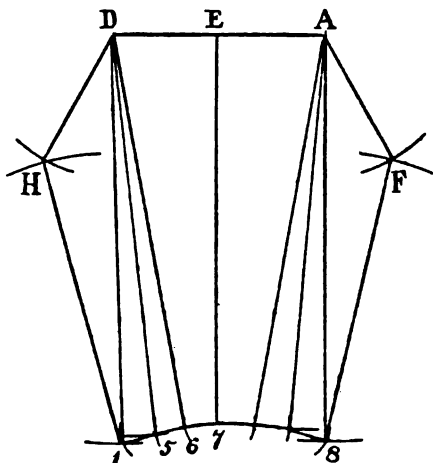


radii respectively equal to C2 (Fig. 40) and the arc 3 to 2 (Fig. 39), describe arcs intersecting in point 2. And with C and 2 as centres and radii respectively equal to C1 (Fig. 40) and the arc 2 to 1 (Fig. 39), describe arcs intersecting in point 1. Through the points 4, 3, 2, and 1 draw an unbroken curved line. With C and 1 as centres and radii respectively equal to G'H' (Fig. 39) and I'H (Fig. 40) describe arcs intersecting in H. Join CH and H1; then 4GCH1 is half the pattern required. The other half can be drawn in like manner, and FBC H148 will be the complete pattern.

The pattern for the FADH178 portion. Draw (Fig. 43) any line DA, and from a point E at or near its centre, draw a line E7 perpendicular to it, and make E7 equal to E'7' (Fig. 39). Make ED and EA each equal to E'D or E'A (Fig. 39), and with D and 7 as centres and radii respectively equal to D6 (Fig. 41) and the arc 7 to 6 (Fig. 39), describe arcs intersecting in point 6. With D and 6 as centres and radii respectively equal to D5 (Fig. 41) and the arc 6 to 5

(Fig. 39), describe arcs intersecting in point 5. And with D and 5 as centres and radii respectively equal to D 1 (Fig. 41) and the arc 5 to 1 (Fig. 39) describe arcs intersecting in point 1. Through the points 7, 6, 5, and 1 draw an unbroken curved line. With D and 1 as centres and

FIG. 43.



radii respectively equal to $E'H'$ (Fig. 39) and $I'H$ (Fig. 40) describe arcs intersecting in H. Join DH, H 1; then 7 E D H 1 is half the pattern required. The other half can be drawn in like manner, and the F A D H 1 7 8 pattern completed.

CASE II.—To draw the pattern for the tall-boy base to be made in two pieces; the seams to correspond with the lines E 7 and G 4 (Fig. 39). In this case it is evident that only one pattern will be required; the E D C G 4 1 7 and E A B G 4 8 7 portions (Fig. 39) being equal.

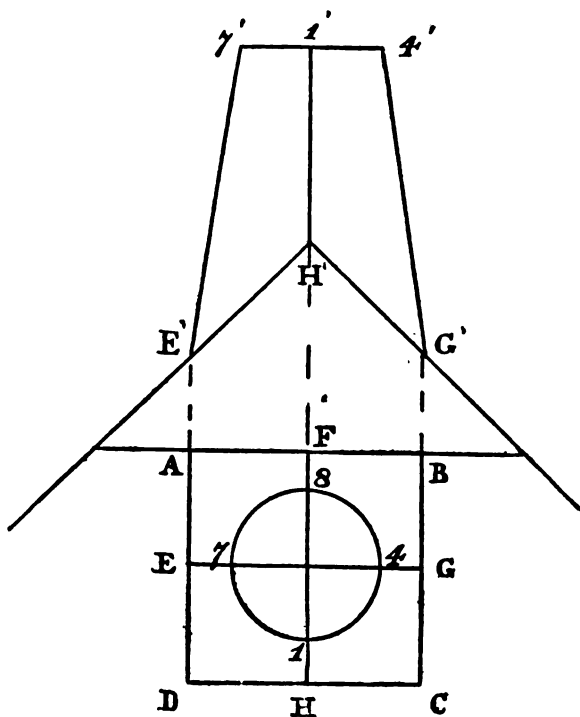
The pattern can be obtained by drawing the G C H 1 4 (Fig. 42) part of the pattern as described in the preceding case and joining to it, on the line H 1, the H D E 7 1 portion of the pattern in Fig. 43. This will give the required pattern.

PROBLEM XXVI.

To draw the pattern for a tall-boy base to fit on the ridge of a roof.

Let $E' 7' 4' G' H'$ (Fig. 44) represent in elevation a tall-boy base on the ridge of a roof, and $A B C D 1 7 8 4$ the plan, drawn as described in the last problem. On referring to the preceding problem, it will be seen that the pattern for the tall-boy base in the present problem will be obtained in exactly similar manner to that of the $F A D H 1 7 8$ portion of Fig. 39, the pattern for which is shown in Fig. 43; the seams will then correspond with the lines $F 8$, and $H 1$, and, as the $E' 7' 1' H'$, and $G' 4' 1' H'$ parts are alike, one pattern only will be required for the body to be made in two pieces.

FIG. 44.



MOULDINGS.

INTRODUCTION.

(97A.) The problems now following treat of mouldings. With these, just as with elbows (p. 239), the essential condition for the formation, with two pieces of moulding, of a joint at any angle, is that the section or end-shape of each piece at the "mitre" (see Problem I., p. 239) or junction shall be the same. This condition should always be borne in mind, and it is fulfilled when, in the plan of the joined pieces on the horizontal plane, the mitre-joint shows as a line bisecting the angle that the two pieces make with one another; for then, as a little consideration will show, there is the same section or cut on each piece. Thus, if this angle is a right angle, that is, if the two pieces of moulding meet "square," the joint line makes an angle in plan of 45 degrees with the internal and external edges of each of the pieces; if the angle of meeting is of 120 degrees, as with six pieces of moulding forming a regular hexagon in plan, each joint line makes an angle of 60 degrees with the internal and external edges of either adjacent piece, and so on.

(98A.) In dealing for pattern-making purposes with the section or shape of a moulding (§ 99A), it is convenient to draw it on two straight lines at right angles, the extremities falling one on either line, and the shape being arranged in respect of one of the lines just as the moulding itself is disposed to the surface to which it is applied; this line is thus part plan line—or part elevation line—of that surface. Looking to Fig. 45 (Problem XXVII.) the section or shape K I H E D C B A is drawn on the indefinite lines A K', K' K, at right angles, one extremity of the shape standing at a point A in A K', and the other extremity at a point K in K' K. The plane of the paper being the plane of the section, then, if A K' represent the surface to which the moulding is applied, A K' is the depth of the moulding, and K' K its outstretch or span. If K' K represent that

surface, then $K'K$ is the depth of the moulding and $K'A$ its span. Thus, that plane of section of a moulding which gives its shape is a plane that contains the lines both of its depth and span.

(99A.) It is necessary to observe that when the section of a moulding or shape of it is spoken of, with no qualifying words, the section is supposed to be on the plane just referred to.

(100A.) The shape $KIHEDCBA$ appears as a line, because metal plate, with which our problems deal, may be regarded as of superficial dimensions only, that is, as having no thickness. In some of the Figs. that follow, with a view of helping the student, the space contained between the line of shape, and the lines of depth and span, is shaded, as in representing a solid moulding; the student will however remember that only the shaped line is the moulding.

The solution of problems is now easy.

PROBLEM XXVII

*Given the shape and length of a piece of moulding
to draw the pattern for it.*

Fig. 45 is the shape of a piece of moulding drawn on the lines $A K'$, $K'K$, and AL (Fig. 45a) its length. Obviously the pattern for the piece of moulding will be a rectangle marked with certain lines, upon which, with the given shape as a guide, the rectangle has to be bent and formed, so as, at its end, to present that shape.

Draw (Fig. 46) any line XX , and from any point A in it draw AY at right angles to XX . From A along XX set off AL equal to AL (Fig. 45a), and from L draw LY' parallel to AY . On AY set off successively distances AB , BC , CD , DE , EF , FG , GH , HI , and IK , equal respectively to AB , BC , CD , DE , EF , FG , GH (F and G are any points in EH , the curved portion of the shape), HI , and IK of Fig. 45. From the points B , C , D , E , F , G , H , I and K draw lines

FIG. 45.

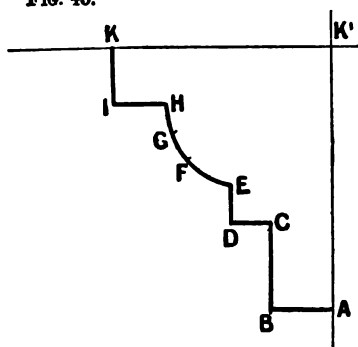


FIG. 45A.

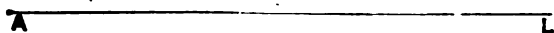
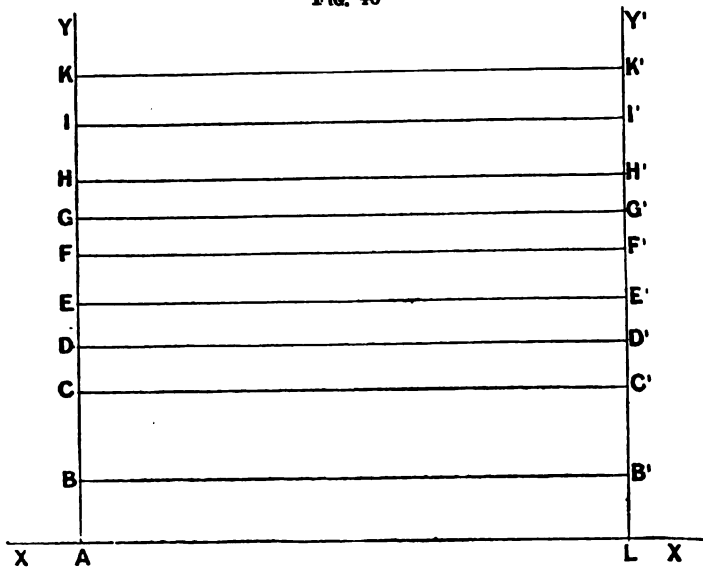


FIG. 46



parallel to AL and terminating in LY' in the points $B', C', D', E', F', G', H', I'$ and K' respectively; $AKK'L$ will be the rectangle pattern required, and the several lines parallel to AL will be the lines by which the pattern is to be bent up and formed to the given shape.

In setting off on AY the distances EF, FG , and GH , the workman must bear in mind that the distances he sets off are chords and not arcs, and that it may be necessary to make allowance for this.

PROBLEM XXVIII.

The shape of a moulding being given, to draw the shape at any angle of section of the moulding, and the pattern for a piece of the moulding cut at that angle. Also, conversely, the section of a moulding at any angle being given to draw the shape of the moulding.

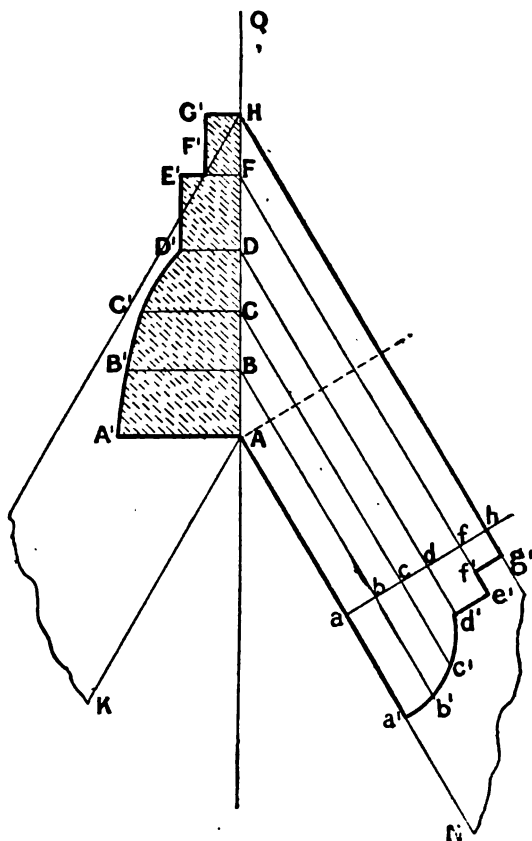
The cases of this problem that usually occur in practice are where the plane of section contains the line of span of the moulding, but not the line of its depth, or contains the line of its depth, but not that of its span, and one particular example of where that plane contains neither line of depth nor line of span.

CASE I.—Where the plane of section contains the line of span of the moulding, but not the line of its depth.

(101A.) Here are seemingly two cases, they are however one and the same, as will be seen from this. Let $h g' f' e' d' c' b' a'$ (Fig. 47), drawn on the lines ah, aN , be the given shape of the moulding, one extremity of it falling on the line ah at h , and the other at a' on aN . If of the lines ah, a' , the line ah represents in plan the surface to which the moulding applies (§ 98A), then ah is the depth of the moulding, and $a'a'$ its span. If $a'a'$ represents that plan line, then $a'a'$ is the depth of the moulding, and ah is its span. Thus the lines $ah, a'a'$, will be respectively depth and span, or span

and depth, according to which of them represents the surface for the moulding, and the plane of section will be described

FIG. 47.



correspondingly. The working drawing for a section at an angle is that which shows the angle.

(102A.) Producing the lines $a'a$ and $g'h$ indefinitely, lines which are perpendicular to ah by construction, and by the

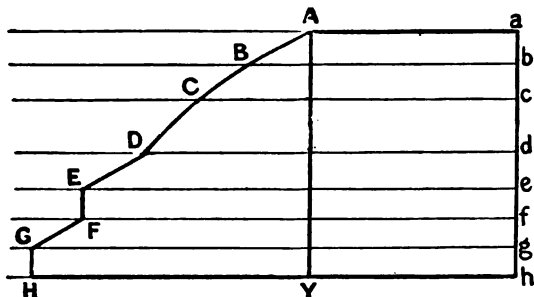
given nature of the moulding ah HA will represent in plan, a piece of the moulding $hg'f'e'd'c'b'a'$. A plane of section of the moulding at any angle NAQ is represented by the line AQ cutting the indefinite lines aA and hH in the points A and H . This plane is perpendicular to the plane of the plan, but does not contain both its lines of span and depth. The plan line of a plane passing through the point A , and containing the lines both of span and depth, would be the line AY .

(103A.) To get the shape of the moulding on the plane of AQ . Produce the line $e'f'$ (also perpendicular to ah by the given nature of the moulding), cutting ah and AQ in f and F respectively, and from the point d' draw a line parallel to the line just drawn, cutting ah and AQ in d and D respectively. In $d'a'$, the curved part of the moulding, take any point e', b' , and from these points draw lines $c'C$, and $b'B$, parallel to $a'A$, cutting ah in c and b , and AQ in C and B , respectively. From A, B, C, D, F , and H , draw lines perpendicular to AH , and make $AA', BB', CC', DD', FE', FF'$, and HG' equal respectively to $aa', bb', cc', dd', fe', ff'$, and hg' . Join $G'F'$ and $E'D'$ by straight lines, and $D'C'B'A'$ by an unbroken curved line; the line $HG'F'E'D'C'B'A'$ will be shape required of the moulding on the plane of section AQ . This shape is often useful as a template.

To draw the pattern for the shape. Draw (Fig. 48) any line ah , and from a point a in it set off distances ab, bc, cd, de, ef, fg and gh , respectively equal to the distances $a'b', b'c', c'd', d'e', e'f', f'g'$, and $g'h$ (Fig. 47). Through a draw a line perpendicular to ah ; make aA equal to aA (Fig. 47), and from A draw a line parallel to ah terminating at a point Y in an indefinite line drawn from h perpendicular to aA . Through b, c, d, e, f , and g , draw indefinite lines bB, cC, dD, eE, fF , and gG perpendicular to ah and cutting the line AY , then the rectangle $aAYh$, with its guide lines between aA and hY parallel to those lines, is the pattern of the length of moulding (see last Problem) $aAYh$ (Fig. 47). On bB, cC , and dD set off distances bB, cC , and dD equal

respectively to b B, c C, and d D (Fig. 47); on e E and f F distances e E and f F each equal to f F (Fig. 47) (the points e' and f' coinciding with each other in plan of the moulding), and on g G, and h H distances g G and h H, each equal to h G (Fig. 47) (the points g' and h' coinciding with each other in plan of the moulding). Through the points A, B, C, D, draw an unbroken curved line, and join D E, E F, F G, and G H by straight lines. Then A B C D E F G H will be the pattern required.

FIG. 48.



(104A.) The pattern for the piece of moulding cut at the angle N A Q (Fig. 47) is also found; it is a A H h (Fig. 48); which pattern, if bent up on the lines d D, e E, f F, and g G, and rounded up between the lines a A and d D, will form the moulding required.

(105A.) In respect of the curve A B C D the caution given in the last problem should be borne in mind. In choosing division points on curves of mouldings, from which, by means of lines to find points for any required section, the workman must be guided by the length and style of a curve. Lines from angular points of a shape, such as h , g , f , e , and d , must always be drawn, as these not only give the angular points in the required section, but are lines on which the pattern of a piece of moulding must be angled up to form the moulding.

The pattern piece of a moulding may be made of any desired length. Length is obtained by simply producing the lines $A a$, $B b$, $C c$, $D d$, $E e$, $F f$, $G g$, and $H h$. If length is not required, but the mitre-line end only of the pattern, this could be found by taking measurements from the line $A Y$ (Fig. 47) instead of $a h$; the pattern thus obtained would be $A B C D E F G H Y$ (Fig. 48).

(106A.) What has been done in this first Case should be noted by the student. The angular points of the moulding, and other chosen points are projected on to one of the lines, $a h$ (Fig. 47) on which the shape is drawn, and thence on to the line in which the moulding is cut. The plane of the cut, whatever its angle, being perpendicular to the surface to which the moulding is applied, span lengths are not affected, but remain unaltered. Depth lengths, however, such as $g' f'$, $e' d'$ (Fig. 47), and the perpendicular length from d' to the line $a a'$ are altered, and the true lengths that these become in the shape of the moulding in the plane of the cut appears in the line $H A$ as $H F$, $F D$, and $D A$ respectively.

CONVERSE PROBLEM.—*The section of a moulding at any angle being given, to draw the shape of the moulding.*

Let $A D' E' F' G' H$ (Fig. 47) be section of a moulding $N A H h$ cut at the angle $N A Q$. Divide $A' D'$ into any number of equal parts, here three, in the points B' and C' , and from the points A' , B' , C' , D' , and E' draw lines perpendicular to $A Q$, cutting that line in the points A , B , C , D , and F . The point F' is, by the nature of the section, a point in $E' F'$; and the line $G' H$ is already perpendicular to $A Q$. From any point a in $N A$ draw $a h$ perpendicular to $N A$, cutting $H h$ in h , and from B , C , D , and F draw lines parallel to $N A$, cutting $a h$ in b , c , d and f . From a on $A N$ set off $a a'$ equal $A A'$; from b , c and d set off $b b'$, $c c'$, and $d d'$ respectively equal to $B B'$, $C C'$, and $D D'$; from f set off $f f'$ and $f e'$ respectively equal to $F F'$ and $F E'$; and from h set off $h g$ equal to $H G'$. Join a' to d' , through b' and c' , by an unbroken curved line, and join $d' e'$, and $f' g'$. Then $a' b' c' d' e' f' g' h$ will be the shape required.

CASE II.—Where the plane of section contains neither line of depth nor line of span of a moulding.

As stated above, only one particular example is to be dealt with; it is useless to here treat the case generally, seeing that other examples very seldom indeed occur in practice. The example being special, it is worked as a whole in Problem XXXII. succeeding.

PROBLEM XXIX.

The shape of a moulding being given, to draw the pattern for joining two pieces of it at any angle.

Let $h g' e' d' a'$ (Fig. 47), drawn on the lines $a h$, $a a'$, be the given shape of the moulding, and $K A N$ the angle at which the two pieces of it are to meet. Bisect the angle $K A N$ (Problem VIII., p. 10), then $A Q$ is the direction of the mitre line, and the two pieces are represented by $K A H$ and $a A H h$. As there is the same section or cut on each piece, the problem now becomes that of drawing the pattern for the piece of moulding, say $a A H h$, cut at the angle $A H h$. This is done by means of the problem preceding.

(107A.) A method used in practice for finding the line $A Q$, that is for bisecting the angle $K A N$, is to set off from A equal distances along $A K$ and $A N$, to join the points thus found by a straight line, and then by means of a square applied to this line to draw a line perpendicular to (square with it) and passing through the point A . This is a perfectly correct procedure.

PROBLEM XXX.

To draw the pattern for an aquarium (or other) base formed of a moulding of one and the same design.

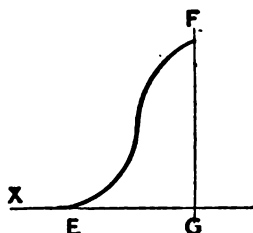
Let $A' B' C' D'$ (Fig. 50) be the plan of the inside edge of the aquarium (or other) base, and $E F$ (Fig. 49), the shape of the moulding, drawn on the lines $G F$ and $G E$; $G E$

representing the surface on which the aquarium stands. Bisect (see Problem VIII., p. 10) each of the angles of the plan (working lines of bisection not shown) by the lines $A'A$, $B'B$, $C'C$, $D'D$. Through either of the angular points, here D' , draw a line perpendicular to $A'D'$; and on it set off $D'f'$ equal to GE (Fig. 49). Through f' draw a line parallel to $A'D'$, and intersecting $A'A$ and $D'D$ in A and D respectively. Through D and A draw lines parallel to $D'C'$, and intersecting $C'C$ and $B'B$ in C and B , and join CB . Then $ABCD$ will be the plan of the outer edge of the aquarium stand or other base, and $A'A$, $B'B$, $C'C$ and $D'D$ the plans of the mitre joints (§ 101A).

If the plans of the outside edge of the aquarium (or other) base be given or first drawn, as $ABCD$, the plan of the inside edge will be thus found. Bisect, first of all, the angles at A , B , C , and D . Next draw a line (not shown in the Fig.) perpendicular to one of the edges, say AD , on its inner side, from any point in it (the line $f'D'$ is such a line, if f' be regarded as not a particular point in AD but as any point in it); on this line set off a distance equal to EG (Fig. 49), and through the extreme point of this draw a line parallel to AD , intersecting the bisecting lines of the angles at D and A in the points D' and A' . Through D' and A' draw lines parallel to DC , intersecting the bisecting lines of the angles C and B in the points C' and B' and join $C'B'$. Then $A'B'C'D'$ is the plan of the inner edge of the stand.

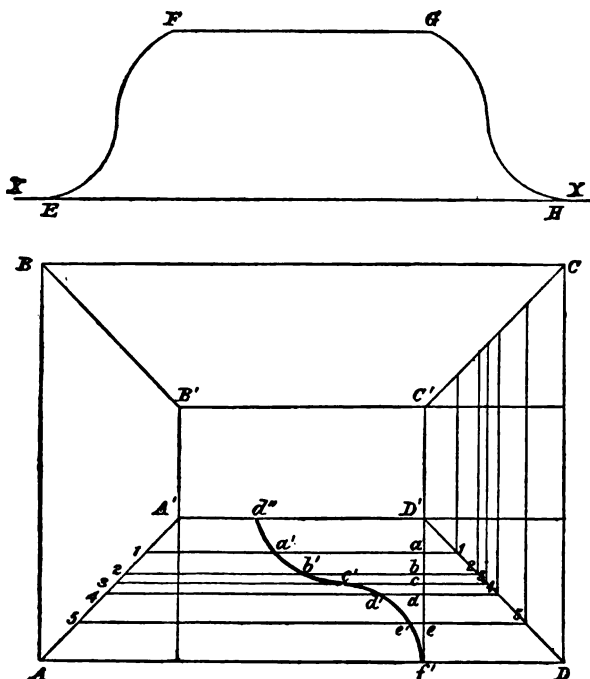
The problem is now essentially completed, and is simply that of drawing the pattern for the EF (Fig. 49) moulding on either mitre plane, which is done by Problem XXIX. It will be a help to the student, however, to work the problem in detail right through. We already have the line $D'f'$ if the inner edge was first drawn; if the outer edge was first

FIG. 49.



drawn, let fall now from D' a line perpendicular to AD , meeting this line in f' ; then $D'f'$, being parallel to the line, not shown, drawn perpendicular to AD on which the distance EG was set off, will be itself equal to EG . Now on $D'f'$, $D'A'$, draw $f'd''$ the required shape of the moulding, disposed towards $D'f'$ as to the surface on which the moulding stands;

FIG. 50.

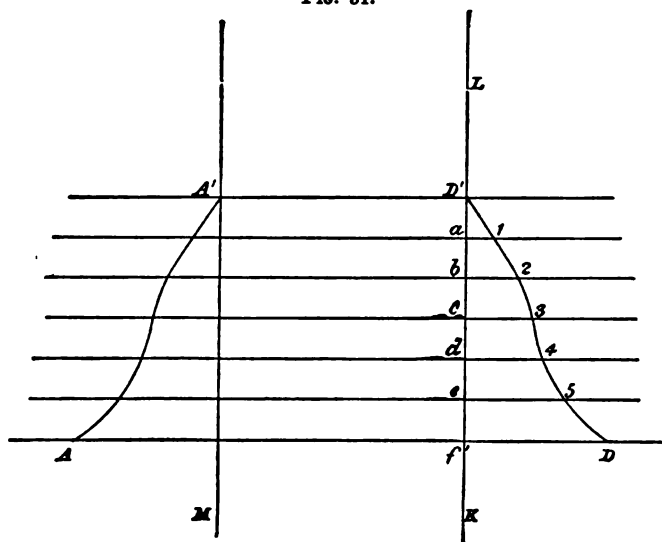


and divide it into any number of parts, equal or unequal (see § 105A, p. 308). The division is here into six equal parts, in the points a' , b' , c' , d' , and e' , through which points draw lines 1 to 1, 2 to 2, 3 to 3, 4 to 4, and 5 to 5 parallel to AD , and terminating in $A'A$ and $D'D$, and from points 1, 2, 3, 4,

and 5, in the line $D'D$ draw lines parallel to DC and terminating in $C'C$.

To draw the pattern for the $A'D'A$ portion of the base. Draw (Fig. 51) any line KL ; and from any point D' in it

FIG. 51.



set off $D'a$, ab , bc , cd , de , and ef' equal respectively to the distances $d''a'$, $a'b'$, $b'c'$, $c'd'$, $d'e'$, and $e'f'$ round the curve $d''c'f'$ (Fig. 50); and through the points D' , a , b , c , d , e , and f' draw lines perpendicular to KL . Make $a1$ equal to $a1$ (Fig. 50), and $b2$, $c3$, $d4$, $e5$, and $f'D$ respectively equal to $b2$, $c3$, $d4$, $e5$, and $f'D$ (Fig. 50); and through the points D' , 1, 2, 3, 4, 5, and D draw an unbroken curved line. Now from D' set off $D'A'$ equal to $D'A'$ (Fig. 50), and through A' draw $A'M$ parallel to KL . From the points in $A'M$, where the lines a , b , c , d , e , and f' cut $A'M$, set off distances to the left of $A'M$ corresponding to the distances $a1$, $b2$, $c3$, $d4$, $e5$, and $f'D$ to the right of the line KL , and

through the points there found draw an unbroken curved line. Then $A'D'AD$ will be the pattern required.

To draw the pattern for the $D'C'DC$ (Fig. 50) portion of the base. It will be at once seen from the plan that this differs from the $A'D'AD$ portion, only in that the distance $D'C'$ is less than the distance $A'D'$; and thus that, if the lines $A'M$ and LK (Fig. 51) are brought closer together, so that $A'D'$ is equal to $D'C'$ (Fig. 50), the pattern so obtained will be the pattern required.

PROBLEM XXXI.

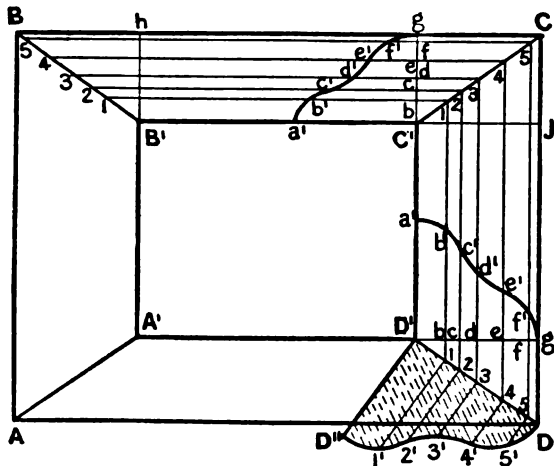
To draw the pattern for an aquarium (or other) base formed of mouldings not of one and the same design.

The problem now before us is that of fitting to a given aquarium or other body, which has to stand in a given position, a position, that is to say, of which the dimensions are fixed, a base made up of mouldings. In the last problem there are no restrictions as to the dimensions of the base.

(108A.) Let $A'B'C'D'$ (Fig. 52) be the plan of the aquarium or other body to which a moulding base has to be fitted, and $ABCD$ the plan of the position the body has to stand in. Join $A'A$, $B'B$, $C'C$, and $D'D$; these lines last drawn will be the plans of the mitre joints (§ 101A). Looking at the plan, it is at once seen that the pieces of moulding on the $B'A'$ and $C'D'$ sides of the body, though equal to each other, are not of the same dimensions as the pieces, also equal to each other, on the $B'C'$, and $A'D'$ sides. An imperative condition of two pieces of moulding properly joining is that the sections of the pieces shall be the same on the mitre-plane (§ 97A). That is to say, that here, for the pieces $A'D'DA$, $C'D'DC$ to properly join notwithstanding the difference of their dimensions, their sections on the mitre-plane of $D'D$ must be the same. When two pieces to be joined are of one and the same design, this condition of same section on the mitre-plane is fulfilled when the mitre line bisects in plan the angle that

the pieces of moulding, as joined, make with one another (§ 97A). That condition has now to be fulfilled when, the two pieces being unequal in dimensions, the mitre does *not* in plan bisect the angle made with each other by the pieces; the line $D'D$, for example, does not bisect the angle $A'D'C'$.

FIG. 52.

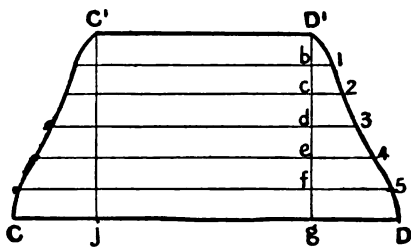


The shape for one of the mouldings may be given or chosen; let it be, say, that for the moulding piece $D'C'CD$, or the piece $B'A'AB$, which is equal to it; and let that shape be $g'e'c'a'$, drawn on the lines $D'g$ (a continuation of the line $A'D'$), and $D'a'$ (part of $D'C'$). Regarding $D'g$ as the line of span of the moulding, $D'a'$ is the line of its depth, that is, of the distance the moulding reaches up on the side of the aquarium or other body. The depth of all four pieces of moulding round the body is the same; it is not affected by the want of correspondence between the plan $A'B'C'D'$ and the plan $ABCD$. From C' on $C'B'$ set off this depth, $C'a'$, equal to $D'a'$, and draw $C'g$ perpendicular to $C'B'$ and meeting the plan line BC of the outer edge of the piece of

moulding $B'C'CB$ in a point g . The line $C'g$ then is the span of this moulding, and we shall now be able to draw on the lines $C'g$, $C'a'$, the shape for it.

Divide $g'e'c'a'$, the chosen shape for the moulding $D'C'CD$, into any number of parts, here six, equal or unequal, in the points b', c', d', e', f' . Through these points draw lines parallel to CD , terminating in the mitre plan-lines $C'C$ and $D'D$, and cutting $D'g$ in b, c, d, e , and f ; the drawn lines being the lines 1 to 1, 2 to 2, 3 to 3, 4 to 4, and 5 to 5. From the points 1, 2, 3, 4, and 5 on the line $C'C$ draw lines parallel to BC , terminating in $B'B$, and cutting $C'g$ in b, c, d, e , and f . From these last points b, c, d, e , and f , set off distances bb', cc', dd', ee' , and ff' , respectively equal to the distances of same letters from the line $D'g$, and through the points a' (on $C'B'$), b', c', d', e', f' , and g on CB draw an unbroken curved line; this line will be the shape for the moulding $B'C'CB$.

FIG. 53.

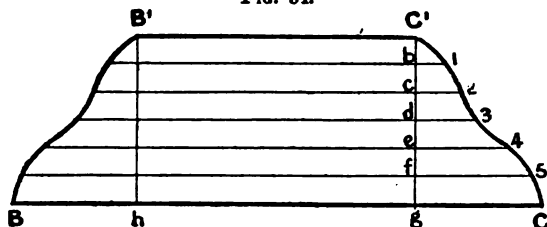


Having the shapes of each of the mouldings, to draw the patterns for the pieces. First that for the $D'C'CD$ pieces. Draw (Fig. 53) any line $D'g$, and from the point D' set off distances $D'b, bc, cd, de, ef$, and fg , equal respectively to the distances $a'b', b'c', c'd', d'e', e'f'$, and $f'g$ of the shape drawn on the lines $D'g, D'a'$ (Fig. 52), and through the points D', b, c, d, e, f , and g draw lines perpendicular to $D'g$. Make gD equal to gD (Fig. 52), and make $f5, e4, d3, c2, b1$ respectively equal to $f5, e4, d3, c2$, and $b1$ from the line $D'g$ (Fig. 52); and from the point D' through the points 1, 2, 3, 4, 5 to D draw an unbroken curved line. Now make gj

METAL-PLATE WORK.

equal to gj (Fig. 52), and through j draw jC' parallel to gD' . Set off jC' equal to gD' , and then obtain points for and draw the curve $C'C$ exactly as with the curve $D'3D$, the two curves being alike. The pattern required will be $C'D'DC$.

FIG. 54.



To draw the pattern for the $B'C'CB$ (Fig. 52) piece of moulding. Draw (Fig. 54) any line $C'g$, and from the point C' set off distances $C'b$, bc , cd , de , ef , and fg , equal respectively to the distances $a'b'$, $b'c'$, $c'd'$, $d'e'$, $e'f'$, and $f'g'$ of the shape drawn on the lines $C'g$, $C'a'$ (Fig. 52), and from the points C' , b , c , d , e , f , and g draw lines perpendicular to $C'g$. Make gC equal to gC (Fig. 52), and make $f5$, $e4$, $d3$, $c2$, $b1$ respectively equal to $f5$, $e4$, $d3$, $c2$, and $b1$ from the line $C'g$ (Fig. 52); and from the point C' through the points 1, 2, 3, 4, 5 to C draw an unbroken curved line. Now make gh equal to gh (Fig. 52), and through h draw hB' parallel to $C'g$. Set off hB equal to gC , and then obtain points for and draw the curve $B'B$ exactly as with the curve $C'3C$; or copy the curve $B'B$ from the curve $C'3C$, the two curves being alike. The pattern $B'C'CB$ will be the pattern required.

The section or shape of either piece of moulding at the mitre plane is obtained as in Problem XXVIII., by drawing (Fig. 52) through the points D' , and 1, 2, 3, 4, 5 on the line $D'D$, lines perpendicular to that line, and making $D'D''$, $11'$, $22'$, $33'$, $44'$, and $55'$, equal respectively to $D'a'$, $b b'$, $c c'$, $d d'$, $e e'$, and $f f'$, and drawing from the point D'' through the extremities of these lines $11'$, $22'$, $33'$, $44'$, and $55'$ to D an unbroken curved line. This section, however, is not needed for the problem.

PROBLEM XXXII.

To draw the pattern for an inclined (raking) moulding, to mitre or join with a given horizontal moulding.

(109A.) This is the case referred to in Problem XXVIII., of section of a moulding by a mitre plane which contains neither the line of depth of the moulding, nor the line of its span. It is easy to work because the mitre plane, so far as the given horizontal moulding is concerned, contains both of those lines. The case is not one, the student must notice, of joining two pieces of moulding *of the same shape* at any angle, one of the pieces being inclined, but of so fitting to one shape of moulding a moulding *of another shape* that the plane of their junction shall contain the line of intersection of the two surfaces to which the mouldings are to apply. As a typical instance of the problem, suppose the mouldings to be gutters, fitting say on a dormer window, or on some part of a roof where one gutter is inclined and the other horizontal. The walls or surfaces (see Fig. 55) to which the mouldings fit are in this instance assumed to be perpendicular to each other, they may however be at any angle; this does not affect the work of the problem. The plane of junction of the mouldings, however, *must contain the line of junction of the surfaces of the walls.*

Let f', e', d', c', b', a (Fig. 55A) drawn on the lines $A'f'$, $A'a$ be the given shape of the horizontal moulding, the lines $A'a$ being part elevation line of the surface to which the horizontal moulding applies; it may in fact be regarded as representing the line of junction of the two flat faces or surfaces of the walls to which the mouldings are to fit. The shape given is not chosen as a good one for a gutter only, but rather as a useful shape to illustrate principles. Hence also the term mouldings is employed instead of gutters, all gutters being mouldings (§ 100A, p. 303). Perpendicular to $A'a$ draw any line XX , produce $A'a$ indefinitely, as aNA , and from any point A in the produced line draw AK at an

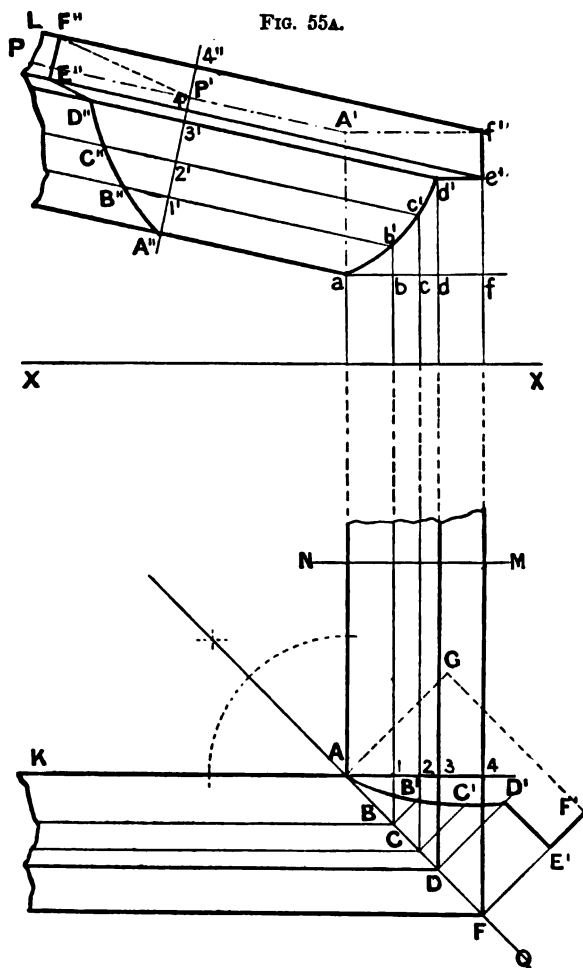
angle in plan equal to that at which the two pieces of moulding are to meet, now a square or right angle as above stated. Bisect the angle KAN by a line AQ , through f' draw $f'MF$ perpendicular to XX , and intersecting AQ in F , and let M be the point in $f'F$ in which a line from any point N in $A'A$ and perpendicular to it cuts $f'F$, and

FIG. 55.



through F draw a line parallel to AK . Next choose division points in the curve $d'a$ as most convenient (§ 105A), the points here taken being c' and b' ; from the points b' , c' , and d' draw lines, each perpendicular to XX , to meet the line AQ , the lines namely $b'B$, $c'C$, $d'D$, and through B , C , and D draw lines parallel to KA . Then $NMF A$ will be the

plan of the horizontal moulding, and F K A that of the inclined moulding, and A F the plan of the mitre-joint (§ 101A).



To draw the elevation of the inclined or raking moulding. From A' draw A' P at an angle to X X equal to the inclina-

tion of the moulding, and from the points f' , e' , and d' (§ 105A), and from c' , b' , and a , draw lines parallel to $A'P$, the $f' L a$ (L being a point in the line drawn from f') is the elevation required.

To draw the shape of the raking moulding. From any point A'' in the line from a parallel to $A'P$ draw a line perpendicular to $a A''$, meeting $f' L$ in $4''$ and cutting the lines drawn parallel to $A'P$ from e' , d' , c' , and b' in the points respectively $4'$, $3'$, $2'$, and $1'$, and continue the line $K A$ to cut the lines $b' B$, $c' C$, $d' D$, and $f' F$ in the points respectively 1 , 2 , 3 , and 4 . As the inclination of the moulding does not affect span dimensions, set off $1' B''$ equal to $1 B$, $2' C''$ equal to $2 C$, $3' D''$ equal to $3 D$, and $4' E''$, $4'' F''$ each equal to $4 F$, the points f' and e' coinciding with each other, in the point F , in plan. Joining the points F'' , E'' , D'' , C'' , B'' , A'' as shown, gives the shape for the inclined moulding, the shape that will mitre accurately in mitre plane $A Q$ with the horizontal moulding.

To draw the pattern for the horizontal moulding $N M F A$. This is done by Problem XXVIII.; in detail it is as follows:—Draw (Fig. 56) any line $A f'$, and from A set off distances $A b'$, $b' c'$, $c' d'$, $d' e'$, and $e' f'$ respectively equal to $a b'$, $b' c'$, $c' d'$, $d' e'$, and $e' f'$ (Fig. 55A), and through the points A , b' , c' , d' , e' , and f' , draw lines perpendicular to $A f'$. Make $f' F$, and $e' E$ each equal to $4 F$ (Fig. 55A), and $d' D$, $c' C$, $b' B$, equal respectively to $3 D$, $2 C$, and $1 B$ (Fig. 55A). From A through the points B , C , and D draw an unbroken curved line, and join $D E$, $E F$. Then $A B C D E F$ is the pattern required. The line $N M$ corresponding to the line $N M$ from any point N in $A A'$ (Fig. 55A) is drawn to indicate that the moulding may be of any length.

To draw the pattern for the inclined moulding $F K A$. Draw (Fig. 57) any line $A'' 4''$, and from A'' set off $A'' 1'$, $1' 2'$, $2' 3'$, $3' 4'$, $4' 4''$ equal respectively to $A'' B''$, $B'' C''$, $C'' D''$, $D'' E''$, $E'' F''$ (Fig. 55A), and through the points A'' , $1'$, $2'$, $3'$, $4'$, and $4''$, draw lines perpendicular to $A'' 4''$. Make $A' a$ equal to $A'' a$ (Fig. 55A), and $1' b'$, $2' c'$, $3' d'$, $4' e'$, and $4'' f'$

respectively equal to $1'b'$, $2'c'$, $3'd'$, $4'e'$, and $4''f'$ (Fig. 55A). Through the a, b', c', d' , draw an unbroken curved line, and join $d'e', e'f'$; then $a b' c' d' e' f'$ is the pattern required.

In the instance where the mouldings form a gutter a straight piece equal in depth to $a A'$ (Fig. 55A) is necessary, with the horizontal moulding, to form a back to the gutter to fit against the wall (see Fig. 55). To the raking gutter

FIG. 56.

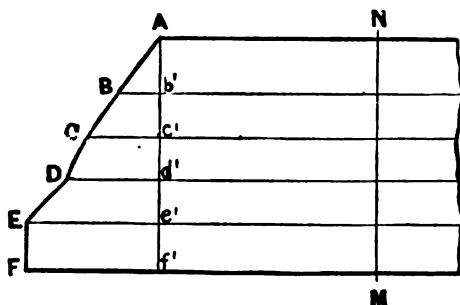
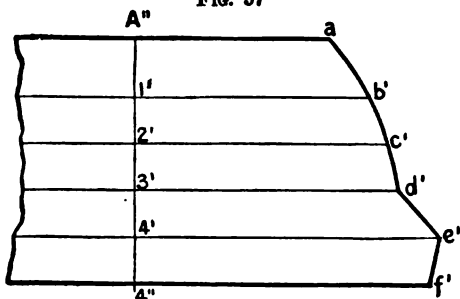


FIG. 57



also a back piece is required (see Fig. 55). The "wall-line," against which the top of the gutter would have to fit, is $A'P$. The straight piece for the back would therefore be equal in depth to $P'A''$ (Fig. 55A), P' being the point of intersection of $A'P$ with $A''4''$. The highest point at the back of the gutter is P' , the highest point in front is F' ; the

line $F''P'$ is drawn to make this clear; and the workman in drawing the pattern must be careful not to use the line $A'P$ except as it has been used in the working of the problem.

Although not needed, it will be useful to show how to obtain the mitre section of the mouldings at AF (Fig. 55A). Draw through point a a line af perpendicular to $A'a$, cutting $b'B$, $c'D$, $d'D$, and $e'F$ in the points b , c , d , and f . Through A , B , C , D , and F draw lines perpendicular to AF , and set off distances FF' , FE' , DD' , CC' , BB' , equal respectively to ff' , fe' , dd' , cc' , and bb' . Join $F'E'$, $E'D'$, and from D' through C' and B' to A draw an unbroken curve; the section sought is $F'E'D'C'B'A'$. The lines AG and GF' represent the lines of depth and span of the horizontal moulding on the plane of junction of the walls or surfaces to which the gutter fixes.

The three problems that immediately follow are examples of penetrations, and in each problem there is intersection of a cone. One example of penetrating bodies has already been dealt with in Problem IV., p. 251, and patterns for cones cut by a flat plane are treated of in Problem XV. (p. 276), XVII. (p. 279), and XIX. (p. 282). The problems referred to should be again worked by the student before proceeding with Problem XXXIII.; especially should he take the course here advised before he attempts Problem XXXV.

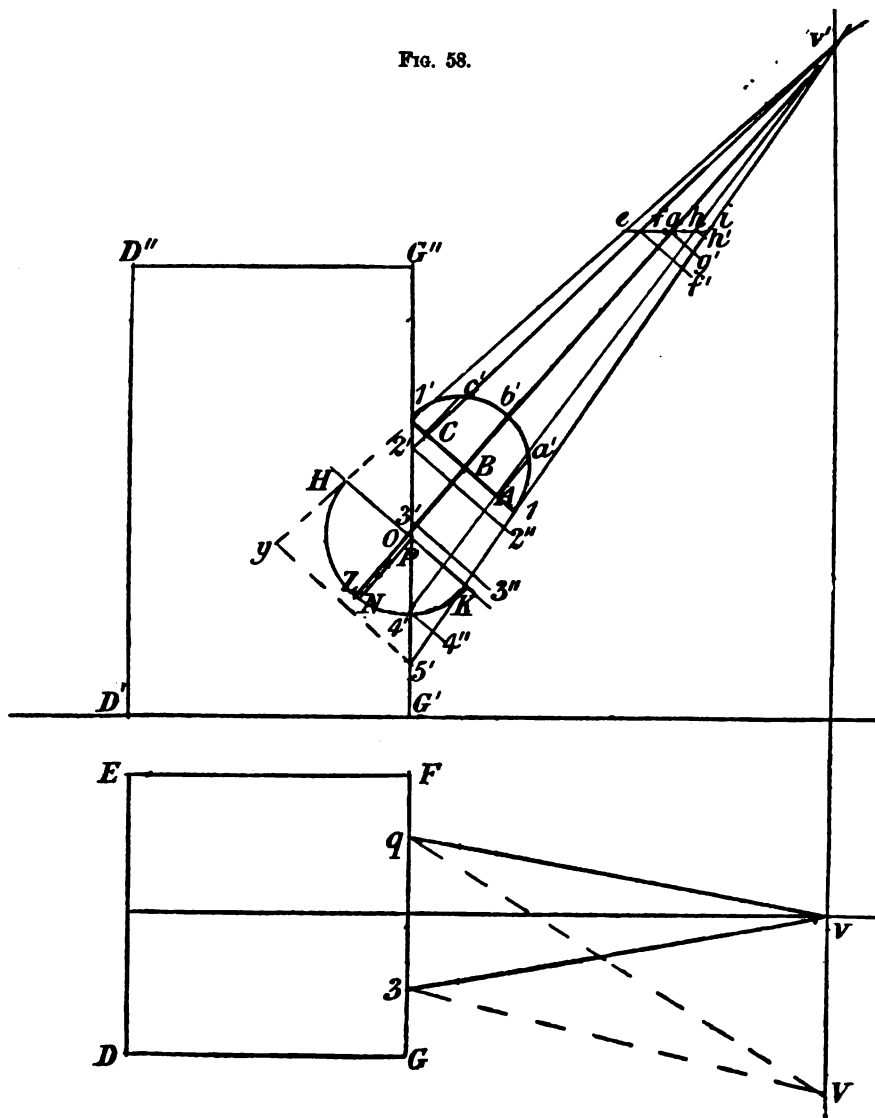
PROBLEM XXXIII.

To draw the necessary patterns where a conical body (part-cone) penetrates a flat surface, but is not square with it.

Examples in point would be where a pipe attaches to a flat surface of a boiler; fixes vertically on the slant of a roof; or, to take a domestic article, forms a spout to a toilet can.

Let $D''G''G'D'$, $EFGD$, (Fig. 58) be respectively elevation and plan of a body, the side of which, $G''G'$ in elevation,

FIG. 58.



F G in plan, is flat, and let $1' e i 5'$ be in elevation the part-cone penetrating the flat side. The plane of the free end $e i$ of the part-cone is here taken to be horizontal, the flat side $G'' G'$ being vertical. The apex of the cone is v' . In the plan the free end of the part-cone is omitted, but the cone, $q v 3$, is shown. As will be seen from the plan and elevation, the cone is out of square *in one direction only* with the flat side of the body. The case in which the cone is out of square *both ways* with the flat surface, that is to say, where it would show in elevation as in the Fig., but in plan as $q V 3$, is not taken, because it comparatively seldom occurs in practice.

The necessary patterns in the present problem are, first, a pattern for the part-cone $1' e i 5'$, and secondly a pattern of the curve of penetration $1' 5'$ in elevation, $q 3$ in plan. The following further construction is required.

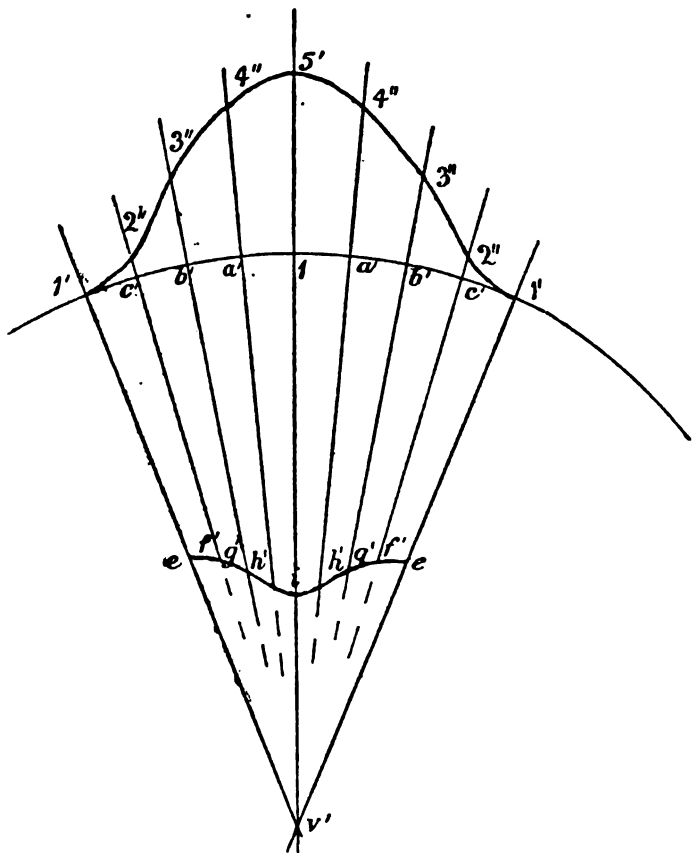
From v' along $v' 5'$ set off $v' 1$ equal to $v 1'$, and join $1' 1$; the line $1' 1$ will represent in edge elevation a circular section of the part-cone. On $1' 1$ describe a semicircle $1' b' 1$, which divide into any convenient number of equal parts (here, to avoid the confusion arising of many lines, four), in the points c', b', a' . Through $c', b',$ and a' draw lines perpendicular to $1' 1$, intersecting it in C, B, and A respectively. Join $v' C$, and produce it to meet $G'' G'$ in point $2'$; similarly join $v' B$, $v' A$ and produce them to meet $G'' G'$ in points $3'$ and $4'$ respectively. Also through $2', 3', 4', f, g$ and h draw lines parallel to $1' 1$, intersecting $v' 5'$ in $2'', 3'', 4'', f', g'$ and h' respectively.

To draw the pattern for the part-cone.

With any point v' (Fig. 59) as centre and radius equal to $v' 1'$ (Fig. 58) describe an arc; at about the centre of it take a point 1, and from v' through point 1 draw an indefinite line. (It is here assumed that the seam of the part-cone will correspond with the line $1' e$, Fig. 58.) Now set off right and left of $v' 1$ distances $1 a', a' b', b' c', c' 1'$ equal respectively to the curve distances of same lettering in Fig. 58. Through each of the points a', b', c' , and $1'$, draw from v' , indefinite lines. Make $v' 5'$ equal to $v 5'$ (Fig. 58), and right and left of $v' 5'$ set off $v' 4'', v' 3'', v' 2''$ equal respectively to the distances $v' 4''$,

$v' 3''$, $v' 2''$ (Fig. 58). Also make $v' i$ equal to $v' i$ (Fig. 58), and right and left of $v' 5'$ set off distances $v' h'$, $v' g'$, $v' f'$, $v' e$ equal

FIG. 59.



respectively to $v' h'$, $v' g'$, $v' f'$, and $v' e$ (Fig. 58). Through the points $1'$, $2''$, $3''$, $4''$, $5'$, $4''$, $3''$, $2''$, $1'$ draw an unbroken curved

line. Also through the points $e, f', g', h', i, h', g', f', e$ draw an unbroken curved line. The figure $e i e 1' 5' 1'$ will be the pattern required.

To find the shape of the hole of penetration.

The case being that of a cone $y v' 5'$ cut by a plane $G'' G'$ which cuts all the generating lines of the cone but is not parallel to its base, the curve of penetration is an ellipse (see § 93A, p. 271-3). The line $1' 5'$ is the length or major axis of the ellipse. Methods of describing an ellipse, length and width being known, have been given at pp. 15-18, and in Problem XXXVII., p. 343. The width of the ellipse is therefore what now has to be found. As the minor axis (width) of an ellipse crosses the major axis at its central point, bisect $1' 5'$ in P ; the point P is then the minor axis in elevation; and we can readily find the length of the line of which it is elevation by drawing the half-base of the cone in which the point lies. Produce $v' 3'$, and through the point P draw a line parallel to $3'' 3'$, intersecting $v' y$ and $v' 5'$ in H and K respectively and the produced $v' 3'$ in O . With O as centre and OH as radius describe a semicircle intersecting $v' 3'$ produced in Z ; then $H Z K$ will be a half base of the cone, turned round, on $H K$ as axis, into the plane of the paper. The line $Z O$ will be a semi-diameter of this base, and the line $P N$ which passes through the point P parallel to $Z O$ and meeting the semicircle in N , will be the half width of the ellipse where P is elevation of the width; will be, that is, half the minor axis required.

If the student will now refer to Problem XIV., p. 274, he will find that this Problem is exactly what has now been explained to him, the semicircle $H Z K$ of Fig. 58 corresponding to the semicircle on the line $A G$ of Fig. 22, p. 275, and the line $P N$ to the line $E F$ of that Fig. If the division of the semicircle $1' 1$ has not been into an even number of parts, the procedure to find $P N$ would be precisely that of finding $E F$ in the Problem referred to.

PROBLEM XXXIV.

To draw the patterns where a conical body (part-cone) penetrates a cylindrical body, the axes of cone and cylinder being in one plane but not at right angles to each other.

Let $D'E'DE$, DFE (Fig. 60) be respectively elevation and part plan of a cylinder, and let $1'v'5'$ be part-elevation, and $E\ v\ 3$ part-plan of the penetrating cone.

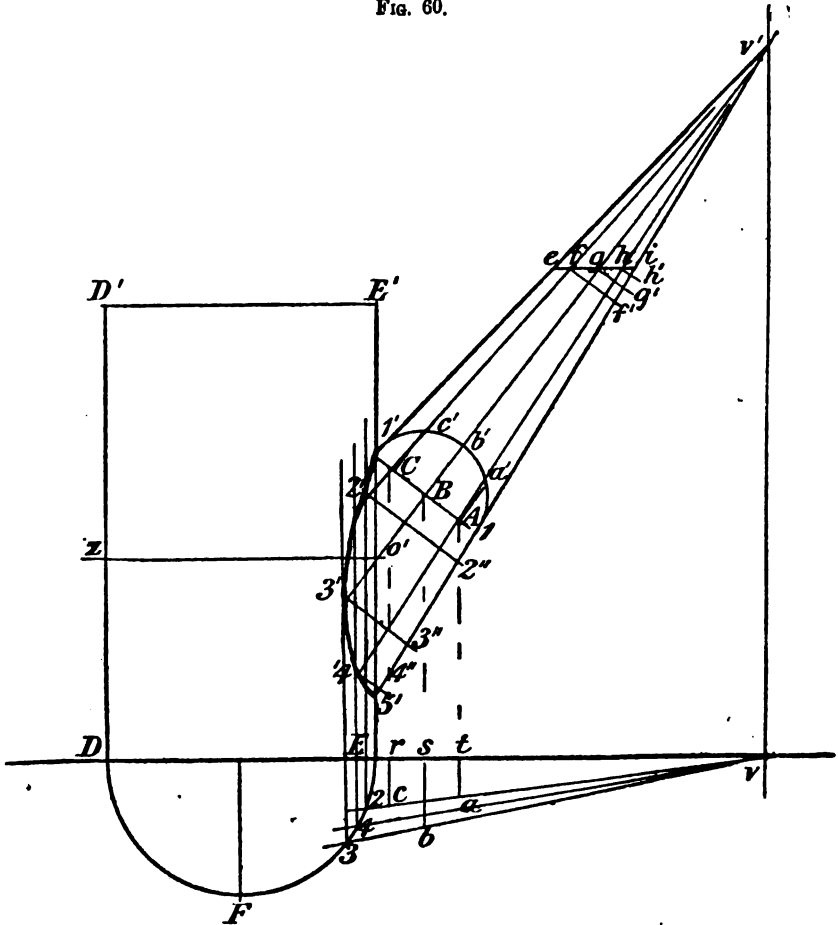
The plane of the free end $e\ i$ of the part-cone is here again horizontal, the cylinder being vertical; but may be at any angle, this affecting only the $e\ i$ end of the part-cone pattern.

As in the last problem there is needed a pattern for the penetrating part-cone, and the shape for the hole, $1'3'5'$ in elevation, $E\ 3$ in half-plan, in the penetrated cylinder. The following further construction is required.

Set off from v' a distance $v'1$ equal to $v'1'$ and join $1'1$; the line $1'1$ will represent in edge elevation a circular section of the cone. On $1'1$ describe a semicircle $1'b'1$, which divide into any convenient number of equal parts (here, to avoid the confusion arising of too many lines, four), in the points c', b', a' . Through $c', b',$ and a' draw lines perpendicular to $1'1$, intersecting it in $C, B,$ and A respectively. Join $v'C$ and produce it; similarly join $v'B, v'A$ and produce them. From the points $C, B,$ and A draw lines perpendicular to vD (plan) and meeting it in the points r, s, t respectively, and produce each of these lines. From r along Cr produced set off rc equal to Cc' , and join vc and produce it to intersect EF in point 2; then $v2$ is the plan of the line $v'2'$. From s along Bs produced set off sb equal to Bb' , and join vb and produce it to intersect EF in point 3; then $v3$ is the plan of the line $v'3'$. From t along At produced set off ta equal to Aa' , and join va and produce it to intersect EF in point 4; then $v4$ is the plan of the line $v'4'$; it falls, the student should notice, inside the line $v3$. The line vE of the given half-plan of the cone is the plan of the line $v'1'$ and also of $v'5'$. The points $E,$

2, 4, 3, are plans of points in the curve of penetration. Through point 3 draw a line perpendicular to D E to inter-

FIG. 60.



sect $v'B$ produced in point $3'$; through point 4 draw a line perpendicular to D E to intersect $v'A$ produced in point $4'$;

and through point 2 draw a perpendicular to DE to intersect $v'C$ produced in $2'$. The points $1', 2', 3', 4',$ and $5'$ are the elevations of points in the curve of penetration, and joining these points by an unbroken line will give the curve in elevation. Through the points $2', 3',$ and $4'$ draw lines parallel to $1'1$ and intersecting $v'5'$ in the points $2'', 3'', 4''$ respectively, and through the points $f, g,$ and h draw lines ff', gg', hh' parallel to $1'1$. Also, parallel to DE draw a line ZO' to measure distances from. The line is here drawn through the point of intersection of $v'B$ produced and $E'E$, but this is not of necessity; it might be drawn through any other point in $E'E$.

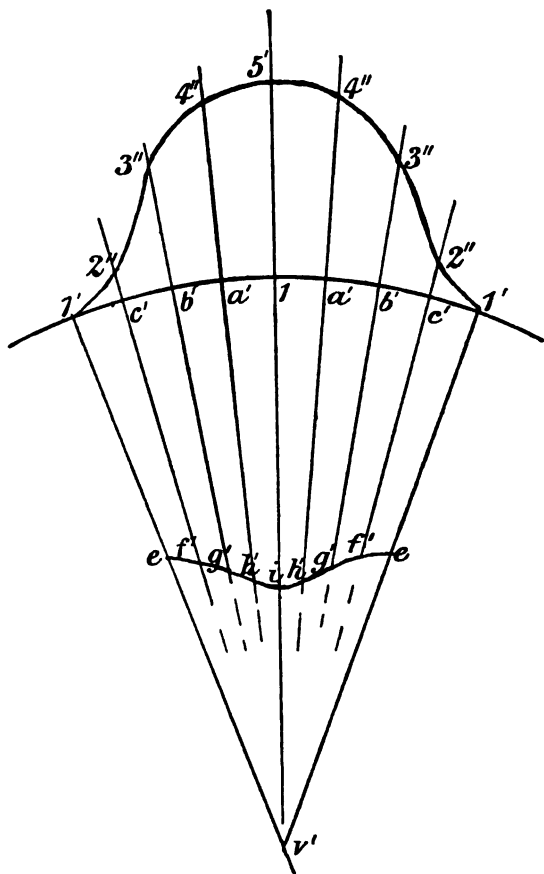
In the previous problem neither plan points were found nor plan lines drawn corresponding to the points and lines just described, for the reason that such points and lines afford no aid at all in the drawing of the shape of the hole of penetration.

To draw the pattern for the part-cone. With any point v' (Fig. 61) as centre and radius equal to $v'1'$ (Fig. 60) describe an arc; at about the centre of it take a point 1, and from v' through 1 draw an indefinite line. (As in the previous problem it is assumed that the seam of the part-cone will correspond with the line $1'e$, Fig. 60.) Now, proceeding point for point exactly as in the previous problem, obtain the figure $eie1'5'1'$, which will be the pattern required.

To draw the shape of the hole of penetration. Essentially the method of doing this has been already given in connection with Problem III., p. 246, the problem of the 'slanting T.' The case there, is that of penetration of a cylinder by a cylinder. Whether the penetration, however, is by a cylinder or by a cone makes no difference as to finding the shape required. For the convenience of the student the working is again here given. Draw (Fig. 62) two indefinite lines $ZO, 1'5'$ intersecting at right angles in O' ; and from O' on ZO set off right and left a distance $O'2$ equal to the distance $E2$ (Fig. 60); to the actual curve distance $E2$ on the cylinder, that is. Similarly, set off right and left of O' the

distances 2 4, 4 3 equal to the distances 2 4, 4 3 respectively (Fig. 60); and through each of the points 2, 4 and 3 draw

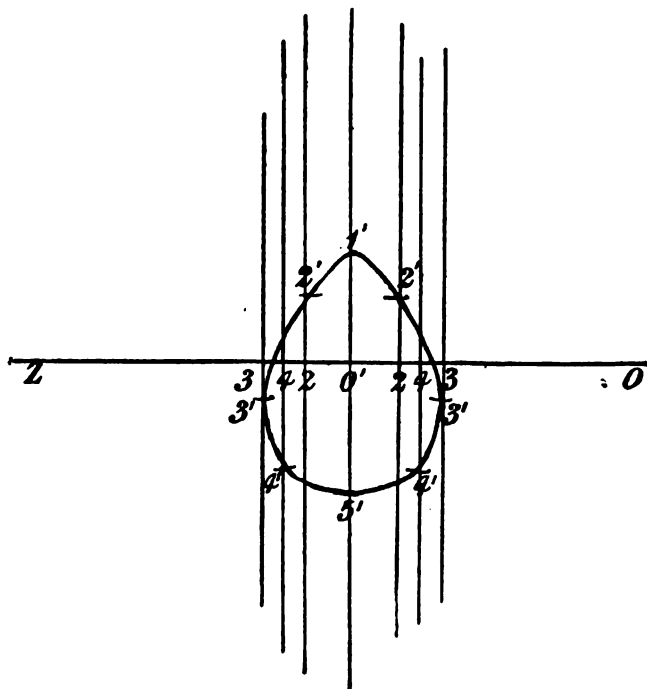
FIG. 61.



lines perpendicular to Z O. From O' on 1' 5' set off O' 1' and O' 5' equal to O' 1', O' 5' (Fig. 60) respectively; also, above

Z O, from each point 2 set off a distance 22' equal to the perpendicular from 2' to Z O' (Fig. 60) and, below Z O, from each point 3 set off a distance 33' equal to the perpendicular

FIG. 62.



distance from 3' to Z O' (Fig. 60), and from each point 4 a distance 44' equal to the perpendicular distance from 4' to Z O' (Fig. 60). Through the points thus obtained draw an unbroken line; this will be the shape required.

PROBLEM XXXV.

To draw the patterns where a conical body (part-cone) penetrates a conical body (part-cone), the axes of the two cones being in one plane, but not at right angles to each other.

Let $O' K' V K$ (Fig. 63) represent a cone frustum, V' being the apex of the complete cone, and $V K Q$ part-plan; and let $1' e i 5'$ be an elevation of the part-cone penetrating $O' K' V K$, with v' for its apex. The Fig. shows the cones in position; the line $V K$ produced is the plan line of the axis of the penetrating cone, the point v being the plan of v' its apex. The plane of the $e i$ end of the penetrating cone is horizontal as before, but may be at any angle, this affecting only the $e i$ end of the part-cone pattern.

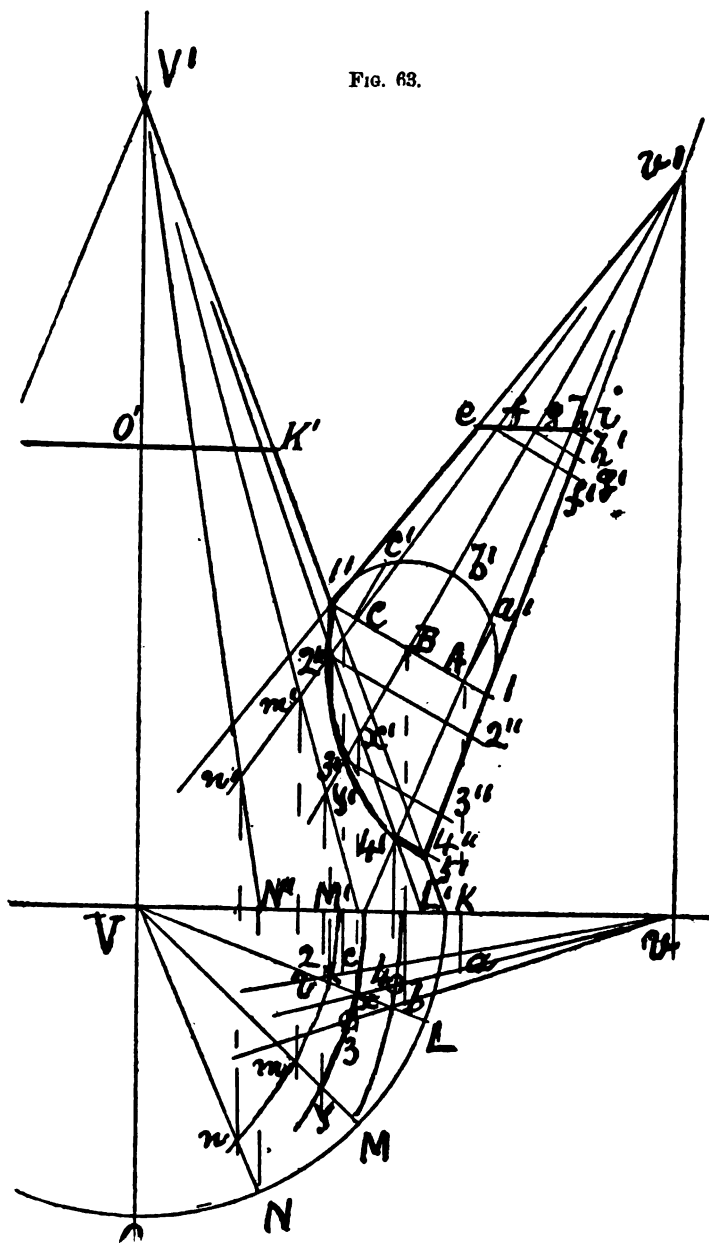
The necessary patterns are a pattern for the penetrating part-cone, and a pattern for the penetrated cone with the shape of the hole of penetration in position upon it. The following further construction, very closely following that of the previous problem, is required.

Set off from v' a distance $v' 1$ equal to $v' 1'$ and join $1' 1$; the line $1' 1$ will represent in edge elevation a circular section of the cone. On $1' 1$ describe a semicircle $1' b' 1$, which divide into any convenient number of equal parts (here four), in the points c', b', a' . Through c', b' and a' , draw lines perpendicular to $1' 1$ intersecting it in C, B , and A respectively. Join $v' C$ and produce it; similarly join $v' B, v' A$ and produce them. From the points C, B , and A let fall lines perpendicular to $V v$ and intersecting it, and produce those lines.

From the point where the perpendicular from A intersects $V v$ set off on the perpendicular a distance equal to $A a'$, and join v with the point a thus found and produce it. Working in like fashion find points b and c and draw lines $v b, v c$ and produce them. Now divide the quadrant of the part-plan of the penetrated cone into any convenient number of equal

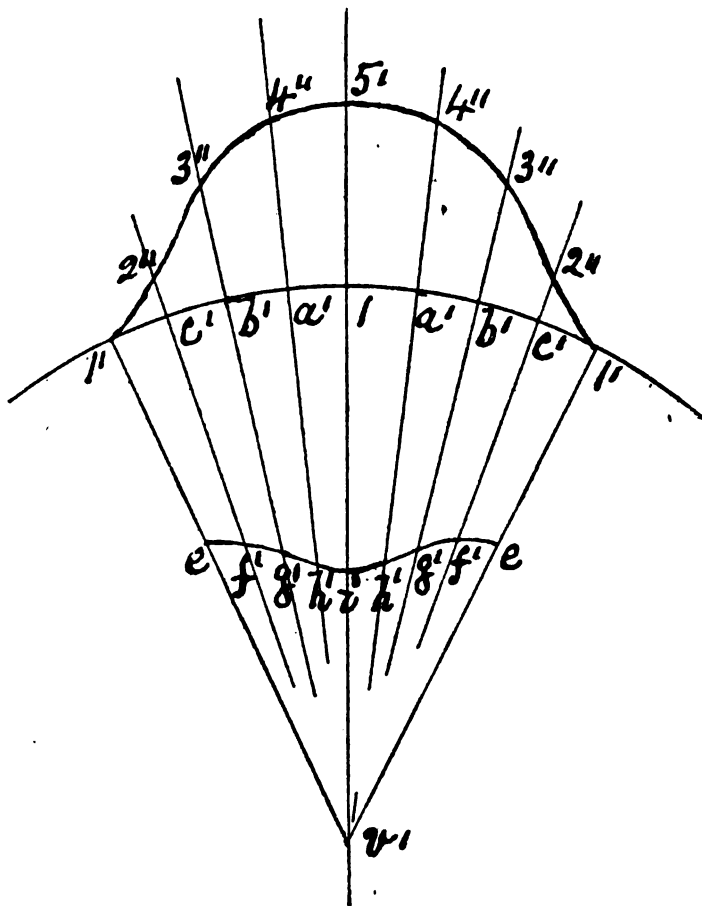
parts (here four), in the points L, M, and N; join L V, M V, N V; and from the points L, M, and N erect perpendiculars, intersecting V v in the points L', M', and N' respectively. Join L' V', M' V', N' V', intersecting v' C produced in the points l' (2' in Fig.), m' and n' respectively. (The point 2' is a point to be presently obtained. It lies, however, so close to l', that l' must be represented by it.) From n', m', and 2' draw lines perpendicular to V v and produce them to intersect V N, V M, and V L in the points n, m and l respectively. Also from the point where v' C produced intersects V' K let fall a perpendicular intersecting V v (the perpendicular is not shown); and through the points n, m, l, and the intersection point just found draw an unbroken curved line. This curve is the plan of the intersection of the part-cone O' K' K V with the plane of v' n', and the point 2 where the curve intersects v c produced is the plan of a point in the curve of penetration of the cones. From point 2 erect a perpendicular, meeting v' n' in the point 2'; this point will be the elevation of point 2 in plan, and we now have two points in the elevation of the curve of penetration, namely the points 1' and 2'. The finding of a third point is similar in the working to the finding of point 2, and is as follows. From the points x' and y' where the line V' B produced intersects V' L' and V' M' respectively, let fall perpendiculars to V v and produce them to intersect V L and V M respectively in the points x and y. From the point where v' B produced intersects V' K let fall a perpendicular intersecting V v (the perpendicular is not shown); and through the points y, x, and the intersection point just found, draw an unbroken curved line. This curve is the plan of the intersection of the part-cone O' K' K V, with the plane of v' y', and the point 3 where this curve intersects v b produced is the plan of the third point in the curve of penetration, the elevation of the point being the point 3' where a perpendicular from point 3 meets v' y'. For a fourth elevation point the working is again similar. From the points (not lettered) where the line v' A produced meets the lines V' L,

FIG. 63.



$V' M'$ respectively, let fall perpendiculars to $V' v$ and, producing them, obtain points of intersection with $V L$ and

FIG. 64.



$V M$ respectively. From the point where $v' A$ produced intersects $V' K$, let fall a perpendicular (the perpendicular

is not shown), and obtain a point of intersection in $V v$. Through the intersection points in $V L$, VM , and $V v$ thus found, draw an unbroken curved line. This curve will be the plan of the intersection of the part-cone $O' K' K V$ with the plane of $v' A$ produced, and the point 4 where this curve intersects $v a$ produced is the plan of the fourth point in the curve of penetration, the elevation of the point being the point 4' where a perpendicular from point 4 meets $v' A$ produced. A fifth point in the elevation is the point 5'. From the point 1', through the points 2', 3', and 4', to 5' draw an unbroken curved line; this curve is the elevation of the curve of penetration. Now through the the points 2', 3', and 4' draw lines parallel to 1' 1, meeting the line $v' 5'$ in the points 2'', 3'', and 4'' respectively; also similar lines through the points f , g , and h meeting $v' 5'$ in f' , g' , and h' respectively.

To draw the pattern for the part-cone.

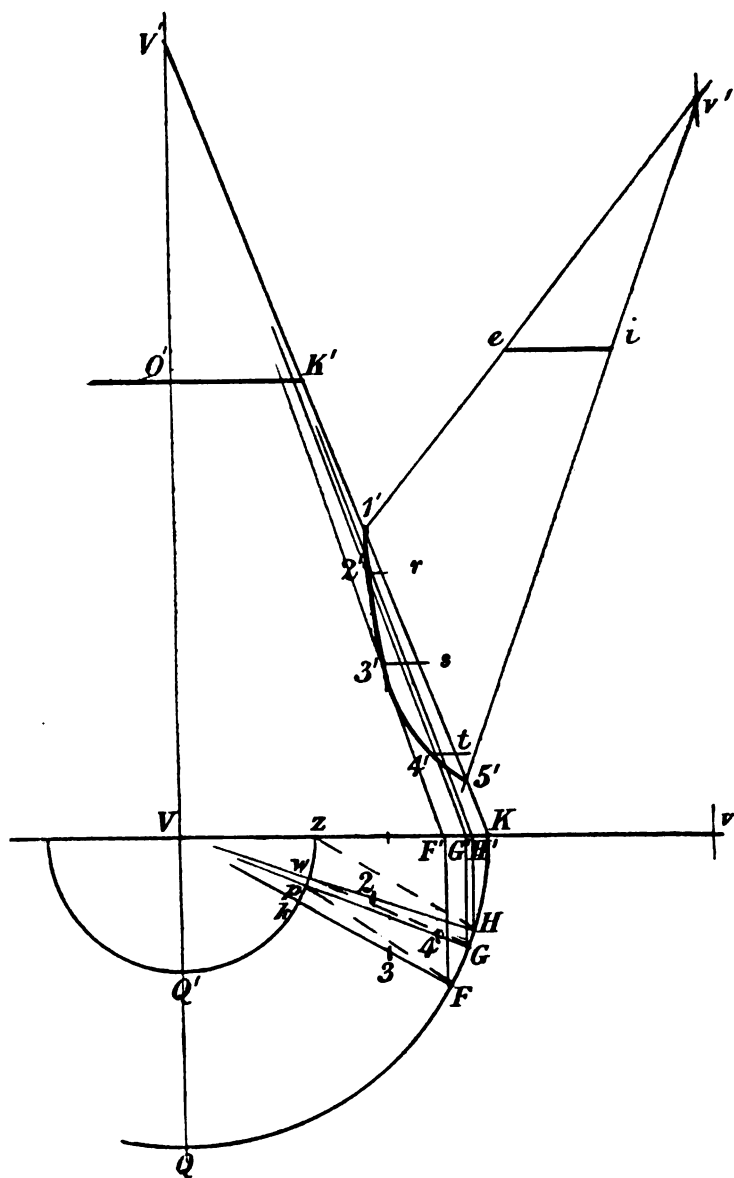
With any point v' (Fig. 64) as centre and radius equal to $v' 1'$ (Fig. 63) describe an arc; at about the centre of it take point 1, and from v' through 1 draw an indefinite line. (As in the previous problem it is assumed that the seam of the part-cone will correspond with the line $1' e$, Fig. 63.) Now, proceeding point for point exactly as in the previous problem, obtain the figure $e : e 1' 5' 1'$, which will be the pattern required.

To draw the shape of the hole of penetration.

In order to make the working clear, and avoid complication of lines, part of Fig. 63 is reproduced in Fig. 65, with which we now have to deal. Join V' with each of the points 2', 3', and 4' and produce $V' 2'$, $V' 3'$, $V' 4'$ to intersect $V K$ in $H' F'$, and G' , respectively. Through F' , G' , and H' draw $F' F$, $G' G$ and $H' H$ perpendicular to $V K$ and intersecting the quadrant $K F Q$ in F , G , and H respectively. Through the points 2', 3', and 4' draw lines 2' r , 3' s , and 4' t parallel to $V K$ and intersecting $V' K$ in the points r , s and t respectively.

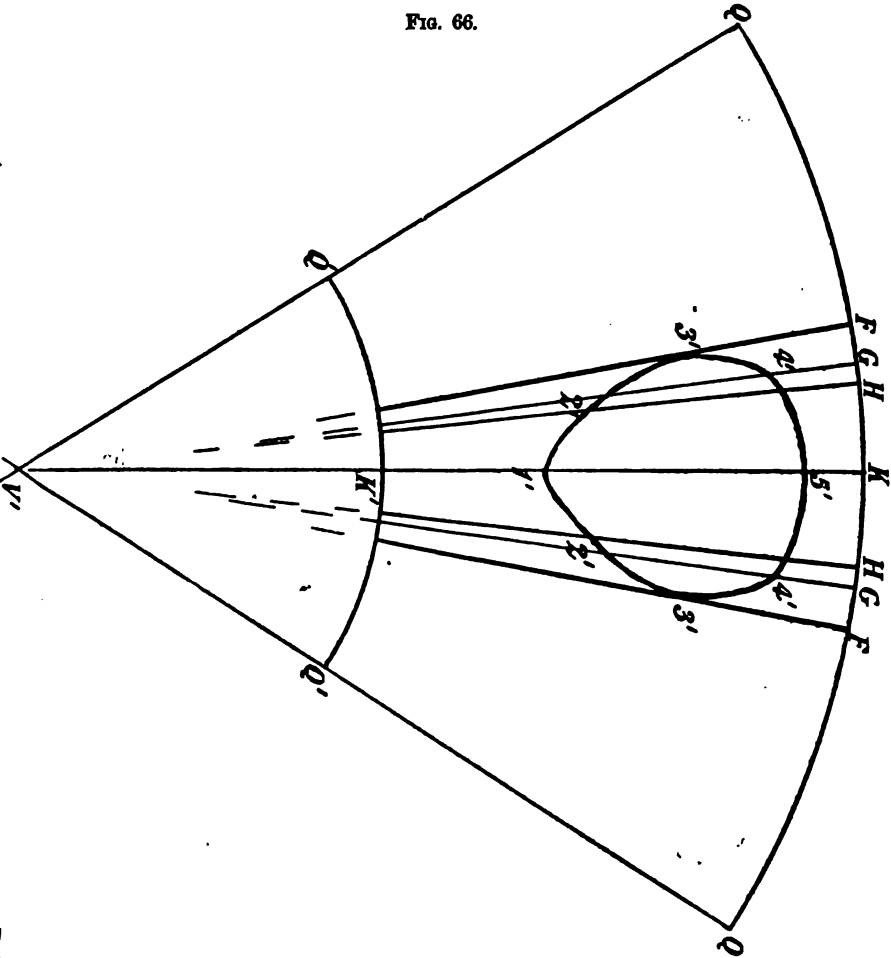
Next, with V' (Fig. 66) as centre and radius $V' K$ (Fig. 65) describe an arc $Q Q$, and in it take any convenient point K , and right and left of K set off distances $K H$, $H G$, $G F$ equal

FIG. 65.



respectively to the curve distances K H, H G, G F (Fig. 65).
Join V' K and right and left of V' K join the points H, G, and

FIG. 66.



F to V'. On V' K from K set off distances K 1', K 5' equal
respectively to K 1', K 5' (Fig. 65); and make the distances

G 4', H 2', F 3' right and left each of them of V' K, equal respectively to the distances K t, K r, K s. Join the points 1', 2', 3', 4', 5', 4', 3', 2', 1' by an unbroken curved line; this will give on a portion of the pattern for the penetrated cone the required shape of the hole of penetration.

If on Q Q from the point F right and left of V' K a distance F Q be set off equal to the curve distance F Q (Fig. 65) and each of the points Q be joined to V', and if further with V' as centre and radius V' K' (Fig. 65) an arc Q' Q' be described intersecting the lines V' Q in the points Q', Q', then the figure Q' Q Q' Q' with the figure 3' 5' 3' 1' within it will be the pattern of half the penetrated cone-frustum with, in proper position with respect to the pattern, the shape of the hole of penetration.

PROBLEM XXXVI.

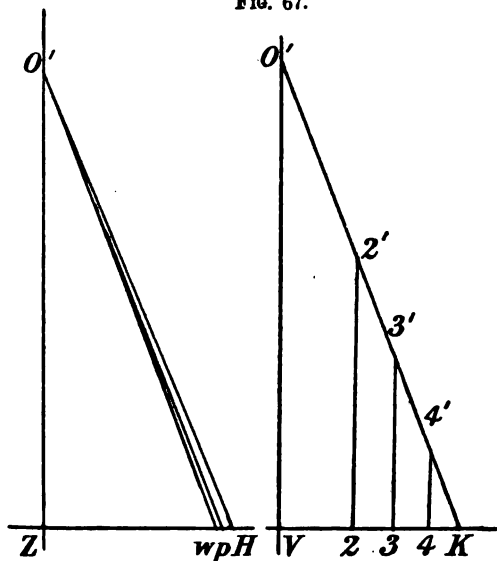
To draw, without long radii, the shape of the hole of penetration.

Large work is now supposed, and consequently that the apex of the cone-frustum is inaccessible. Having, however, the necessary dimensions as to frustum and penetrating cone (either the ends and height of the frustum, or ends and slant), we can draw (see Problems VI. and VII., pp. 37 and 39) the part elevation O' K' K V (Fig. 63) of the frustum and its part plan V K Q, and also the elevation 1' e i 5' of the penetrating cone in position. The procedure to draw the hole of penetration by short radius starts with the plan points of the curve of penetration. The method of finding these (points 2, 3 and 4 of last problem), differs from that there described only in the way of obtaining the lines from L, M and N intersecting O' K'. The lines in Fig. 63 were drawn to the cone apex: with our frustum, now, we have no apex. Draw, therefore, with V as centre, and O' K' as radius a quadrant intersecting the lines V L, V M and V N (none of the construction here described is shown; the student will, however, have no difficulty in mentally following on Fig. 63 what has to be done); and, from the points of inter-

section, erect perpendiculars to intersect $O'K'$. Joining these points of intersection in $O'K'$ with L' , M' and N' will give the lines required.

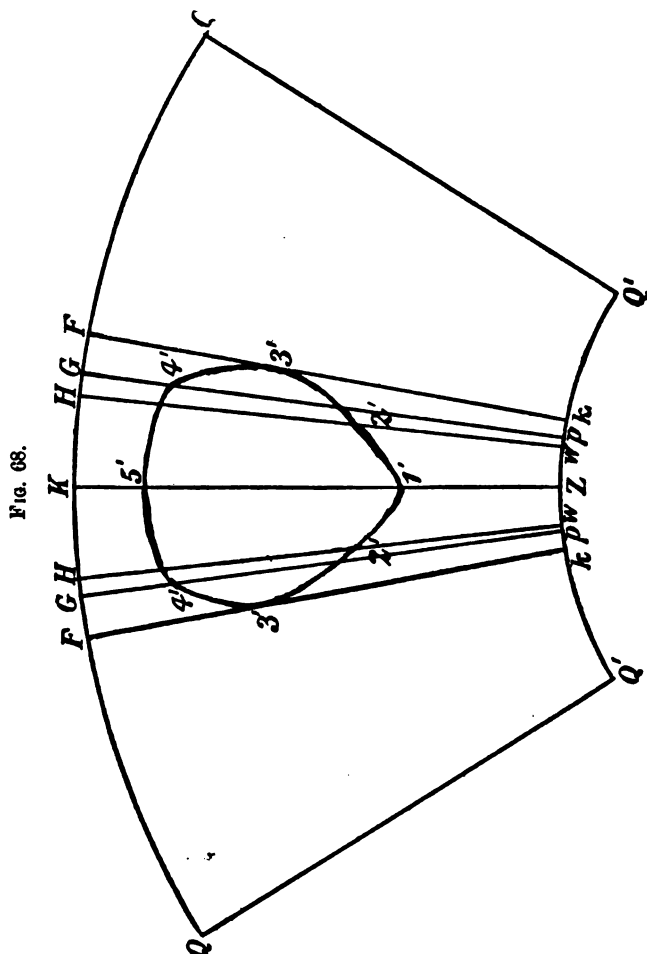
The plan points having been found, draw (Fig. 65), (the part reproduction of Fig. 63) from V through the plan points lines intersecting the quadrant KQ in H , F and G , VH being through point 2, VF through 3, and VG through 4. The quadrant zQ' in Fig. 65 is the quadrant the student was just above instructed to mentally draw in Fig. 63, and this quadrant intersects the lines now drawn in w , p and k respectively. Join $H z$, $G w$, $F p$.

FIG. 67.



Next (Fig. 67) draw two lines ZO' , ZH , perpendicular to each other and intersecting in Z ; make ZO' equal to VO' (Fig. 65), ZH equal to zH (Fig. 65), Zw equal to Gw (Fig. 65), and Zp equal to Fp (Fig. 65); and join $O'H$, $O'p$, $O'w$; then $O'H$, $O'w$ and $O'p$ are the true lengths of

the diagonals represented in plan by zH , Gw and Fp respectively. Now (Fig. 67) draw two lines VO' , VK perpen-



dicular to each other and intersecting in V , make VO' equal to VO' (Fig. 65), make VK equal to zK (Fig. 65), and join

O' K. Then O' K is the true length of z K, w H, p G, k F, and Q' Q (Fig. 65). Make V 2, V 3, and V 4 equal respectively to w 2, k 3, and p 4 (Fig. 65), and through the points 2, 3, and 4 draw lines perpendicular to V K and intersecting O' K in the points 2', 3', and 4' respectively. This gives the points 2', 3', and 4' in their true positions on their generating lines, positions corresponding respectively with the positions of the points r , s , and t on V' K (Fig. 65) as obtained in long radius method, last Problem.

To set out the hole of penetration.

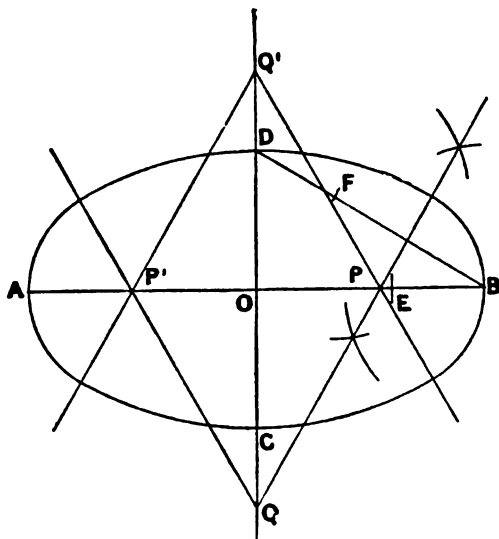
Draw (Fig. 68) a line K Z equal to O' K (Fig. 65), and with Z and K as centres, and radii respectively equal to O' H (Fig. 67) and K H (Fig. 65), describe right and left of K arcs intersecting in H, and with H and Z as centres and radii O' K (Fig. 67), and z w (Fig. 65), describe right and left of Z arcs intersecting in w . Also with w and H as centres and radii O' w (Fig. 67), and H G (Fig. 65), describe right and left of K arcs intersecting in G; and with G and w as centres and radii O' K (Fig. 67) and w p (Fig. 65), describe right and left of Z arcs intersecting in p . With p and G as centres and radii O' p (Fig. 67) and G F (Fig. 65), describe right and left of K arcs intersecting in F; and with F and p as centres and radii K O' (Fig. 67) and p k (Fig. 65), describe arcs intersecting in k . Join right and left of K Z the points H w , G p , F k and Q Q. On K Z set off K 5' and K 1' equal respectively to K 5' and K 1' (Fig. 65); on each line H w set off H 2' equal to K 2' (Fig. 67), on each line G p set off G 4' equal to K 4' (Fig. 67), and on each line F k set off F 3' equal to K 3' (Fig. 67). Now through the points 1', 2', 3', 4', 5', 4', 3', 2' to 1' draw an unbroken line, and this will give the shape of the hole required. Also through F, G, H, K, H, G, F draw an unbroken curved line, and through k , p , w , Z, w , p , k . The arcs F Q and the arcs k Q' are obtained as shown in Problem VIII., p. 41. Join the extremes of these arcs by the lines Q Q'; we then have complete the pattern of the half-frustum O' K' K V with the hole of penetration in position upon it.

PROBLEM XXXVII.

To draw an oval, its length and width being given.

For long narrow ovals this method is better than that of Problem XII. p. 13.

FIG. 69.



Draw (Fig. 69) two lines AB , CD , perpendicular to one another, and intersecting in O . Make OA and OB each equal to half the length, and OC and OD each equal to half the width, of the oval. From O mark off OE equal to OD . Join DB ; make DF equal to EB , and bisect BF by a line perpendicular to it, and cutting the axes AB , and CD produced, in points P and Q respectively; then P will be the centre for an end curve, and Q the centre for a side curve. Make AP' equal to EB , and OQ' , on CD produced, equal to OQ ; join $Q'P$, $Q'P$, and $Q'P'$. With Q as centre and radius QD describe an arc, meeting the produced lines $Q'P'$ and QP ; with Q' as centre and same radius describe an arc

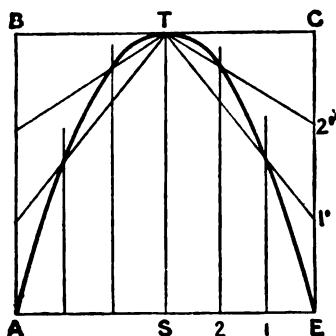
meeting the produced lines $Q'P'$ and $Q'P$; and with PB as radius and P and P' successively as centres, describe arcs to meet those just drawn; this will complete the oval.

PROBLEM XXXVIII.

To draw a parabola (§ 94A, p. 278), the axis and width (see Problem XVII., p. 279) being given.

Draw (Fig. 70) two lines TS and AE , at right angles and meeting in S . Make ST equal to the given axis, SE equal to

FIG. 70.

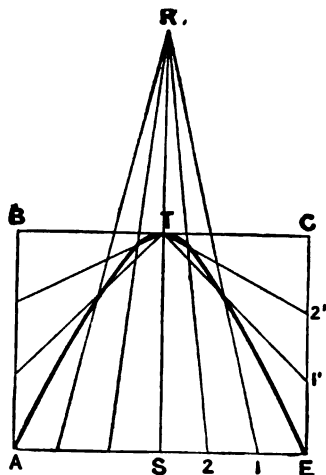


half the given width (double ordinate, see Fig. 25, p. 279), and SA equal to SE . Through E and A draw EC and AB perpendicular to AE , and through T draw BC parallel to AE and cutting AB and EC in points B and C respectively. Divide SE into any number of equal parts (here three) in the points 1 and 2 , and divide EC into the same number of equal parts that SE is divided into, in the points $1'$ and $2'$. Through points 1 and 2 draw lines parallel to TS , and join $1'T$ and $2'T$; then the intersection of $T2'$ with the line drawn through point 2 will be a point on the parabola, as also will be the intersection of $1'T$ with the line through point 1 . Through T and the points just obtained, draw to E an unbroken curved line; repeat the construction on the other side of TS , and the required parabola is completed.

PROBLEM XXXIX.

To draw an hyperbola (§ 95A, p. 273), the length, major axis and width (double ordinate) being given.

FIG. 71.



Draw (Fig. 71) two lines RS and AE at right angles and meeting in S . Make RT equal to the given major axis, ST equal to the given length of the hyperbola (see Fig. 27, p. 282). Make SE equal to half the given width (double ordinate), and SA equal to SE . Through E and A draw EC and AB perpendicular to AE , and through T draw BC parallel to AE and cutting AB and EC in points B and C respectively. Divide SE into any number of equal parts (here three) in the points 1 and 2 , and divide EC into the same number of equal parts that SE is divided into, in the points $1'$ and $2'$. Join $1R$, $2R$, $1'T$ and $2'T$; then the intersection of $1R$ and $1'T$ will be a point in the hyperbola, as will also be the intersection of $2R$ and $2'T$. Through T and the points just obtained draw to E an unbroken curved line; repeat the construction on the other side of SR , and the required hyperbola is completed.

BOOK IV.

CHAPTER I.

PIPE BENDING.

BEFORE taking up problems on the construction of bends from flat sheet-metal, it will not be outside the scope of this volume to devote some little space to a consideration of the methods to employ, and tools to use, in bending ordinary metal tubing, as this is a knowledge oftentimes wanted in the sheet-metal workshop, and is by no means common knowledge there. The workman who is unacquainted with these methods and tools finds great difficulty in giving to a straight piece of tubing a desired curvature; in, that is, so shaping it, that the completed bend shall be free from ridges, dents, or 'kinks,' and not flattened in the throat; the perfection of a bend, of course, being when the internal diameter or 'bore' of the tube is the same throughout.

What has to be done in the way of tube bending in the ordinary workshop must be done with the appliances that are usually found there, or can easily be made there. The tools necessary for the production of bends, specially smooth and circular in section, such as those of wind musical instruments, cannot be made in the ordinary sheet-metal workshop; it will be well, however, to also give a brief description of the working of these bends.

The method of bending a pipe or tube varies with the metal of it, its diameter, and the curvature required. Ordinary gas barrel, of the smaller sizes, may readily be bent without any special preparation to curves, even of small radius. A piece of iron, preferably round iron, being gripped in a vice, horizontally or vertically as most convenient, and

used as a fulcrum or bending post, the barrel may be shaped round it to the curve required. A slight curvature should first be given, then, changing the place of contact of barrel and fulcrum, a further curvature; and so on. A piece of wood, if of sufficiently large diameter to bear the strain it will be put to, may be similarly used. If the barrel is of so large diameter that its bending against a piece of iron in the manner described is beyond the workman's strength, it should be made red-hot where the bend is to come. Thus softened, the workman will find the bending comparatively easy. In bending iron barrel in this way there is always a slight flattening in the throat of the bend. In such work, however, as iron barrel is generally used for, this flattening is not of consequence; the thickness of the barrel and the toughness of the metal prevent any great amount of flattening.

The procedure necessary to bend tubes made of metals softer than iron—of copper, brass, zinc, tin, lead, or 'compo,' for instance—is less simple, and to prevent buckling, puckering or flattening, in the throat of a bend, such tubing must be 'loaded.' The substances used for loading are various, and the choice between them depends upon the metal of which the tube to be bent is made, and greatly upon the finish and symmetry required in the particular work to be done. Lead, resin, pitch, and resin and pitch in equal parts, are the substances most in favour amongst workmen. When 'set' after being melted and run into tubing, they are found to bend without breaking as the bending of a tube progresses, and to offer the needful resistance to change of section of it. A spiral spring not closely coiled, or a piece of cane or solid rubber, may often be advantageously used to 'load' soft metal pipe with. Either of these loadings can be pulled through the made bend, and will serve again and again as loading. A tightly-rolled piece of paper will often serve, even for brass tubing; if used for brass tubing, it can be burnt out if need be. On an emergency, and if the ends of the piece of tube to be bent are tightly corked or otherwise sealed up, and the look of the finished work is not of particular im-

portance, a tube may be loaded with sand, or even with water. The reason why sand and water, as loading, are suitable only for an emergency, is that the plugging at one of the tube ends often gives way in the course of the bending.

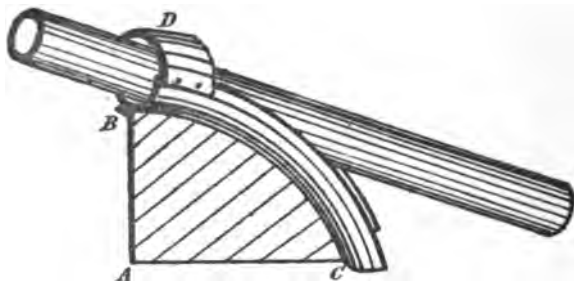
The melting-point of lead is 323° C., and, as a loading substance for brazed brass tube, lead has this disadvantage—that when melting it out of a bent tube, there is danger lest any weak spot in the brazed seam should crack or open up, and special care needs to be taken to warm up the tube slowly and equally in melting out the lead because of this. When the lead has been run out of a bent tube, little particles of lead often remain in the tube adhering to the surface. To dislodge these, the tube should be again warmed up, to a temperature a little higher than that of the melting point of lead, and the open end struck smartly on the bench, or with a piece of wood, the tube being held with a pair of pliers, or otherwise as may be convenient. Resin or pitch, as loading substances, leave behind a thin adherent film after being melted out of a tube. This must not be forgotten when choice of a loading substance has to be made. If it is imperative that the inner surface of a bent tube should be clean, then neither of these substances can be used with a tube of soft metal, as the film has to be *burnt* off, which would mean spoiling the tube. They may be used with copper or brass tubing, when, for the reason that the throat of the bend need not be perfectly circular in section, it is desired that the loading substance shall not offer any great resistance to bending.

Brass or copper drawn tubing should be annealed before being bent. In loading a piece of pipe with either lead, pitch, or resin, two or three layers of brown paper should be wrapped round one end of it and securely tied. If lead is the loading material, the tube should be rigidly fixed vertically, with its closed end embedded in sand, so that molten lead may not run out to do mischief. The lead is poured from an ordinary plumbers' ladle. And in loading with pitch and resin the tube should at least rest and be

secured with its closed end on some solid substance, to prevent leaking out of the hot pitch or resin.

The appliances commonly used in bending light tubes are shown in Figs. 1 and 3. In Fig. 1, B C is a channel-piece of stout metal, say brass, of semicircular section. It may be made by cutting a tube into halves longitudinally, and giving to one of the pieces the desired B C curve, by stretching with blows of a hammer each of the longitudinal edges of the piece. To this channel-piece, at one end of it, a semicircular strap D is soldered or riveted, or both, so as to form at this end of the channel a complete ring-piece. A B C is a stout piece of metal, shaped, one edge of it, to the

FIG. 1.

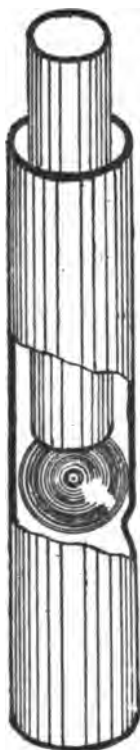


B C curve, and firmly soldered to the throat of the channel piece. The tool in use is gripped in a vice by its A B C piece, one end of the pipe to be bent is placed in the ring-piece D, as shown in the Fig., and the bending is effected in the forcing of the pipe by hand pressure into the B C channel.

Should there be a dent in the straight piece of tube to which curvature has to be given, it may be removed as shown in Fig. 2. A brass or steel ball being forced into the tube, so as to rest on the edge formed by the dent, may be made to pass right through by striking it smartly and repeatedly with a second tube of slightly smaller diameter than the dented tube. Slight dents in a lead-loaded tube or bend may be worked

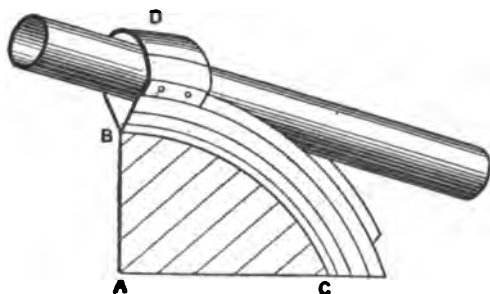
up by means of light blows of a round-faced hammer on either side of the dent; this treatment will force the dent outwards.

Fig. 2.



The appliance shown in Fig. 3 is in all respects the same as that represented in Fig. 1, except that the channel piece B C is of V section instead of semicircular section. Zinc tubes used for 'bead' for baths and other articles are bent without being loaded; and in bending these unloaded tubes a V-shaped

Fig. 3.



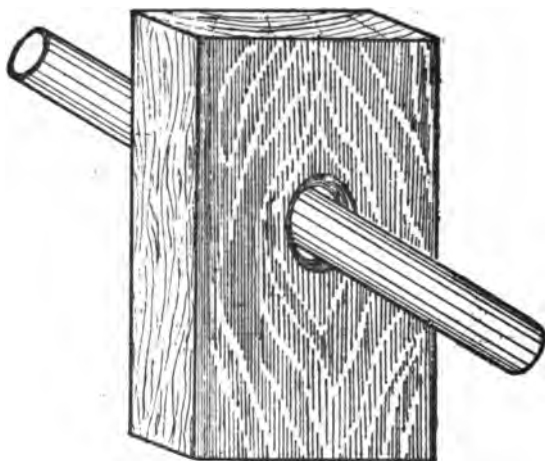
channel-piece answers best, for the reason that the flat sides of it give support to the tube, and help to keep its circular section and to prevent flattening in the throat.

In Figs. 4 to 9, appliances for bending stout tubes are represented. Fig. 4 shows a piece of hard wood with a hole through it, in which is a piece of pipe ready for bending. The hole is just large enough to admit the tube, and the edges of the hole on either side of the wood are rounded, so that in bending the tube there are no sharp edges in the wood to cause dents in the tube. In using the tool, the wood block is fixed in a vice.

Fig. 5 represents an appliance for bending pipes into coils.

A circular block of wood is cut away, part of it, so as to form a shouldered shank, as seen in the Fig., the shank being squared to fit a hole in a bench, or other convenient place. An iron staple is driven into the block, or an iron plate to which a staple has been fixed is screwed on to it. The illustration shows how to start a bend. In making coils, the diameter of the circular portion of the block is that of the hollow of the coil.

FIG. 4.



For making sharp bends in small tubes, the arrangement represented in Fig. 6 is employed. Two iron staples are driven into a stout wooden block, a small movable steel mandrel A A is passed through them, and fixed by wedges B B. The loaded tube is put under the mandrel and bent against it to the curve required.

When a number of pieces of tube of equal diameter, which have been bent to one curve, are required to be of true circular section, and to have their outside surfaces smooth and free from marks, moulds of cast brass are made use of, in

which the tubes can be placed for the necessary manipulation. The moulds are in two pieces and alike in general

FIG. 5.

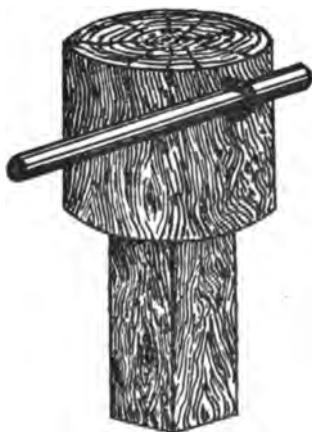
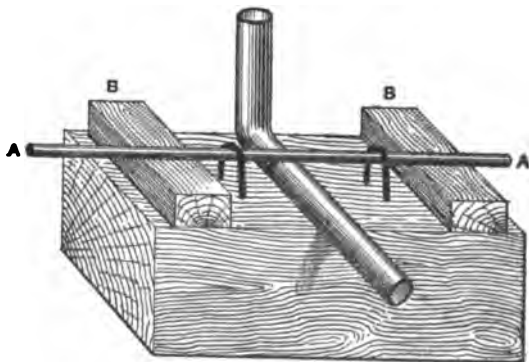


FIG. 6.

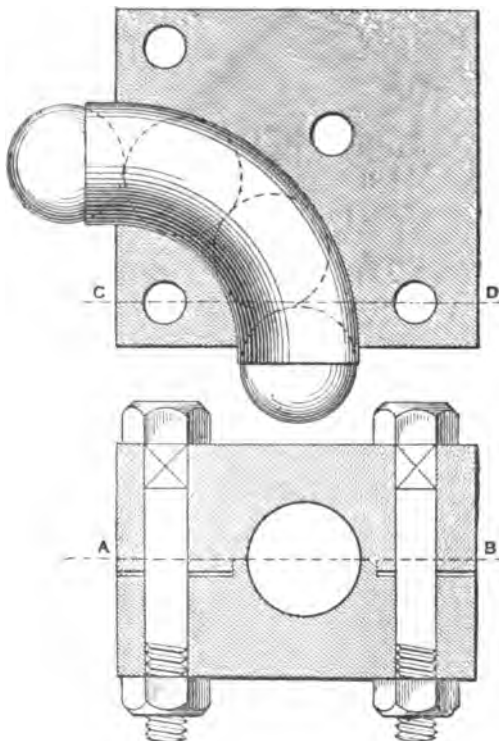


shape, and each piece is channelled; the channelling being such, that when the two pieces of a mould are put together

the channels form a hollow chamber which will hold one of the pieces of bent tube. The grooves are made quite smooth, and each groove of a mould is of the same section throughout,

FIGS. 7 and 8.

Sectional Elevation Section on line A.B



Cross Section on line C.D.

that section being a semi-circle. In the section (Fig. 8) of two half-moulds bolted together, it will be seen that the hollow formed by the two grooves is circular; the circle is

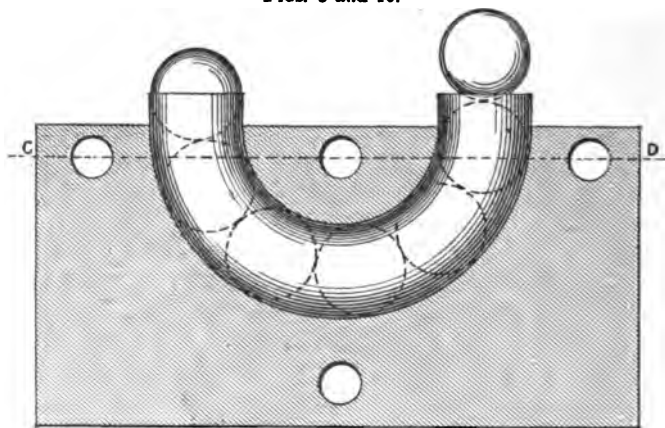
of the same diameter as the outside of the tube that is being worked. Fig. 7 is a section of the upper of the two half-moulds shown in Fig. 8, a section on the line A B; the mould is for a quadrant bend. The through-holes, shown in Fig. 7, are for the bolts by which the mould-halves are fixed together. In the Fig., a piece of tube is represented lying in position in the half-mould. The section, Fig. 8, is in the plane of the line C D of Fig. 7, which plane passes longitudinally through the middle of the bolts that go in the holes on the line C D.

The moulds are thus used. A piece of tube as rough-bent is placed in the groove of a half-mould, the other half-mould is then placed in position upon it, and the two pieces are firmly bolted together. Brass or steel balls (steel by preference) turned quite smooth and true, and of diameter equal to the inside of the tube being worked, are then forced through the tube. One ball having been placed in the tube, it is driven well in by light blows with the mallet. A second ball is placed in the tube on the first, and the mallet again used. In malleting the second ball into the tube, the first ball is driven still further in. A third ball is now applied and dealt with as the first two were, and so on to whatever number of balls are necessary to go right through the tube, the number of balls being determined by the length of the quadrant. A glance at Fig. 7, in which the balls are shown, will make the working quite clear. As the balls fit very closely, all the irregularities that the tube presents are forced outwards against the smooth sides of the mould, and the pressure of the substance of the tube by the balls against the mould groove being continuous, a perfectly smooth exterior surface is given to the tube as well as a true circular section. Should there have been any convexities outside the tube, these will have been already forced down when the tube was tightened up in the mould. The balls should be lightly dusted with black lead before using.

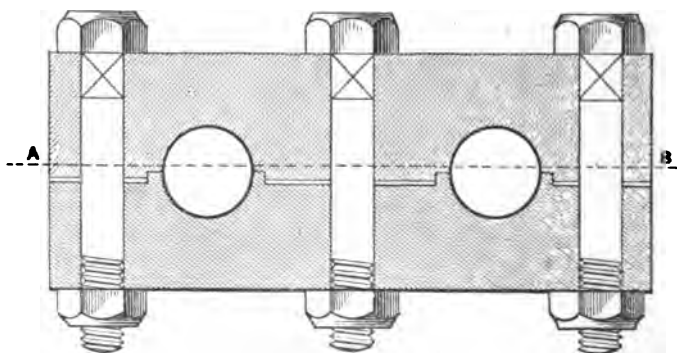
A semicircular bend is worked in like fashion. Fig. 10 is a

section of two half-moulds for such bend bolted up. Fig. 9 is a section of the upper of the two half-moulds shown in Fig. 10, a section on the line A B. The through-holes of Fig. 9 are

FIGS. 9 and 10.



Sectional Elevation. Section on line A.B.



Cross Section on line C.D.

the bolt holes. A semicircular piece of tube lying in the groove of the half-mould is seen in the Fig. The section, Fig. 10, is in the plane of the line CD of Fig. 9, which plane

passes longitudinally through the middle of the bolts that go in the holes on the line C D. The manipulation of a piece of tube, rough-bent into a semicircle and placed in a mould such as represented, is exactly the same as has been explained for a quadrant tube.

Manifestly, a piece of the same tube can be bent into an arc greater than a semicircle with the semicircular mould. The rough bend, as required, being first made, and a semicircular piece of it having been smoothed, the bolts would be loosened and the tube shifted so as to bring a further portion of the rough bend into the hollow of the mould. This being then again bolted up, a driving through of the steel balls, like to that which has been described, will smooth out the further arc. Similarly, an arc greater than a quadrant, or a semicircular arc, or an arc greater than a semicircle, can be worked in the quadrant tool. An arc greater than a semicircle can also be manipulated in the semicircular tool; and with one or other of the tools, or with both. S curves, compounded or two quadrants of two semicircles, or of semicircle and quadrant, can be readily fashioned. And the grooves of the moulds made use of need not be of one radius, though they must always be of one section. An S, for example, could be made up of a large semicircle and a small one.

It may be said in respect of all the pipe bending that has been so far considered, that the result depends more on the tool than on the workman, or, at any rate, as much on the tool as on the workman. Bending *lead* pipe, however, is quite another matter, and has not been specially referred to. Here the result depends far more on the workman than on the tool, and the bending of lead pipe is an important part of the sheet-lead worker's (plumber's) training, and calls for its own tools. The general particulars of pipe bending that have been given will enable a workman in sheet-metal, other than lead, to bend a piece of lead pipe on occasion when a job he is engaged upon may require it, which is all that can fittingly be taught in a book such as this, which is for the most part geometrical.

Bends which are to be of such 'bore,' metal, and thickness, that they cannot, by any of the methods that have been described, be conveniently formed from pipes that correspond in diameter, metal, and thickness, have to be made up out of flat sheet-metal, in pieces, two or more. Various problems in such bends now follow on.

CHAPTER II.

PATTERNS FOR PIPE-BENDS.

A SPECIAL prefatory word is necessary to these problems. Bodies of compound curvature, such as those now under consideration, will not develop into the flat; patterns in the flat for these, therefore, can only be approximate, and the sheet-metal after being cut to a pattern, must be worked into the desired shape by extension and alteration of surface, where required, with the hammer. It may be said of the patterns for such bodies as we are now considering, that they are geometrical compromises, and that the fewer the pattern-pieces for any one such body the greater the compromise. The direction of compromise in each particular instance is decided by experience. As a general rule, a pattern should be started if possible from a central line, and the hammer work left for the surfaces outside of this line. Especially should this course be followed if there is in the body for which pattern has to be made a central line which lies in a flat plane—in a plane, that is, like the surface of the metal plate on which the pattern is to be laid out; because then that line will not be disturbed (will be a neutral line) in the subsequent manipulation. The full meaning of these remarks will be more apparent as we proceed.

PROBLEM I.

To draw the pattern for a bend of equal circular section.

Six cases will be treated of—five in this problem, and one in the problem following.

And, the 'quadrant' bend being a very common one, and an extreme instance, it will be taken as typical. For any other length of arc, however, the working will be exactly similar.

CASE I.—The bend to be made up of two like pieces, each of them semicircular in section. That is, the seams of the bend are to follow the curve of it, one in the middle of the throat, the other in the middle of the back. Manifestly but one pattern-piece is required.

First draw a side elevation and a semicircular section of the half-bend for which pattern is required, as follows. Draw (Fig. 12) two lines $Q 1''$, $Q 7''$ at the same angle to each other as that which the given arc for the bend subtends: here an angle of 90° , that being the angle subtended by a 'quadrant.' On $Q 1''$ set off $Q 1$ equal to the given radius for the inner arc of the bend, and with Q as centre and $Q 1$ as radius describe the quadrant $1 7$; this quadrant will be, in elevation, the inner curve or throat line of the half-bend. Now set off along $Q 1''$ from point 1 a distance $1 1''$ equal to the diameter of the given bore of the bend, and with Q as centre and $Q 1''$ as radius describe the quadrant $1'' 7''$; this quadrant will be, in elevation, the outer curve or back line of the half-bend; and the whole figure $1'' 7'' 7 1$ will be a side elevation of it. Bisect $1'' 1$ in A , and through A at right angles to $Q 1''$ draw a line $A O$. From point A on the under side of $Q 1''$ set off on the line just drawn a distance $A O$ equal to $A 1$, through O draw a line $d D$ parallel to $Q 1''$, and with O as centre and $O A$ as radius describe the semicircle $d A D$; this will be a semicircular section of the half-bend. Now divide each of the quadrants $A d$, $A D$ into any, the same, number of equal parts (here three) in the points b, c and B, C respectively, and with Q as centre and radius $Q A$ describe the quadrant $A P$.

With what line should the laying-out of the pattern be started? Draw (Fig. 11) a front elevation $A P 7'' 7 1$ of the half bend, and continue the line $A P$ of it indefinitely both

XX (that is, lying wholly in the plane XX), being the quadrant line AP (Fig. 12). Now the line AP is the line formed by the contact points of *all* the semicircular sections of the half-bend; and if the pattern for the half-bend is fully drawn on the plane XXXX, Fig. 12 (the figure 1" 7" 7 1 in Fig. 12 is not pattern, but elevation, it must be remembered), this line AP of it remains on that plane right through from the drawing of the pattern for the half-bend to the completion of it by shaping up—is, in fact, a 'neutral' line. The line being thus a constant both as to form and position, and also being central, presents itself as the line with which the drawing of the pattern should be started. The other quadrant lines of the side elevation (the lines 7" 1", 7 1) are still quadrant lines in the finished half-bend, but the lines for them that were drawn on the plane XXXX, the flat surface of the sheet metal, *have not remained on that plane*; they have in the shaping up been transferred to the plane of division of the bend, to the plane that, appearing edgeways in Fig. 11, shows there as the line YY. To make this quite clear. The extreme points 7², 7 (Fig. 11) of the line 7²P 7, which is the *pattern* line for the 7" P 7 end (Fig. 12) of the half-bend, are also the extreme points 7", 7 of, respectively, the quadrants 7" 1", 7 1; and as this line becomes, after the shaping up of the pattern, the curve 7" P 7 (Fig. 11), it is evident that the points 7², 7 on the XX plane have been transferred to the plane YY. And so with all the rest of each of these two quadrants. The quadrant of which P is an extremity has alone remained in the XX plane.

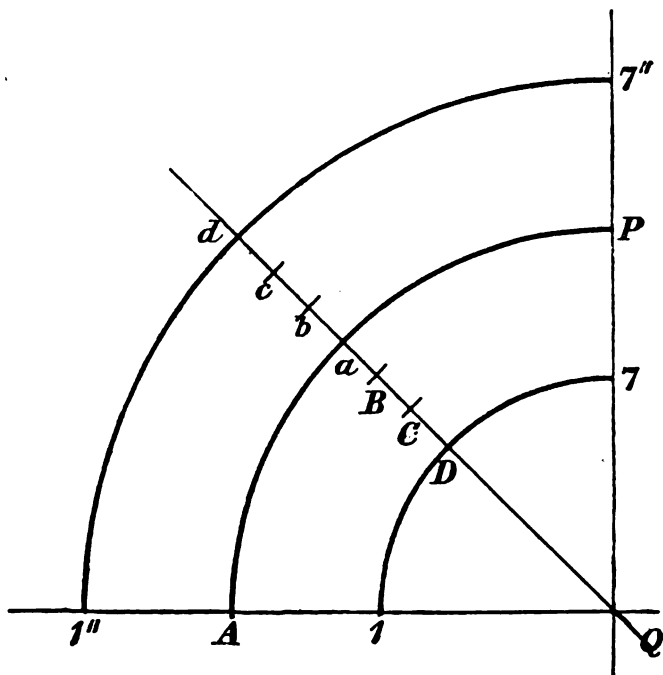
To draw the pattern for the half-bend.

Draw (Fig. 13) at right angles to each other two lines Q 1", Q 7". From Q set off QA equal to QA (Fig. 12), and with Q as centre and radius QA describe the quadrant AP. This line, as we just above found, is the neutral line with which to start the pattern. Bisect AP in point *a*; join Q *a* and produce it; and from *a* towards Q set off one after another the distances *a* B, B C, C D equal respectively to the distances

METAL-PLATE WORK.

A B, B C, C D (Fig. 12), and, away from Q, also successively, the distances $a b$, $b c$, $c d$ equal respectively to the distances $A b$, $B c$, $C d$ (Fig. 12). Through point d with radius $Q d$ describe the arc $1'' 7''$ terminating in the points $1''$ and $7''$ in $Q 1''$ and $Q 7''$ respectively, and through point D with $Q D$ as

FIG. 13.



radius describe the arc $1 7$ terminating in the points 1 and 7 in $Q 1''$ and $Q 7''$ respectively; then $1'' d 7'' 7 D 1$ will be the pattern required. Bent up, stretched and hollowed judiciously with the hammer, and trimmed, it will shape into the half-bend of the elevation and section.

Bends thus made cannot be recommended where any con-

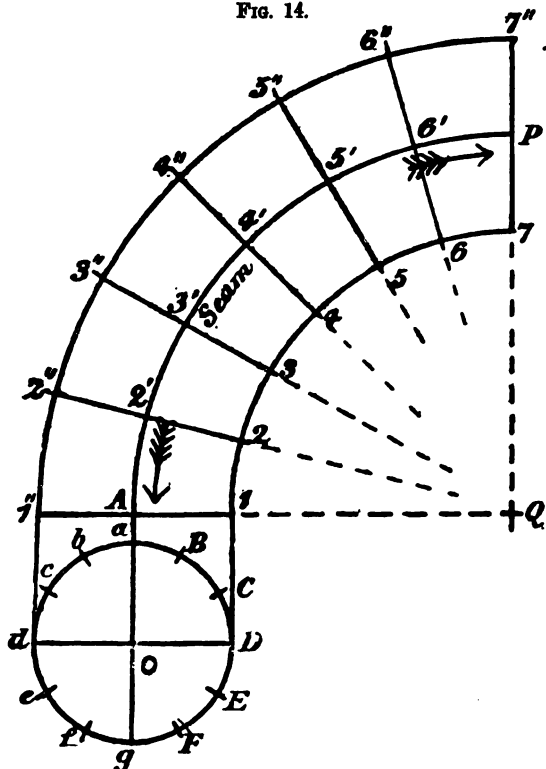
siderable internal pressure is to be resisted ; as, on account of the great amount of stretching required at the throat of the bend, the metal becomes thin and weak there. In a few small sizes, half-bends stamped up are now commercial articles, two stampings making up the complete bend.

CASE II.—The bend is to be made up of two pieces, with the seams following the curve of the bend, centrally on its opposite sides. Two pattern-pieces will be required.

Draw (Fig. 14) two lines $Q1''$, $Q7''$ at right angles to each other (the bend being quadrant) and intersecting in Q . On $Q1''$ set off $Q1$ equal to the given radius of the bend, and with Q as centre and $Q1$ as radius describe the quadrant 17 ; this quadrant will be in elevation the throat of the bend. Now set off along $Q1''$ from point 1 a distance $11''$ equal to the diameter of the required bore of the bend, and with Q as centre and $Q1''$ as radius, describe the quadrant $1''7''$; this quadrant will be in elevation the outer curve of the bend; the figure $1''7''71$ will be a side elevation of the bend. Next draw through the points $1''$, 1 and perpendicular to $Q1''$, two lines $1''d$, $1D$; through any point d in $1''d$ draw dD parallel to $Q1''$; with O the mid-point of dD as centre, and Od as radius describe the circle $d a D g$; and through O draw a line gA perpendicular to $Q1''$, cutting the circle just described at g and a and meeting $Q1''$ at A . The circle $a d g D$ will be a circular section, anywhere, of the bend; the semicircle $a D g$ will be a section anywhere of its throat half, and the semicircle $a d g$ of its back or 'saddle' half. Now with Q as centre and QA as radius describe the quadrant AP ; this quadrant will be the seam of the bend, in elevation; divide the quadrant 17 into any number of equal parts (here six) in the points $2, 3, 4, 5, 6$; and from Q draw lines through these points (the lines are only partly drawn in the Fig.), meeting the quadrant AP in the points $2', 3', 4', 5', 6'$, and the quadrant $1''7''$ in the points $2'', 3'', 4'', 5'', 6''$; which will be a division of the quadrants AP and $1''7''$, each of them, into also six equal parts. Further, divide the arcs $a d, d g$,

$g D$, $D a$, each of which is one quarter of the circumference of the circular section of the bend, into, each of them, any (the same) number of equal parts (here 3) in the points b and c for the arc $a d$, and the points e and f , F and E , C and B for the arcs $d g$, $g D$, and $D a$ respectively.

FIG. 14.



One pattern-piece will be required for the throat portion $A 4' P 7 4 1$ ($a D g$ in section) of the bend, and another for the saddle portion $A 4' P 7'' 4'' 1''$ ($a d g$ in section) of it.

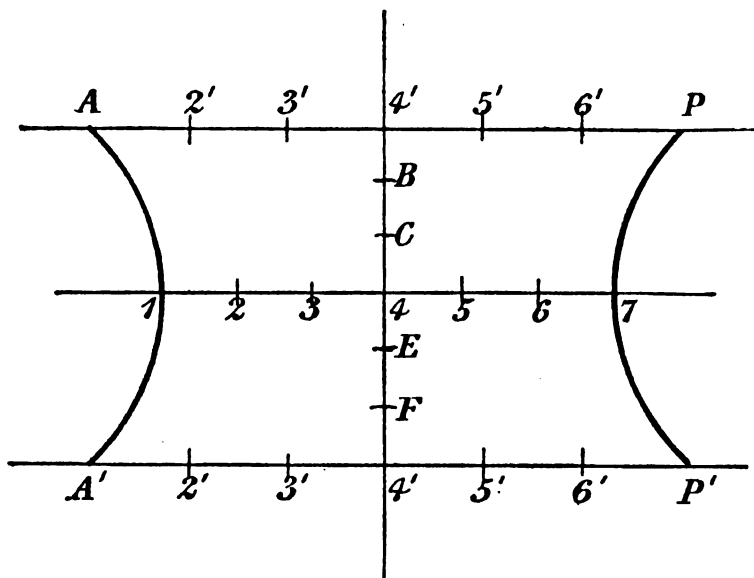
To draw the pattern for the $A 4' P 7 4 1$ portion.

The line to commence it upon is easily decided. If the finished throat portion of the bend were laid on a flat surface, it would rest with its $A 4' P$ edge (or opposite corresponding edge) on that surface. But either of these edge lines is an extremity line, and quite unsuited, therefore, to start a pattern with. And there is no line whatever that can be drawn in the flat for this throat half pattern that would lie against a surface in the shaped up throat piece; no line that would be 'neutral,' that is. This being the case, manifestly the next best thing to do is to start the pattern with a line of single curvature only, that is a line which, in making up from the flat to the half-round, bends wholly in one plane. The only line which answers to this condition is the central line 1 4 7; with this line then, developed of course into a straight line, we will begin the required pattern.

Draw (Fig. 15) an indefinite straight line 1 7; through any point 4 in it draw at right angles a line $4' 4'$. From point 4 to the left of the line $4' 4'$ set off successively the distances 4 3, 3 2, 2 1 equal respectively to the distances 4 3, 3 2, 2 1 (Fig. 14), and to the right of $4' 4'$ the distances 4 5, 5 6, 6 7 equal respectively to the distances 4 5, 5 6, 6 7 (Fig. 14); the straight line 1 7 (Fig. 15) will then be equal in length to the quadrant curve 1 7 of Fig. 14. Now from the point 4 above the line 1 7, set off one after another the distances 4 C, C B, B $4'$ equal respectively to the distances D C, C B, B a (Fig. 14), thus making the line 4 C B $4'$ equal in length to the arc D a (Fig. 14); and, below 1 7 the distances 4 E, E F, F $4'$ equal respectively to the distances D E, E F, F g (Fig. 14), thus making the line 4 E F $4'$ equal in length to the arc D g (Fig. 14). The whole line $4' 4'$ will then be equal in length to the semicircle $a D g$, which is the semicircular section of the throat half of the bend. Through each point $4'$ draw indefinite lines parallel to 1 7; from the point $4'$ on the B C side of 1 7 set off one after another distances $4' 3'$, $3' 2'$, $2' A$ equal respectively to the distances $4' 3'$, $3' 2'$, $2' A$ (Fig. 14), and, also successively, the distances $4' 5'$, $5' 6'$, $6' P$ equal respectively to the distances $4' 5'$, $5' 6'$, $6' P$ (Fig. 14); and from

the point 4' on the E F side of 17, set off successively distances 4'3', 3'2', 2'A' equal respectively to the distances 4'3, 3'2', 2A' (Fig. 14), and the distances, also one after another, 4'5', 5'6', 6'P' equal respectively to the distances 4'5, 5'6', 6'P (Fig. 14). The straight lines A P, A' P' of Fig. 15 will then be each equal in length to the curve line A P (Fig. 14), which is the line of the seam of the bend.

FIG. 15.

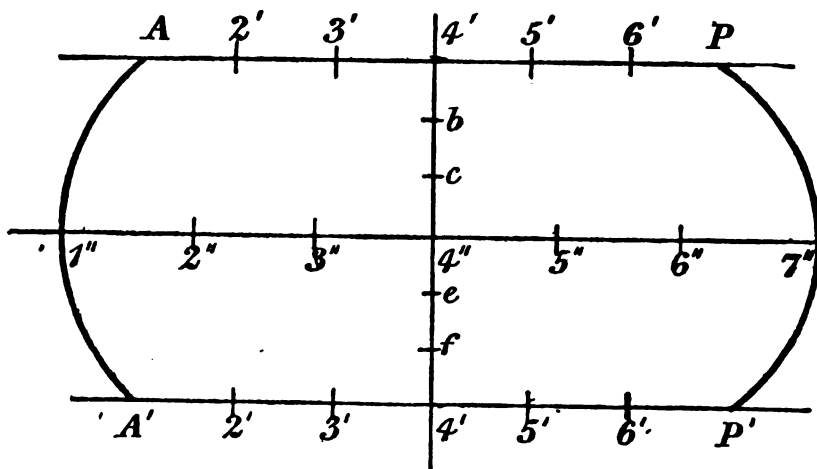


From A to A' draw an arc passing through the point 1, and from P to P' draw an arc passing through the point 7. The figure A 4' P 7 P' 4' A' 1 will be the pattern required. Bent up (the bending of the line 1 4 7 is of single curvature only), got into form by stretching where needed with the hammer, and trimmed, it will shape into the throat piece for the quadrant bend.

To draw the pattern for the $A\ 4'\ P\ 7''\ 4''\ 1''$ (Fig. 14) portion of the bend. Just as for the throat-half we commenced the pattern with the central line $1\ 4\ 7$ developed, we start the saddle-half pattern with the line $1''\ 4''\ 7''$ developed.

Draw (Fig. 16) an indefinite straight line $1''\ 7''$. On it, from any point $4''$, set off successively to the left of $4''$ the distances $4''\ 3''$, $3''\ 2''$, $2''\ 1''$ equal respectively to the distances $4''\ 3'$, $3''\ 2'$, $2''\ 1''$ (Fig. 14), and to the right of $4''$ successively the distances $4''\ 5''$, $5''\ 6''$, $6''\ 7''$ equal respec-

FIG. 16.



tively to the distances $4''\ 5''$, $5''\ 6''$, $6''\ 7''$ (Fig. 14); the whole line $1''\ 7''$ will then be equal to the quadrant curve $1''\ 7''$ (Fig. 14). Through point $4''$ draw a line $4'\ 4''$ perpendicular to $1''\ 7''$, and from the point $4''$ set off one after another the distances $4''\ c$, $c\ b$, $b\ 4'$ equal respectively to the distances $d\ c$, $c\ b$, $b\ a$ (Fig. 14), thus making the line $4''\ c\ b\ 4'$ equal to the arc $d\ a$ (Fig. 14); and, also successively, the distances $4''\ e$, $e\ f$, $f\ 4'$ equal respectively to the distances $d\ e$, $e\ f$, $f\ g$ (Fig. 14), thus making the line $4''\ e\ f\ 4'$ equal to the arc $d\ g$

(Fig. 14). The whole line $4' 4'$ will then be equal in length to the semicircle adg , which is the semicircular section of the back half of the bend. Through each point $4'$ draw indefinite lines parallel to $1'' 7''$; from the point $4'$ on the bc side of $1'' 7''$ set off one after another distances $4' 3', 3' 2', 2' A$ equal respectively to the distances $4' 3', 3' 2', 2' A$ (Fig. 14), and, also successively, the distances $4' 5', 5' 6', 6' P$ equal respectively to the distances $4' 5', 5' 6', 6' P$ (Fig. 14); and from the point $4'$ on the ef side of $1'' 7''$ set off successively distances $4' 3', 3' 2', 2' A$ equal respectively to the distances $4' 3', 3' 2', 2' A$ (Fig. 14), and the distances, also one after another, $4' 5', 5' 6', 6' P$ equal respectively to the distances $4' 5', 5' 6', 6' P$ (Fig. 14). The straight lines $AP, A'P'$ of Fig. 16 will then be each equal in length to the curve line AP (Fig. 14), which is the line of the seam of the bend. From A to A' draw an arc passing through the point $1''$, and from P to P' draw an arc passing through the point $7''$. The figure $A 4' P 7'' P' 4' A' 1''$ will be the pattern required. Bent up (the bending of the line $1'' 4'' 7''$ is of single curvature only), got into form where needed with the hammer, and trimmed, it will shape into the saddle-piece for the quadrant bend.

Making thus, in two pieces, is of advantage when the seams have to be brazed, as the pieces when lying on a flat surface will butt up against each other.

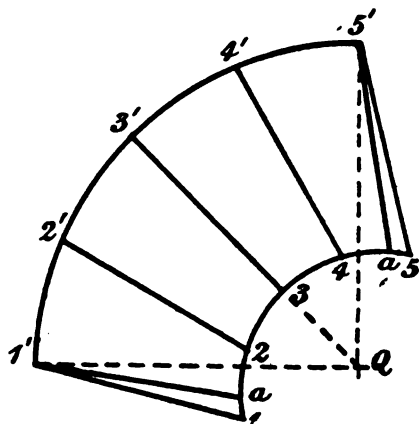
CASE III.—The bend to be made up of four pieces; the seams, all of them, to follow the curve of the bend; one to be in the middle of the throat, one in the middle of the back, and the other two central on the opposite sides of the bend. Two pattern-pieces will be required, one for each half-throat of the bend and the other for each half-back of it.

This Case is a sort of combination of the two previous Cases, inasmuch as the seams come in throat, back *and* sides. Patterns for the pieces of which the bend is to be made up might be obtained either by dividing the half-bend

undergo curvature at all; in Case II. there is no such line. From this it is evident that we had better work from the Case I. neutral line. Making-up bends in four pieces is necessary for large work principally, and it is desirable with such work to construct the half-throat pattern in the way that is shown just below. Let us now deal with the problem.

Draw (Fig. 17) a side elevation $1'' 5'' 5' 1$ of the bend and a circular section agm of it, in the manner described in Case II. Also draw the central quadrant $1' 5'$; divide the arc $1 5$

FIG. 18.



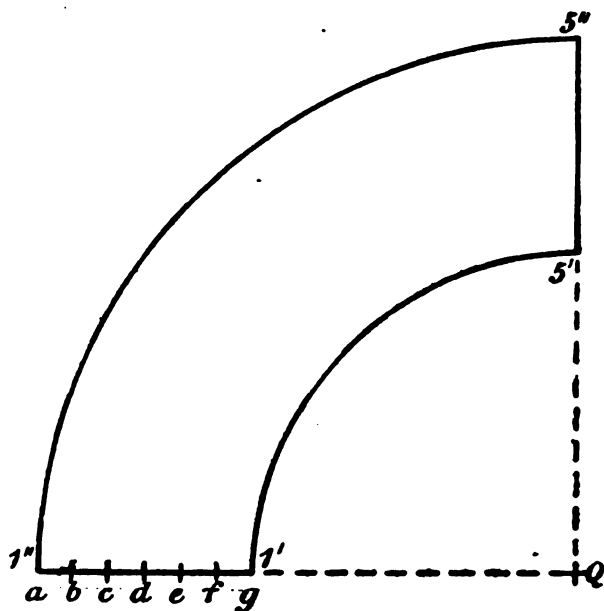
into any number of equal parts (here four) in the points 2, 3, 4; and draw lines from Q through these division points, cutting the central quadrant in the points 2', 3', 4' respectively. and the outer curve of the elevation in the points 2'', 3'', 4'' respectively. Also divide the quadrant ag of the circular section into any number of equal parts (here six) in the points b, c, d, e and f .

To draw the pattern for the half-throat piece $1' 3' 5' 5' 3' 1$.

Draw (Fig. 18) two lines $Q 1'$, $Q 5'$ perpendicular to each other and intersecting in Q. With Q as centre and $Q 1'$

(Fig. 17) as radius, describe an arc $1'3'5'$ cutting $Q1'$, $Q5'$ in points $1'$ and $5'$ respectively. With this quadrant arc, equal to the quadrant $1'3'5'$ (Fig. 17) we start the pattern. First divide the curve $1'3'5'$ into the same number of equal parts that the curve $1'3'5'$ (Fig. 17) is divided into, in the points $2'$, $3'$ and $4'$. Now join the middle point of the quadrant, (in this

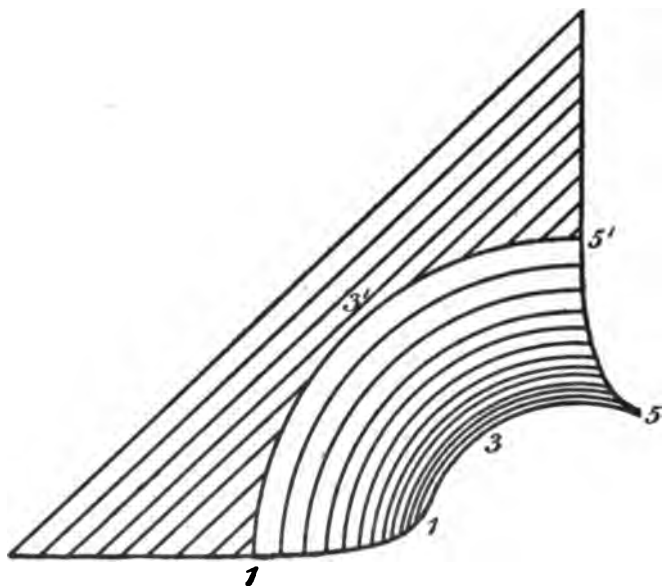
FIG. 19.



instance the mid-point coincides with the point $3'$, the arc having been divided into an even number of parts), with the centre Q , and from point $3'$ along $3'Q$ set off a distance $3'3$ equal to the length of the quadrant arc ag (Fig. 17). Now with $4'$ and 3 as centres and radii respectively the quadrant-length ag and the distance 34 (Fig. 17), describe arcs intersecting in point 4 ; and with $5'$ and 4 as centres and radii

respectively the quadrant-length ag and the distance 4 5 (Fig. 17) describe arcs intersecting in point 5. Working in exactly like manner mark points 2 and 1. Through the points 1, 2, 3, 4 and 5 draw an unbroken curved line, and join 4' 4, 5' 5, 2' 2 and 1' 1; we then have as pattern the figure 1' 3' 5' 5 3 1. In practice, however, because of the stretching of the metal and consequent gain of surface coming of

FIG. 20.

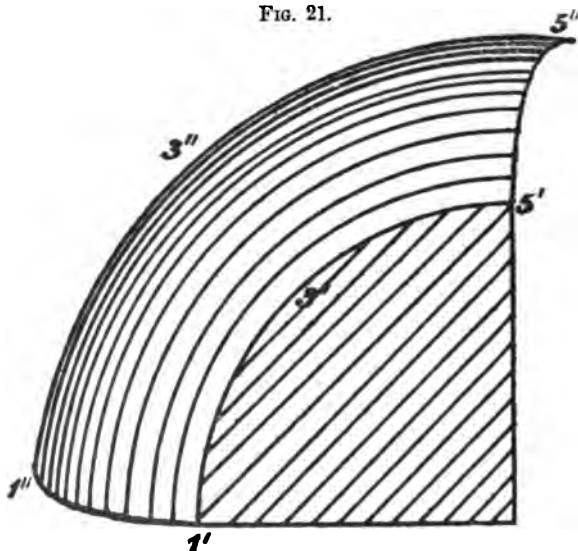


the working-in of the throat curve, it is necessary to cut off pieces 1 1' a , 5 5' a at the ends of the pattern, which thus becomes 1' 3' 5' a 3 a . No rule can be laid down for marking these pieces; so much depends upon the individual workman, on experience, and on the metal that is being worked.

To draw the pattern for the 1" 3" 5" 5' 3' 1' portion of the bend.

Draw (Fig. 19) two indefinite straight lines $Q1''$, $Q5''$, and with Q as centre and radius $Q1'$ (Fig. 17) describe the quadrant $5'1'$ cutting $Q1''$ and $Q5''$ in the point $1'$ and $5'$ respectively. From the point $1'$ set off successively the distances $1'f$, $f e$, $e d$, $d c$, $c b$, $b 1''$ equal to the distances $g f$, $f e$, $e d$, $d c$, $c b$, $b a$ (Fig. 17), and with Q as centre and radius $Q1''$ describe the quadrant $5''1''$; then the figure $1''5''5'1'$ will be the pattern required.

FIG. 21.

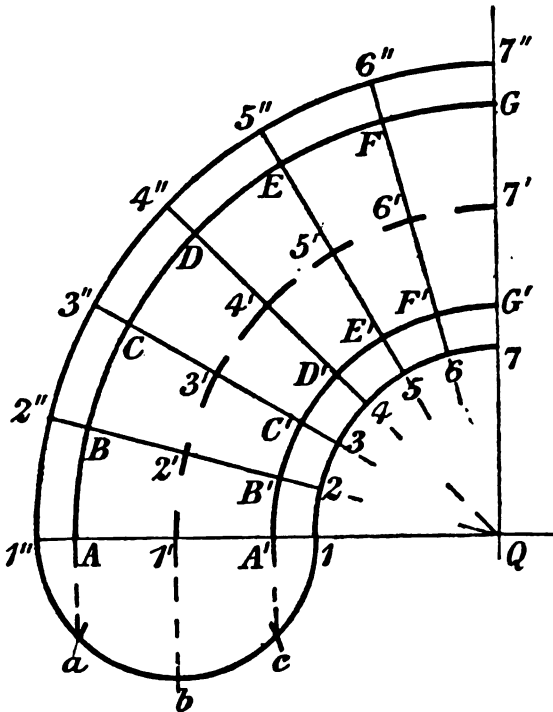


In Fig. 20 the half-throat pattern piece (Fig. 18) is represented as worked into shape. A portion of the flat sheet on which the pattern is supposed drawn is shown as left attached, to illustrate that the $1'3'5'$ curve of the pattern (Fig. 18) is still intact when the shaping up by the hammer is completed. In Fig. 21 the half-back pattern piece (Fig. 19) is represented with a portion of the flat metal sheet left attached, and similarly here the same curve, the $1'3'5'$ curve, remains intact.

CASE IV.—The bend to be made in four pieces; two of these being part-sides and alike, the third being a piece for the throat, and the fourth a piece for the back; the seams coming two on each side of the bend.

Draw (Fig. 22) two lines $Q 1''$, $Q 7''$ at right angles to each other and intersecting in Q . With Q as centre and radius

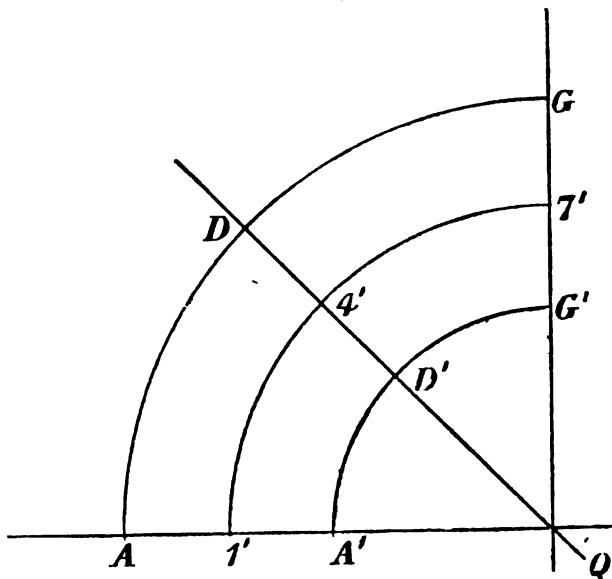
FIG. 22.



$Q 1$ equal to that of the given curve for the throat of the bend describe the quadrant $1 7$. Along $Q 1''$ from point 1 , set off a distance $1 1''$ equal to the diameter of the circular

section of the bend, and with Q as centre and $Q 1''$ as radius describe the quadrant $1'' 7''$; then the figure $1'' 7'' 7 1$ will be a side elevation of the bend. Bisect $1 1''$ in $1'$; from $1'$ draw a line $1' b$ perpendicular to $Q 1''$; and with $1'$ as centre and radius $1' 1$ describe the semicircle $1' b 1$ cutting the line $1' b$ in b . Now divide each of the quadrants $1'' b$, $1 b$ into two equal parts in the points a and c respectively, and from a and

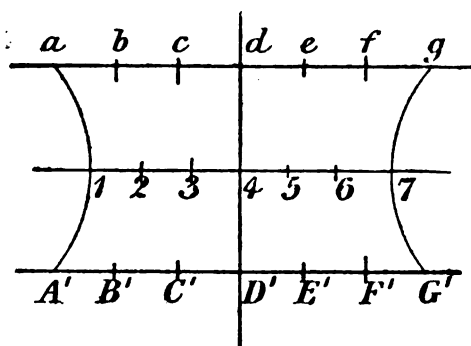
FIG. 23.



c draw $a A$, $c A'$ perpendicular to $Q 1''$ and cutting it in the points A and A' respectively. With Q as centre and radius successively $Q A$, $Q 1'$, $Q A'$ describe the quadrants $A G$, $1' 7'$, $A' G'$; the quadrants $A G$, $A' G'$ will represent the seams of the bend. Divide the quadrant $1'' 7''$ into any convenient number of equal parts (here six) in the points $2''$, $3''$, $4''$, $5''$, $6''$, and join each of these points with Q , the line $2'' Q$ intersecting the quadrants $A G$, $1' 7'$, $A' G'$ in the points B , $2'$, and B'

respectively, and the lines 3" Q, 4" Q, 5" Q, and 6 Q, intersecting these same quadrants similarly and giving the points C, 3', C'; D, 4', D'; E, 5', E'; and F, 6', F'. The four pieces for the bend will be as follows:—two part-side pieces alike, represented each, in elevation, by the figure A D G G' D' A', and in transverse section by the line *a b c*; a throat piece, in side elevation A' D' G' 7 4 1, but of course, in actuality, double that, extending, that is, from the seam A' D' G' to the seam on the opposite side of the bend, that corresponds with the seam A' D' G', (represented also in half-section by the line *c 1*); and a back-piece, in half-section *a 1'*, and in side elevation A D G 7" 4" 1".

FIG. 24.



To draw the pattern for the side-piece.

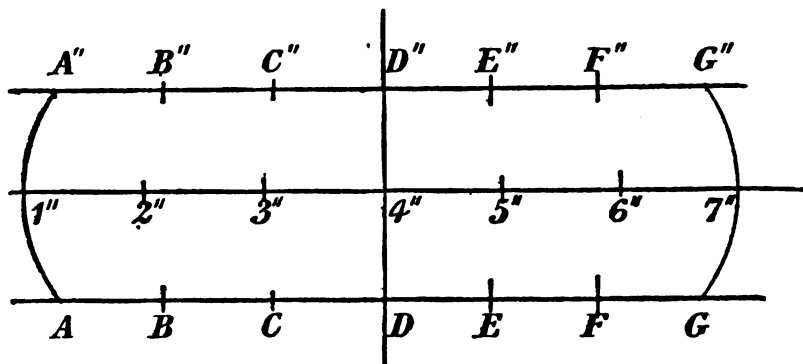
Draw (Fig. 23) two lines Q G and Q A perpendicular to each other and intersecting in Q. From A along Q A set off a distance Q 1' equal to Q 1' (Fig. 22), and with Q 1' as radius describe the quadrant 1' 4' 7'. Also from 1' set off distances 1' A', 1' A equal respectively to the distances *b c*, *b a*, (Fig. 22), with Q A' as radius describe the quadrant A' D' G', and Q A as radius describe the quadrant A D G; the figure A D G G' D' A' will be the pattern required.

To draw the pattern for the throat piece.

Draw (Fig. 24) an indefinite straight line 1 7. From a point

4 about central in it set off to the left distances 4 3, 3 2, 2 1 equal respectively to the distances 4 3, 3 2, 2 1 (Fig. 22), and, to the right, distances 4 5, 5 6, 6 7 equal respectively to the distances 4 5, 5 6, 6 7 (Fig. 22). Through point 4 draw an indefinite line perpendicular to 1 7; above it set off 4 d and below it 4 D' equal each to the curve distance 1 c (Fig. 22); and through d and D' draw indefinite straight lines parallel to 1 7. Now to the left of $d D'$ set off from d the distances $d c$, $c b$, $b a$, and from D' the distances $D' C'$, $C' B'$, $B' A'$ equal, each set, to the distances $D' C'$, $C' B'$, $B' A'$ (Fig. 22); and to the right of $d D'$ set

FIG. 25.



off from d distances $d e$, $e f$, $f g$ and from D' distances $D' E'$, $E' F'$, $F' G'$ equal respectively to the distances $D' E'$, $E' F'$, $F' G'$ (Fig. 22). Join the points a 1 A' by an unbroken curved line, also the points g 7 G' ; the figure $a g 7 G' A' 1$ will be the pattern required.

To draw the pattern for the back-piece.

Draw (Fig. 25) an indefinite straight line 1" 7". From a point 4" about central in it set off to the left distances 4" 3", 3" 2", 2" 1" equal respectively to the distances 4" 3", 3" 2", 2" 1" (Fig. 22) and, to the right distances 4" 5", 5" 6", 6" 7" equal respectively to the distances 4" 5", 5" 6", 6" 7" (Fig. 22).

Through point 4" draw an indefinite line perpendicular to 1" 7"; above it set off 4" D" and below it 4" D equal each to the curve distance 1" a (Fig. 22); and through D" and D draw indefinite straight lines parallel to 1" 7". Now to the left of D" D set off from D" the distances D" C", C" B", B" A", and from D the distances D C, C B, B A equal, each set, to the distances D C, C B, B A (Fig. 22); and to the right of D" D from D" the distances D" E", E" F", F" G", and from D the distances D E, E F, F G, equal, each set, to the distances D E, E F, F G (Fig. 22). Join the points A" 1" A by an unbroken curved line; also the points G" 7" G; the figure A" G" 7" G A 1" will be the pattern required.

CASE V.—The bend to be made up of circular segments.

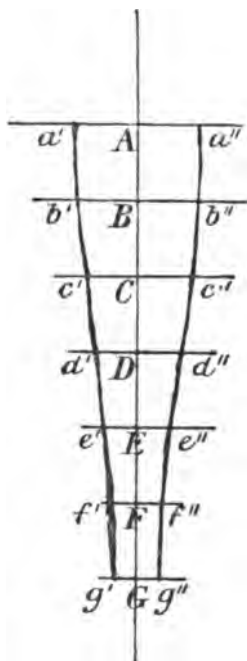
Draw (Fig. 26) two indefinite lines Q a', Q 7' at right angles to each other and intersecting in Q. With Q as centre and radius equal to that of the given quadrant for the throat of the bend describe the quadrant g' 7 meeting the lines Q a', and Q 7' in the points g' and 7 respectively. From g' set off a distance g' a' equal to the given diameter of the circular section of the bend, and with Q as centre and Q a' as radius describe a quadrant a' 7' meeting Q a' and Q 7' in a' and 7' respectively; then g' a' 7' 7 will be a side elevation of the bend. Now divide the quadrant a' 7' into as many equal parts as the bend is to have segments, here six; the figure g' a' a" g" shows one of the segments, the remaining segments are indicated by lines, not lettered. To find the pattern necessary for one of the circular segments is to find the pattern for all.

To find the segment pattern.

On the line g' a' describe the semicircle g' d a', which divide into any convenient number of equal parts (here six) in the points, f, e, d, c and b, and from each of these points draw lines perpendicular to g' a' and intersecting it in the points f', e', d', c' and b'. Bisect the arc a' a" and join the point of intersection with Q: also draw a' a" the chord of the arc a' a", and from the points b', c', d', e', f' and

$G g'$ (or $G g''$) (Fig. 26), and similarly make $F f'$ and $F f''$, $E e'$ and $E e''$, $D d'$ and $D d''$, $C c'$ and $C c''$, $B b'$ and $B b''$, $A a'$ and $A a''$, equal respectively, pair for pair, to $F f'$ (or $F f''$), $E e'$, $D d'$, $C c'$, $B b'$, and $A a'$ (Fig. 26). From the point g' through f' , e' , d' , c' and b' to a' draw an unbroken curved line; also from g''

FIG. 27.



through f'' , e'' , d'' , c'' and b'' to a'' ; the figure $g' a' a'' g''$ will be the pattern for the half of one of the circular segments of the bend. The like work repeated, but reversed in order, on the other side of the line $a' a''$, will give, in the entire figure so completed, the pattern for a whole segment of the bend.

PROBLEM II.

To draw the pattern for a lobster-back cowl.

Draw an indefinite line AO (Fig. 28), and from any point O in it draw a perpendicular OT . With O as centre and radius equal to the diameter of cowl required, describe an arc $AA'T$, cutting OT in T . Produce OT downwards, and make OQ equal to the depth of the rim required; through Q draw QP , parallel to AO , and from A draw AP perpendicular to AO and cutting QP in P ; $AOQP$ is the elevation of the rim. To draw the hood $TSRO$, draw TS perpendicular to OT , make TS and OR of the required dimensions for the hood, and join SR . This completes the elevation $PASROQ$ of the cowl. Now divide the arc AT into the same number of equal parts that the shell of the lobster-back is to have segments. Here we take the number of segments as three; A' and G are the points of division of the arc. Joining these points to O we have in GOT , $A'OG$, and AOA' the elevations of the segments of the shell.

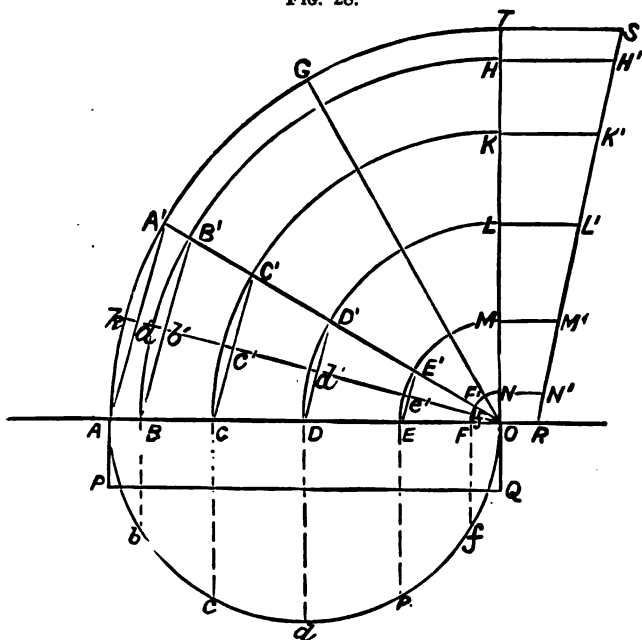
To draw the pattern for the segments.

On either of the lines AO , $A'O$, GO , TO describe a semi-circle (it is here described on AO), and divide it into any convenient number of equal parts (here six) in the points b, c, d, e and f . Through b, c, d, e and f draw lines perpendicular to AO , and cutting it in points B, C, D, E and F respectively. With O as centre and radii OB, OC, OD, OE and OF successively, describe arcs $BB'H, CC'K, DD'L, EE'M$ and $FF'N$, cutting $A'O$ in the points B', C', D', E' and F' respectively, and TO in the points H, K, L, M and N respectively. We now find the pattern for the segment AOA' ; the pattern for that segment, the segments being all equal, will be the pattern for all.

Bisect the arc AA' in k and join kO . Join AA', BB', CC', DD', EE' and FF' by straight lines, cutting kO in points a', b', c', d', e' and f' respectively. Next draw an indefinite line $a'O$ (Fig. 29), and on it set off $a'b', b'c', c'd', d'e', e'f'$

and $f'O$, each equal to $A b$ (Fig. 28), one of the equal parts into which the semicircle $A d O$ is divided, and through each of the points a', b', c', d', e' and f' draw lines perpendicular to $a'O$. Make $a'A', b'B', c'C', d'D', e'E'$ and $f'F'$ respectively equal to $a'A', b'B', c'C', d'D', e'E'$ and $f'F'$ (Fig. 28); also make $a'A, b'B, c'C, d'D, e'E$ and $f'F$ respectively equal to

FIG. 28.



$a'A, b'B, c'C, d'D, e'E$, and $f'F$ (Fig. 28). Join A, B, C, D, E, F and O , and also A', B', C', D', E', F' and O' , by unbroken curved lines, then $A O A'$ will be half the pattern required, which can be completed by drawing on $A A'$ a figure exactly the same as $A O A'$.

To draw the pattern for the hood $TSRO$.

Draw (Fig. 30) an indefinite line TO , and on it set off

TH, HK, KL, LM, MN and NO, each equal to $A b$ (Fig. 28), one of the equal parts of semicircle $A d O$. Through each of the points T, H, K, L, M, N and O draw lines perpendicular to TO; and make TS, HH', KK', LL', MM', NN' and OR respectively equal to TS, HH', KK', LL',

FIG. 29.

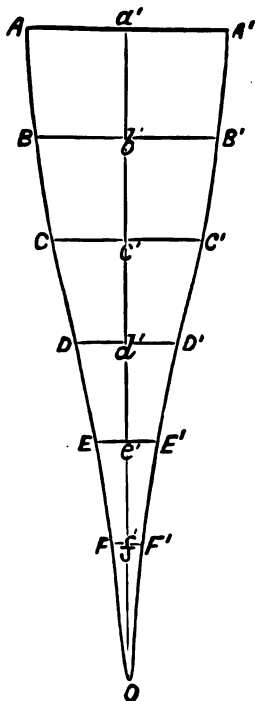
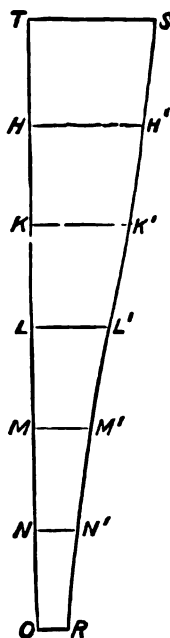


FIG. 30.



MM', NN' and OR (Fig. 28). Join S, H', K', L', M', N' and R by an unbroken curved line; TSRO will be the pattern for one half of the hood; duplicating this on the line TS will complete the pattern.

If closer approximation is necessary the lines $a' O$ (Fig. 29)

and T O (Fig. 30) should each of them be made equal to half the circumference of a circle having A O (Fig. 28) for its diameter, and then divided into as many equal parts as semi-circle A d O (Fig. 28) is divided into. In practice, for convenience in making up, it is necessary to cut off the point O (Fig. 29) of the pattern. How much shall be cut off must be left to the experience of the workman; it varies according to the size of the "boss" used to cover the throat. Also, in practice, to ensure neatness of fitting, the rim at O (Fig. 28) is notched out a little. This, again, is a matter in which experience must be the workman's guide.

The rim being cylindrical, its pattern needs no description beyond saying that it is a rectangle, one side of which is equal to the depth of the rim, and the other side equal to the circumference of a circle having A O for its diameter.

NOTE.—In practice the usual sizes of rim A O Q P and hood T S R O are as follows:—

Diameter of Cowl.	Depth of Rim.	Size of Hood at Widest and Narrowest Parts.
8 inches.	2 inches.	4 inches and 1 inch.
9 "	2½ "	4½ " 1½ "
10 "	2¾ "	4¾ " 1¾ "
11 "	2¾ "	5 " 1½ "
12 "	3 "	5½ " 1¾ "

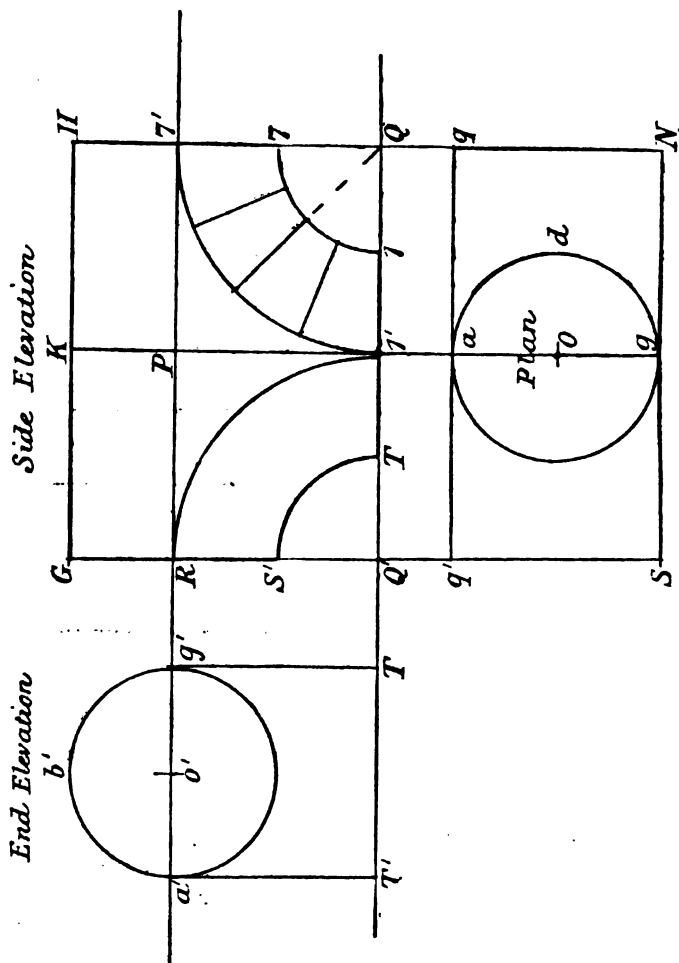
PROBLEM III.

To draw the pattern for a round neck T-piece to connect equal circular pipes.

Draw (Fig. 31) two indefinite lines K g, 7' a' at right angles to each other and intersecting in P, and set off from P right and left along 7' a', and towards g on line K g distances P 7', P R and P 1' equal each to the diameter of the pipe with which the T is to connect. Through the point 1'

draw an indefinite line $Q T'$ parallel to $7' a'$, and through the points $7'$ and R draw lines $H N$, $G S$ parallel to $K g$ and intersecting $Q T'$ in Q and Q' respectively. Bisect $7' Q$ in

FIG. 31.



point 7, and as $7'Q$ is equal to $P1'$, each of the divisions $7'7$, $Q7$ will be equal to half the diameter of the pipe that the T connects with. From point $7'$ along $7'H$ set off $7'H$ equal to $7'7$, from point R on GS set off distances RS' , RG each equal to $7'7$, and join GH . Now with Q and Q' as centres and radius $Q7$ describe the quadrants 71 and $S'T$ respectively, with Q as centre and radius $Q7'$ describe the quadrant $7'1'$, and with Q' as centre and same radius describe the quadrant $R1'$. Then $T S' G H 71$ will be a side elevation of the round-neck T. On $7'a'$ take any convenient point O , and with O as centre and $Q7$ as radius describe the circle $a'b'g'$ cutting $7'a'$ in the points a' and g' . From points a' and g' draw lines $a'T'$, $g'T$ perpendicular to $7'a'$ and meeting $Q'T'$ in the points T' and T respectively. Then $T'a'b'g'T$ will be an end elevation of the T. On Kg take any convenient point O , and with O as centre and $Q7$ as radius, describe the circle adg , cutting Kg in the points a and g . Through point a draw a line $q'q$, parallel to $Q'T'$ and cutting GS and HN in points q' and q respectively, and through g draw a line SN parallel to $Q'T'$, and cutting GS in HN in S and N respectively. Then $q'aqNgS$ will be a plan of the T.

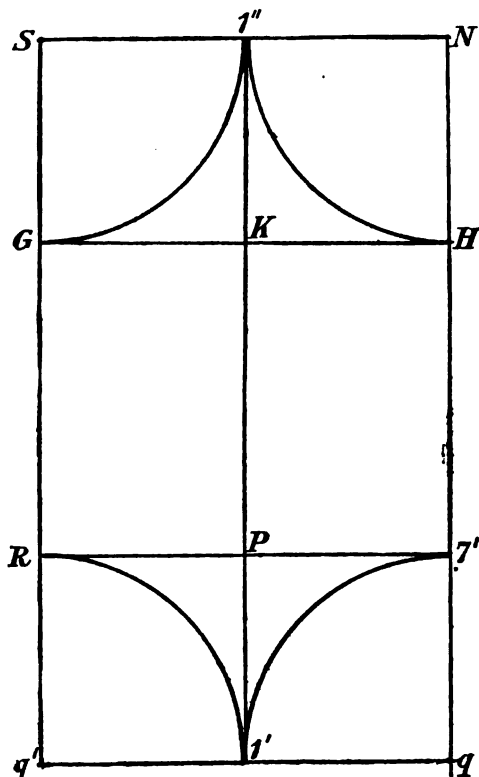
To draw the pattern for the $RGH7'1'$ portion of the T.

Looking at the side and end elevations, it will be seen that the $RGH7'$ portion of the T is a semicylinder of $a'b'g'$ section, and that the $R1'7'1'$ part of it lies wholly in the plane of $a'T'$, and the portion on the opposite side of the T corresponding to $R1'7'1'$ wholly in the plane of $g'T$. The pattern for the $RGH7'1'$ piece of the T can consequently be obtained as follows.

Draw (Fig. 32) two lines $R7', 1'1''$ at right angles to each other and intersecting in P . From P along $P1''$ set off PK equal to the developed length of the arc adg (Fig. 31), that is equal to semicircumference of the circular pipe. Through K draw GH parallel to $R7'$; and from P set off distances $P1', P7', PR$ each equal to $P1'$ (Fig. 31), and from K a distance $K1''$ also equal to $P1'$ (Fig. 31). Through the

points $1'$, $1''$ draw $q'q$ and SN parallel to $R7'$; and through the points $7'$ and R draw lines qN , $q'S$ parallel each to $1'1''$, cutting $q'q$ in q and q' , and GH in G and H , and SN in S and N . With q' , q , N and S successively as centres

FIG. 32.



and radius $q'R$ describe the quadrants $1'R$, $1'7'$, $1''H$ and $1''G$. The figure $1'R G 1''H 7'$ will be the pattern required.

Looking at the side elevation and plan in Fig. 31, it will

be seen that the $1'7'71$, $1'RST$, portions of the T are halves of bends of equal circular section (the $1'7'71$ portion has drawn on it lines converging to Q in order to show the beginning of the construction of the pattern). In the case of bends, the quadrant curves $1'7'$, $1'R$, and corresponding quadrants on opposite side of the T would have been neutral lines (see p. 362). Here they are extreme curves of the throats of the T and of the portion of it for which we have just drawn the pattern. According to the dimensions of the pipe that is being dealt with, the throats may be made either in one or in two pieces. If in one piece the method of drawing the pattern is given in Case II. of Problem I. (p. 364); if in two pieces the method of procedure will be found in Case III., p. 369, of same Problem.

PROBLEM IV.

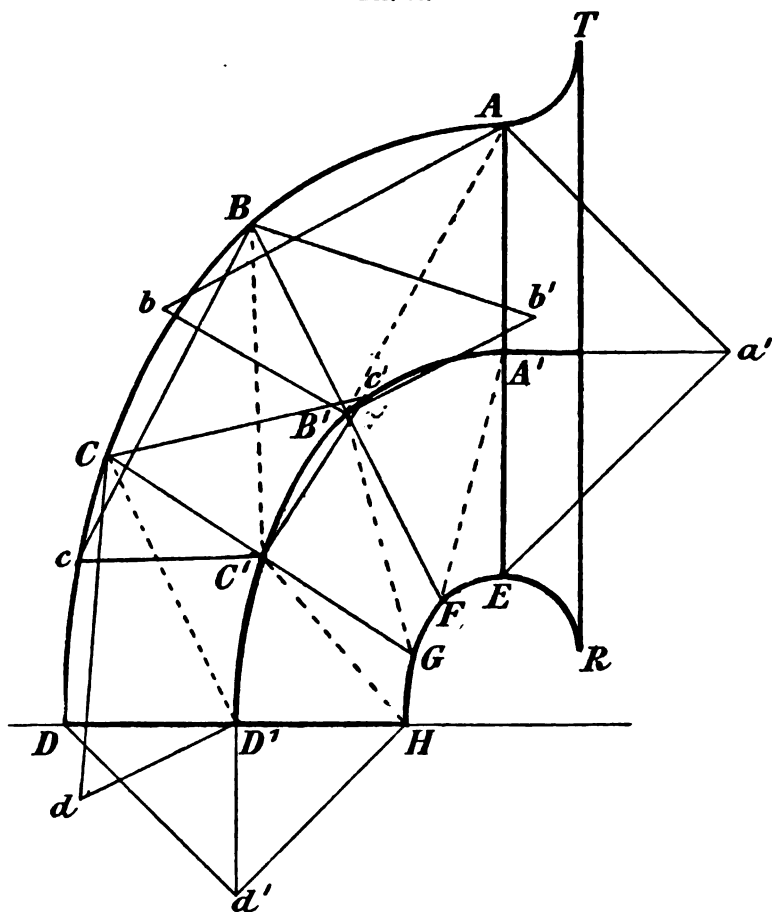
To draw the pattern for a curved elbow (bend) of square section, to join two unequal square pipes.

CASE I.—Where the throat of the bend is one of the angles of the square.

Let A D H E (Fig. 33) be a part side elevation of the bend, that is to say, its extreme lines, the A E end being the larger. The A E R T portion of the Fig. is unconnected with this Problem. Bisect A E in A', from A' draw A' a' perpendicular to A E and make A' a' equal to A' A, and join A a', a' E; then A a' E will be a half-section of the larger end of the bend. Similarly draw D d' H a half-section of the smaller end of it. To complete the side elevation, divide each of the curves A D, E H into any convenient number of equal parts (here three) in the points B, C, and F, G, respectively. Join B F and C G, and bisect these lines in B' and C' respectively. From point A' through points B' and C' draw to D' an unbroken curved line; the figure A D D' H E A' will be the complete side elevation of the bend. The bend will be made up of

four pieces ; the seams will correspond with the lines of the angles of the bend, that is, with $A B C D, E F G H, A' B' C' D'$,

FIG. 33.



and the line not seen in the elevation that is opposite to $A' B' C' D'$. Two patterns will be required, one for the

$A B C D D' C' B' A'$ and corresponding piece, and one for the $A' B' C' D' H G F E$ and corresponding piece.

To draw the pattern for the $A B C D D' C' B' A'$ pieces.

We already have, in $A a'$, and $E a'$, the true lengths of $A A'$, and $E A'$ respectively; also, in $D d'$, $H d'$, the true lengths of $D D'$, and $H D'$, respectively. To find the true lengths of $B B'$, and of $F B'$, equal to it, through B' draw $B' b'$ perpendicular to and equal to $B B'$, and join $B b'$; then $B b'$ will be the true length required. To find the true lengths of $C C'$, and of $G C'$, equal to it, through C' draw $C' c'$ perpendicular to and equal to $C' C$, and join $C c'$; then $C c'$ will be the true length required. Now join $A B'$, $B C'$, $C D'$ and find their true lengths. Through B' draw $B' b$ perpendicular to $A B'$ and equal to $B' b'$, and join $A b$; through C' draw $C' c$ perpendicular to $B C'$ and equal to $C' c'$, and join $B c$; and through D' draw $D' d$ perpendicular to $C D'$ and equal to $D' d'$, and join $C d$; then $A b$, $B c$, and $C d$ will be respectively the true lengths required.

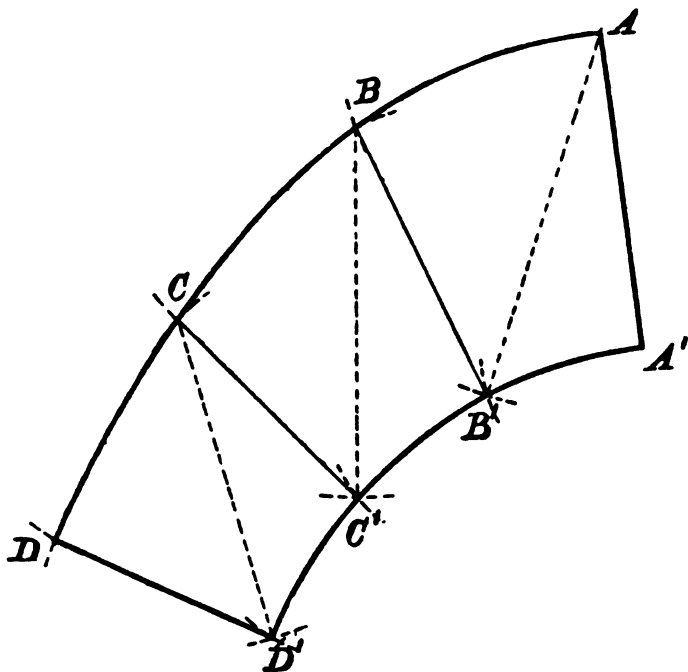
Next draw (Fig. 34) a line $A A'$ equal to $A a'$ (Fig. 33), and with A and A' as centres and radii respectively equal to $A b$ and $A' B'$ (Fig. 33) describe arcs intersecting in B' ; and with B' and A as centres and radii respectively equal to $B b'$ and $A B$ (Fig. 33) describe arcs intersecting in B . With B and B' as centres and radii respectively equal to $B c$ and $B' C'$ (Fig. 33) describe arcs intersecting in C' ; and with C' and B as centres and radii respectively equal to $C c$ and $B C$ (Fig. 33) describe arcs intersecting in C . Also with C and C' as centres and radii respectively equal to $C d$ and $C' D'$ (Fig. 33) describe arcs intersecting in D' ; and with D' and C as centres and radii respectively equal to $D d'$ and $C D$ (Fig. 33) describe arcs intersecting in D . Join $D D'$; through D , C , B , A draw an unbroken curved line. Also draw an unbroken curved line through D' , C' , B' , A' . Then $D C B A A' B' C' D'$ will be the pattern required.

The other lines drawn in Fig. 34 correspond to the lines of same lettering in Fig. 33; they are not a necessary part of the construction.

To draw the pattern for the $A'B'C'D'HGFE$ pieces.

Join $A'F$, $B'G$, and $C'H$; and, to avoid confusion of lines, find their true lengths apart from the elevation. Draw (Fig. 35) from any point O the lines OA' and OF perpendicular to each other, and equal respectively to $A'a'$ and

FIG. 34.



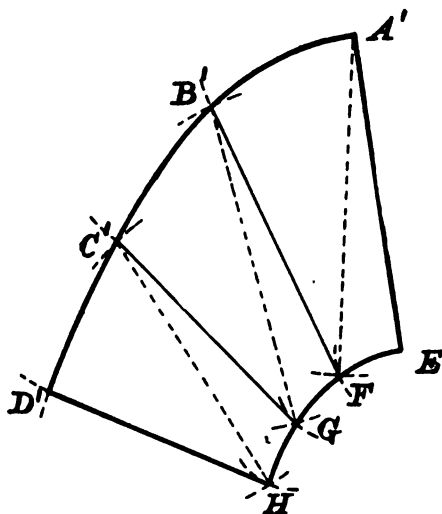
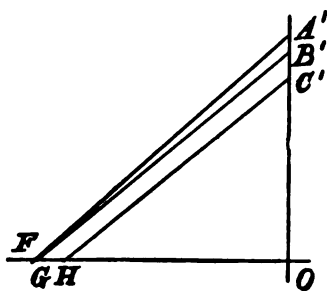
$A'F$ (Fig. 33), and join $A'F$; then $A'F$ will be the true length of $A'F$ (Fig. 33). Now set off on OA' distances OB' , OC' , equal respectively to $B'b'$, $C'c'$ (Fig. 33), and on OF distances OG , OH , equal respectively to $B'G$, $C'H$ (Fig. 33), and join $B'G$, $C'H$. Then $B'G$ will be the true length of $B'G$ (Fig. 33), and $C'H$ the true length of $C'H$

(Fig. 33). In Ea' , Hd' (Fig. 33), we already have the true lengths of EA' , HD' respectively, and in $B'b'$, $C'c'$, the true lengths of $B'F$ and $C'G$.

Now draw EA' (Fig. 36) equal to Ea' (Fig. 33), and with A' and E as centres and radii respectively equal to $A'F$ (Fig. 35) and EF (Fig. 33) describe arcs intersecting in F ; and with F and A' as centres and radii respectively equal to $B'b'$ and $A'B'$ (Fig. 33) describe arcs intersecting in B' .

FIG. 35.

FIG. 36.



With B' and F as centres and radii respectively equal to $B'G$ (Fig. 35) and FG (Fig. 33) describe arcs intersecting in G ; and with G and B' as centres and radii respectively equal to $C'c'$ and $B'C'$ (Fig. 33), describe arcs intersecting in C' . Also with C' and G as centres and radii respectively equal to $C'H$ (Fig. 35) and GH (Fig. 33) describe arcs intersecting in H ; and with H and C' as centres and radii respectively equal to Hd' and $C'D'$ (Fig. 33) describe arcs intersecting in D' . Join HD' ; through D' , C' , B' , A' draw an

unbroken curved line. Also draw an unbroken curved line through H, G, F, E. Then D' C' B' A' E' F' G' H will be the pattern required.

As to the other lines drawn in Fig. 36, see the remarks made in connection with the preceding pattern. The patterns necessary for the formation of the bend A B C D D' H G F E A' are now completed.

CASE II.—Where the throat of the bend is one of the sides of the square.

Let A a e E, D d h H (Fig. 37) be the half sections of two square pipes to be joined, A a e E being half section of the larger, and D d h H half section of the smaller, and let A B C D H G F E be the side elevation of the bend to be constructed to join the two pipes. The bend will be made up in four pieces, and the seams will, as in Case I., correspond with the angles of the bend, that is, with A B C D, H G F E, and the lines not seen in the elevation, that are opposite to the mentioned lines. The back and throat of the bend being perpendicular to the plane of the paper, are represented respectively by the single curve lines. Three patterns will be required, one for the A D H E and opposite side, one for the back A B C D, and one for the throat E F G H.

To draw the pattern for the A D H E piece.

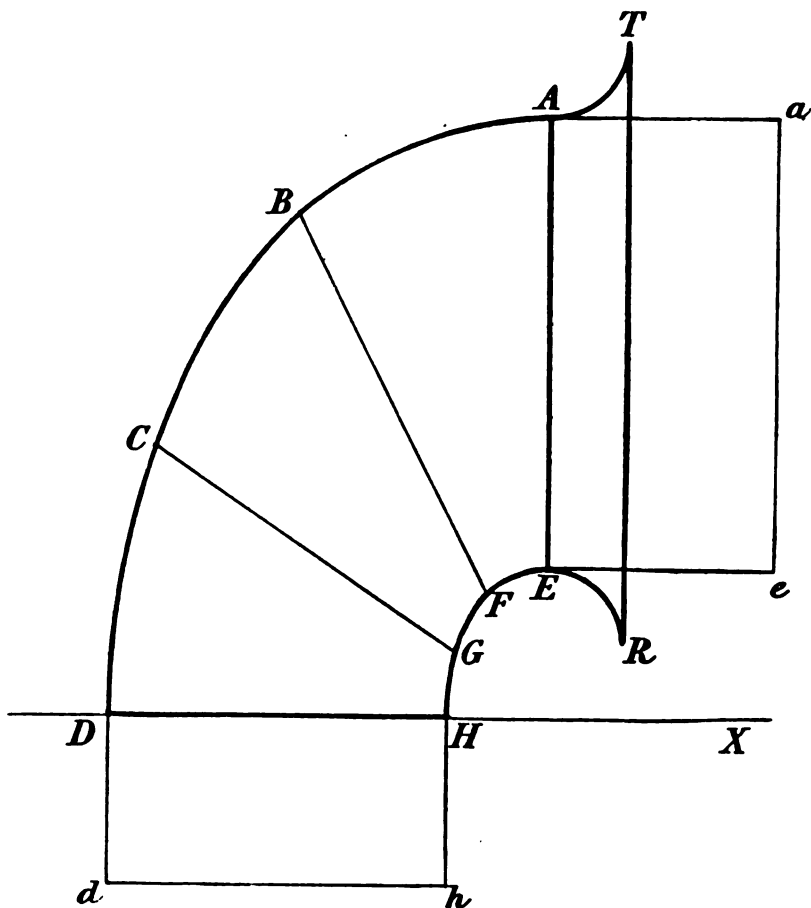
This pattern is in the main a repetition of the side of the bend as seen in the elevation, that is A D H E is the pattern required. A slight extension, however, should be made to the pattern beyond the lines A E and D H, seeing that whilst A D H E, the *elevation* of the bend, is in the plane of the paper, A D H E, the *side* of the bend, is not in that plane but at an angle to it.

To draw the pattern for the back A B C D of the bend.

Divide each of the curves A B C D, E F G H into any convenient number of equal parts (here three) in the points B, C, and F, G respectively, and join B F, C G. Now draw (Fig. 38) any line A D, and from any point A in it set off succes-

sively distances AB, BC, CD , equal respectively to the distances AB, BC, CD round the curve $ABCD$ (Fig. 37),

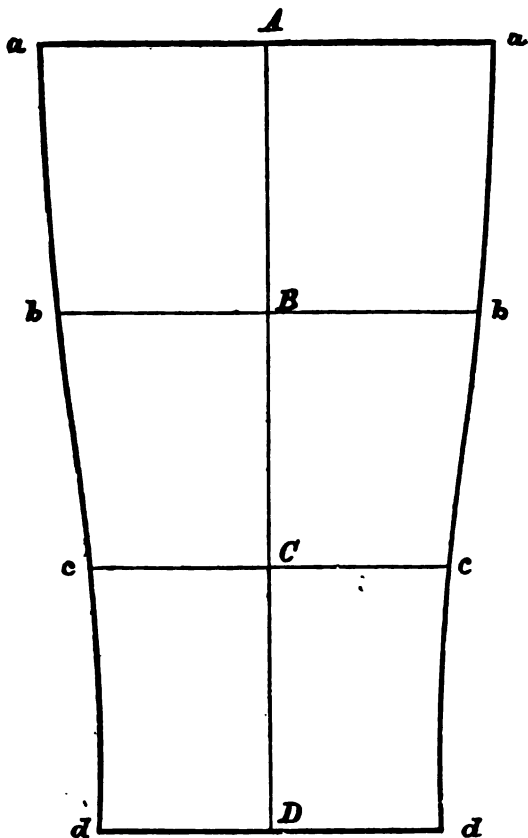
FIG. 37.



and through the points A, B, C and D draw lines perpendicular to AD . Set off right and left of AD , a distance Aa

equal to Aa (Fig. 37); Bb equal to half BF (Fig. 37); Cc equal to half CG (Fig. 37); and Dd equal to Dd (half

FIG. 38.

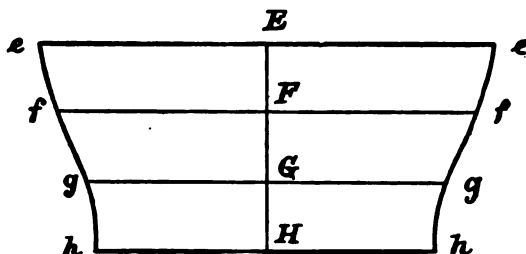


DH) (Fig. 37). Through each set of points a, b, c, d , draw an unbroken curved line. Then the enclosed figure $ad d a$ is the pattern required.

To draw the pattern for the throat $EFGH$ of the bend.

Draw (Fig 39) any line EH , and from any point E in it set off successively distances EF , FG , GH equal respectively to the distances EF , FG , GH round the curve $EFGH$ (Fig. 37), and through the points E , F , G and H draw lines perpendicular to EH . Set off right and left of EH , a distance

FIG. 39.



Ee equal to Ee (Fig. 37); Ff equal to half BF (Fig. 37); Gg equal to half CG (Fig. 37); Hh equal to Hh (half DH) (Fig. 37). Through each set of points e, f, g, h , draw an unbroken curved line. The enclosed figure $ehhe$ is the pattern required. The patterns necessary for the formation of the bend $ABCDHGF E$ are now complete.

PROBLEM V.

To draw the pattern for a ventilator of square section.

CASE I.—Where the throat of the ventilator is one of the angles of the square.

Let $TACDHFER$ (Fig. 33) represent the ventilator in side elevation; the seams of it are to be at the angles. The portion $ADHE$ is a curved tapering elbow or bend of square section. On DH draw $Dd'H$, half section of the square at DH , and on AE draw $Aa'E$ half section of the square where the mouthpiece is attached. These half sections may

be regarded as the half sections of two unequal square pipes, and the drawing of the pattern for the A D H E portion of the ventilator now becomes that of the setting out a pattern for a square tapering bend, the throat of which is one of the angles of the square, to join the two unequal square pipes whose half sections are D d' H and A a' E. Divide each of the curves A C D, H F E into any convenient number of equal parts (here three) in the points B, C and F, G respectively, and join B F and C G cutting the elevation curve A' D' in the points B' and C'; the lines B F and C G will be respectively bisected in the points B' and C'. The further procedure for the pattern will be exactly as shown in Case I. of preceding Problem.

To complete the ventilator. It finishes off with a moulding, of which the curve A T is a transverse section. The student will have no difficulty in dealing with this mouth of the ventilator; he will find every instruction in Problem XXX., p. 310.

PROBLEM VI.

To draw the pattern for a tapering circular bend.

The methods given are approximate, as indeed all ordinary practical methods must be for problems of this description.

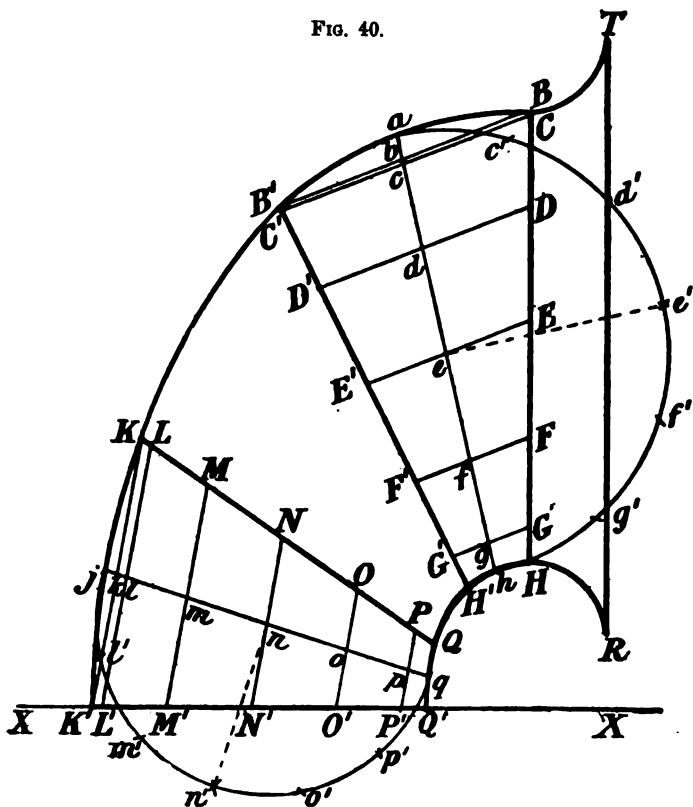
CASE I.—The bend to be made up of circular segments.

Let K' K B' B H' H' Q' Q' (Fig. 40) represent in elevation a tapering circular bend. (The B T R H portion of the Fig. is unconnected with this Problem.) Divide each of the curves B K', and H Q' into as many equal parts as it is determined there shall be segments, here say three parts, the points of division being B', K, and H', Q respectively. Joining B' to H', and K to Q, we have in B B' H' H, B' K Q H', and K K' Q' Q elevations of the three segments of which the ventilator is to be made up.

To draw the pattern for the segment B B' H' H.

Bisect the arcs $B'B$ in a and $H'H$ in h , and join ah ; also join $B'B$, intersecting ah in b . On ah describe a semicircle $a'e'h$ and divide it into any number of equal parts (here six) in the points c', d', e', f' and g' . From these points draw lines

FIG. 40.



perpendicular to ah , cutting ah in the points c, d, e, f and g respectively (to avoid confusion these lines are not shown in the figure), and through c, d, e, f and g draw $C'C, D'D, E'E, F'F$, and $G'G$ parallel each to $B'B$.

Next draw (Fig. 41) an indefinite line $b h$, and set off on it distances $b c, c d, d e, e f, f g$ and $g h$, each equal to $a c'$ (Fig. 40), that is, to one of the divisions of the semicircle $a e' h$, and through b draw $B' B$ at the same angle to $b h$ that $B' B$ (Fig. 40) is to $b h$. Now through c, d, e, f, g and h (Fig. 41) draw $C' C, D' D, E' E, F' F, G' G$ and $H' H$ parallel each to $B' B$. Make $b B, c C, d D, e E, f F, g G$ and $h H$ equal respectively to $b B, c C, d D, e E, f F, g G$ and $h H$ (Fig. 40). Also make $b B', c C', d D', e E', f F', g G'$ and $h H'$ equal respectively to $b B', c C', d D', e E', f F', g G'$ and $h H'$ (Fig. 40). Joining B, C, D, E, F, G and H , and B', C', D', E', F', G' and H' by unbroken curved lines gives in $B H H' B'$ the pattern required.

To draw the pattern for the segment $K' K Q Q'$.

Bisect the arc $K' K$ in j and $Q' Q$ in q , and join $j q$; also join $K' K$, intersecting $j q$ in k . On $j q$ describe a semicircle $j n' q$ and divide it into any number of equal parts (here six) in the points $l', m', n', o',$ and p' . From these points draw lines perpendicular to $j q$, cutting $j q$ in the points l, m, n, o and p respectively (these lines are not shown in the figure), and through $l, m, n, o,$ and p draw $L' L, M' M, N' N, O' O$ and $P' P$, parallel each to $K' K$.

Next draw (Fig. 42) an indefinite line $k q$, and set off on it distances $k l, l m, m n, n o, o p$ and $p q$, each equal to $j l'$ (Fig. 40), that is, to one of the divisions of the semicircle $j n' q$, and through k draw $K' K$ at the same angle to $k q$ that $K' K$ (Fig. 40) is to $k q$. Now through l, m, n, o, p and q (Fig. 42) draw $L L', M M', N N', O O', P P'$ and $Q Q'$ parallel each to $K K'$. Make $k K, l L, m M, n N, o O, p P$ and $q Q$ equal respectively to $k K, l L, m M, n N, o O, p P$ and $q Q$ (Fig. 42). Also make $k K', l L', m M', n N', o O', p P'$ and $q Q'$ equal respectively to $k K', l L', m M', n N', o O', p P'$ and $q Q'$ (Fig. 42). Joining K, L, M, N, O, P and Q , and K', L', M', N', O', P' and Q' by unbroken curved lines gives in $K Q Q' K'$ the pattern required.

In practice it is found desirable to leave the ends $H' H$ (Fig. 41) and $Q' Q$ (Fig. 42) of the pattern full long. It is

easier to cut a piece off if a pattern is too long than to add a piece if too short.

The pattern for the segment $K B' H' Q$ (Fig. 40) need not

FIG. 41.

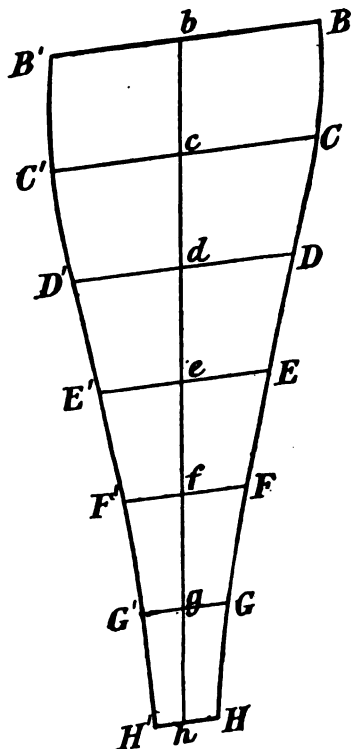
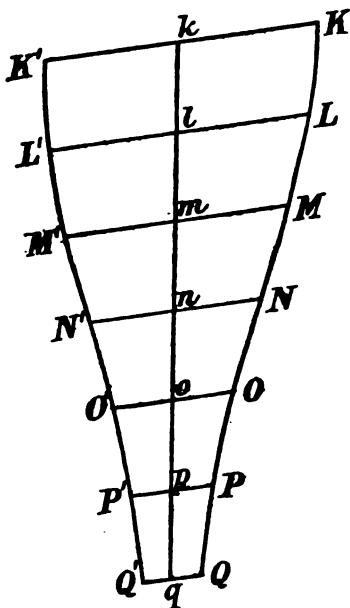


FIG. 42.



be drawn here. To add the construction for it would make a confusion of lines in the Fig. The pattern can be obtained in similar manner exactly to that by which the patterns are obtained for the other two segments.

The patterns drawn are of course halves only of the full

circular segments ; and the remark that concludes Case V. of Problem I., p. 381, applies also here. The dimensions of the bend decide whether its segments shall be in one or two pieces each. The present Problem corresponds with that of Case V. just referred to.

CASE II.—The bend to be made in four pieces, namely, two part-sides (alike), one back-piece, and one throat-piece. The seams will follow the curve of the bend, two on each side of it.

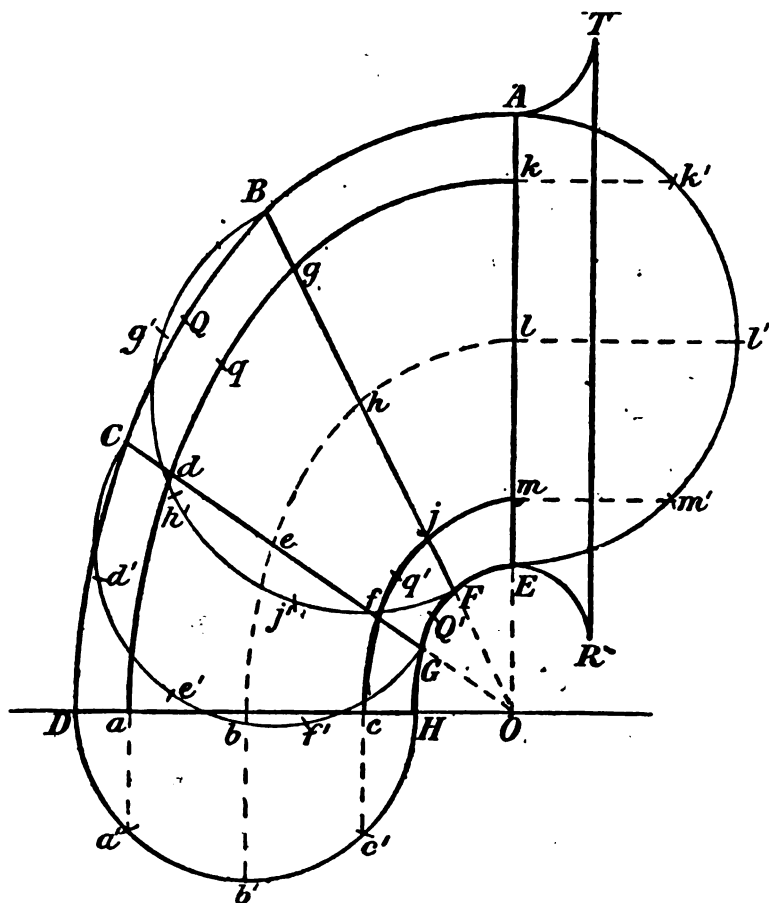
Let D A E H (Fig. 43) be a side elevation of the bend. Divide the arc D A into any convenient number of equal parts, here three, in the points C and B ; and the arc H E into the same number, in the points G and F ; and join C G, B F. On each of the lines D H, C G, B F, A E describe a semicircle, and divide each semicircle into any convenient number of equal parts, here four, in the points a', b', c' for the semicircle on D H, the points d', e', f' for the semicircle on C G, the points g', h', j' for the semicircle on B F, and the points k', l', m' for the semicircle on A E. From the points a', b' and c' draw lines perpendicular to D H, intersecting it in the points a, b and c . Similarly obtain the points d, e, f on C G, g, h, j on B F, and k, l, m on A E. From a through d and g to k draw an unbroken curved line ; also from b through e and h to l , and from c through f and j to m . In shaping up a part-side piece the line $b e h l$ will be a neutral line. The seams of the bend will correspond with the lines $a d g k$ and $c f j m$.

To draw the pattern for a part-side piece.

Draw (Fig. 44) two lines $O a', O k'$ at right angles to each other. Also draw a line $O d'$ at the same angle to $O a'$ that $O C$ is to $O D$ (Fig. 43), and a line $O g'$ at the same angle to $O k'$ that $O B$ is to $O A$ (Fig. 43). On $O a'$ set off a distance $O b$ equal to $O b$ (Fig. 43) ; on $O d'$ a distance $O e$ equal to $O e$ (Fig. 43) ; on $O g'$ a distance $O h$ equal to $O h$ (Fig. 43) ; and on $O k'$ a distance $O l$ equal to $O l$ (Fig. 43). From the point b through e and h to l draw an unbroken curved line ;

this line is a reproduction of the curved line $b e h l$ (Fig. 43). From b set off $b'c'$, $b'a'$ equal to $b'c'$, $b'a'$ (Fig. 43); from e set

FIG. 43.



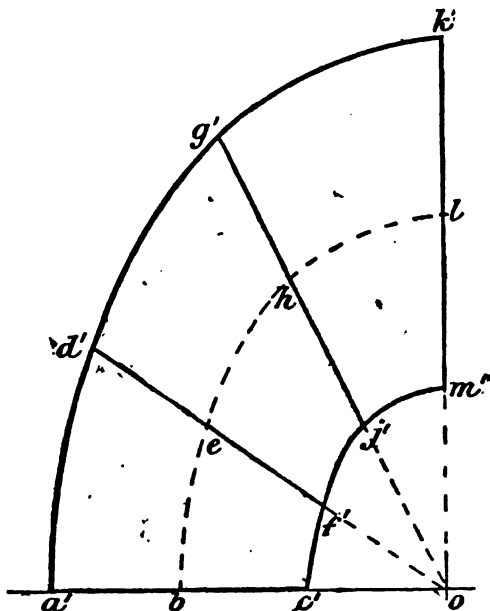
off $e'f'$, $e'd'$ equal to $e'f'$, $e'd'$ (Fig. 43); from h set off $h'j'$, $h'g'$ equal to $h'j'$, $h'g'$; and from l set off $l'm'$, $l'k'$ equal to $l'm'$, $l'k'$

(Fig. 43). From a' through d' and g' to k' draw an unbroken curved line, and from c' through f' and j' to m' draw an unbroken curved line; the figure $a' d' g' k' m' j' f' c'$ will be the pattern required.

To draw the pattern for the back-piece.

Bisect the curve $B C$ (Fig. 43) in Q , and the curve $g d$ in q . Now draw (Fig. 45) an indefinite straight line $A D$, and

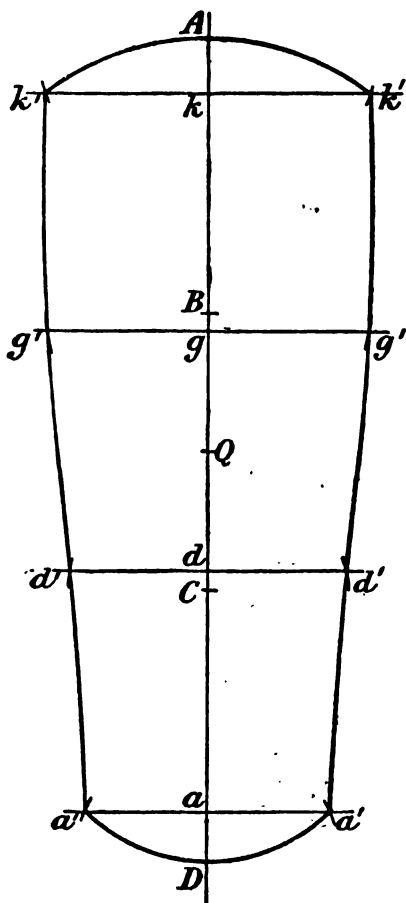
FIG. 44.



about midway in it take a point Q . From Q on $Q A$ set off a distance $Q B$ equal to $Q B$ on the curve $C B$ (Fig. 43), and from B on same line set off $B A$ equal to $B A$ (Fig. 43). Also from Q on $Q D$ set off a distance $Q C$ equal to $Q C$ (Fig. 43) and from C on same line a distance $C D$ equal to $C D$ (Fig. 43). Next from Q set off $Q g$ equal to $q g$ (Fig. 43), and

from g set off gk equal to gk (Fig. 43); and again from Q a distance Qd equal to qd (Fig. 43), and from d a distance da

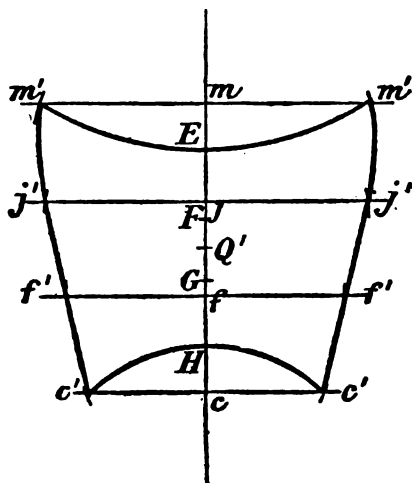
FIG. 45.



equal to da (Fig. 43). Through the points k , g , d and a draw indefinite lines perpendicular to AD . With A as

centre and radius $A k$ (Fig. 43) describe arcs intersecting the line through k in the points k', k' ; with B as centre and radius $B g'$ (Fig. 43) describe arcs intersecting the line through g in the points g', g' ; with C as centre and radius $C d'$ (Fig. 43) describe arcs intersecting the line through d in the points d', d' ; and with D as centre and radius $D a'$ (Fig. 43) describe arcs intersecting the line through a in the points a', a' . From k' through g' and d' to a' , on each side of the line

FIG. 46.



$A D$, draw an unbroken line; also from k' on the left of $A D$ an unbroken line through A to k' on the right of $A D$; and similarly from the point a' an unbroken line through D to a' . The figure $D a' k' A k' a'$ will be the figure required.

To draw the pattern for the throat-piece.

Bisect (Fig. 43) the curve $F G$ in the point Q' and the curve $f j$ in q' . Now draw (Fig. 46) an indefinite straight line $m c$, and about midway in it take a point Q' . From Q' on $Q' m$ set off a distance $Q' F$ equal to $Q' F$ (Fig. 43), and from

F a distance $F'E$ equal to $F'E$ (Fig. 43); also from Q' set off $Q'G$ equal to $Q'G$ (Fig. 43), and from G set off GH equal to GH (Fig. 43). Next from Q' set off $Q'j$ equal to $q'j$ (Fig. 43), and jm equal to jm (Fig. 43); and again from Q' a distance $Q'f$ equal to $q'f$ (Fig. 43), and from f a distance fc equal to fc (Fig. 43). Through the points m, j, f and c draw lines perpendicular to mc . With E as centre and radius Em' (Fig. 43) describe arcs intersecting the line through m in the points m, m' ; with F as centre and radius Fj' (Fig. 43) describe arcs intersecting the line through j in the points j, j' ; with G as centre and radius Gf' (Fig. 43) describe arcs intersecting the line through f in the points f, f' ; and with H as centre and Hc' (Fig. 43) as radius describe arcs intersecting the line through c in the points c, c' . From the point m through j' and f' to c' , on each side of mc , draw an unbroken line; also from m' on the left of mc an unbroken line through E to m' on the right of mc ; and similarly from the point c' an unbroken line through H to c' . The figure $Hc'm'E m'c'$ will be the figure required.

PROBLEM VII.

To draw the pattern for a circular tapering ventilator.

Let (Fig. 43) $DATREH$ be a side elevation of the ventilator. The preceding Problem deals with the $DAEH$ portion of it, which portion is a circular tapering bend. The $ATRE$ mouth-piece may be stretched out of flat sheet-metal or be made as a cone-frustum, and then stretched as required.

BOOK V.

CHAPTER I.

METALS AND THEIR PROPERTIES; ALLOYS; SOLDERS;
SOLDERING FLUXES.

In the pages that follow, we deal almost exclusively with the metals that are used largely in plate or sheet; our business being the setting-out of patterns on sheets or surfaces.

CHARACTERISTIC FEATURES.

(92.) Metals are natural elementary substances, as far as is known. They are *opaque* (not transparent), reflect light from their polished surfaces, and have a characteristic lustre, known as the *metallic lustre*. With the exception of mercury, they are all solid at the ordinary temperature of the atmosphere.

(93.) Silver, tin, lead, mercury, antimony, zinc, cadmium, and bismuth, have a whitish or greyish colour. Gold stands alone as a metal having a yellow colour; copper is the only red metal.

(94.) Metals differ widely in their behaviour under the influence of heat; some, as tin and lead, are fusible below red heat; others, as copper, gold, and silver, fuse readily in ordinary furnaces; nickel, iron, and manganese fuse with great difficulty; platinum is practically infusible. Arsenic, cadmium, zinc, and mercury are *volatile*, that is, vaporise easily. An interesting example of volatility is that of zinc, which when at a bright red heat takes fire, burns with a greenish flame, and *oxidises* (unites with the oxygen gas

of the atmosphere), being thereby converted into a dense white flocculent substance, called formerly 'philosopher's wool.'

(95.) The fracture of metals is often characteristic; we get crystalline, granular, fibrous, silky, and other fractures. We have fractures in iron, depending upon its kind, of the first three forms. In copper we find the silky fracture.

PROPERTIES; SPECIFIC GRAVITY; MELTING-POINTS.

Metals have various properties. Some remarks on these, and other particulars respecting metals, now follow.

(96.) *Malleability*.—A property which is possessed by metals in varying degree is that of malleability, that is, of permitting extension of surface without rupture, by, for example, hammering, pressure, or rolling. Gold, which is capable of being hammered into leaves of extreme thinness, is the most malleable of all metals. Other metals, though malleable to a considerable degree, require to be *annealed* (heated red and allowed to cool down slowly) once or even several times during the operation of rolling out, or extending by the hammer as in raising and stretching. Copper is an example; though, curiously enough, copper is equally malleable whether, after heating, it is allowed to cool slowly or is cooled suddenly by dipping while at red heat in cold water. Zinc is in its most malleable condition at a temperature a little above the boiling-point of water: when less than half as hot again as this, it is brittle and unworkable.

(97.) Of the theory of annealing nothing definite appears to be known; but it is supposed that on rolling out or hammering a piece of metal the particles or molecules of which the metal is composed, become strained and disarranged ('the grain closed'), and the metal is hardened; and that on heating, the metal expands, and the strain being removed, the molecules rearrange themselves. This, however, does not explain many matters connected with

annealing; for example, why one heated metal is hardened by being suddenly dipped in cold water, and another metal softened when treated in the same manner.

The following, which illustrates the effect of 'hammer hardening' on iron, may be of interest:—

(98.) In 1854, at the meeting of the British Association in Liverpool, a paper on the crystallisation of iron under certain circumstances was read by Mr. Clay (Mersey Iron and Steel Works), who stated that he selected a piece of good, tough, fibrous bar-iron, which he heated to a full red heat, and then hammered by light, rapid, tapping blows, until it was what is called 'black-cold.' After complete cooling he broke it, and found that the structure of the iron was entirely changed, and that, instead of bending nearly double without fracture, as it should have done, and breaking with a fine silky fibre when fracture did occur, an entire alteration had taken place, and the bar was rigid, brittle, and sonorous, incapable of bending in the slightest degree, and breaking with a glassy, crystallised appearance. By simply heating the bar to full red-heat again, the fibre was restored exactly as before.

(99.) *Tenacity*.—The property in metals of resisting being torn asunder by a tensile or stretching force, is called *tenacity*.

The tenacity of metals varies with their purity and molecular condition, as due to modes of treatment or preparation: for example, the tenacity of steel is much influenced by its 'temper,' and that of cast iron made by the *cold-blast* process is greater than when the process is that of the *hot-blast*.

(100.) *Ductility*.—The property of being permanently lengthened by a tensile or stretching force, as in wire drawing, is called *ductility*. All the malleable metals are more or less ductile, though the most malleable metals are not necessarily the most ductile; ductility being influenced more by tenacity than by malleability.

The table shows how some of the metals, starting from gold (see 'Malleability,' above, § 96), rank under the headings given.

<i>Malleability.</i>	<i>Ductility.</i>	<i>Tenacity.</i>	
Gold Silver Copper Tin Platinum Lead Zinc Iron Nickel	Gold Silver Platinum Iron Copper Aluminium Zinc Tin Lead	Steel Iron Copper Platinum Silver Gold Zinc Tin Lead	

(101.) *Conductivity.*—Of all solids, metals are the best conductors of heat. The order of conductivity for a few important metals, beginning with the best conductors, is—silver, copper, gold, tin, iron, lead, platinum, and bismuth.

(102.) *Welding.*—An important property of some of the metals is that pieces can be ‘welded’ together, that is, incorporated with each other. Iron at a white heat is in a pasty condition and can be ‘welded’; that is, if two white-hot and clean surfaces of iron be brought into contact and pressed or hammered together, they thus are ‘welded,’ that is, become part of one and the same mass. If lead and gold in a fine state of division be strongly pressed together at the ordinary temperature of the atmosphere, they will form one mass.

(103.) *Hardness.*—The comparative hardness of metals is usually estimated by the force required to draw the metals into wires of equal diameter. In order of hardness we have—steel, iron, copper, silver, tin, antimony, and lead.

(104.) *Specific Gravity.*—Some substances are, in their nature, more weighty bulk for bulk than others. Thus, a cubic inch of lead is heavier than a cubic inch of iron; and a cubic inch of iron than a cubic inch of water.

By their *specific gravity* the weights, relatively to each other, of substances, are known. The standard of comparison is an equal bulk of pure distilled water, and if the specific gravity of a body is, say, 2, this means that it is twice as heavy as the same bulk of water.

The specific gravity of platinum is 21 ; platinum therefore bulk for bulk is 21 times heavier than water.

The specific gravity of antimony is 6·7, and a cubic foot of pure distilled water weighs very nearly 1000 ounces. Therefore a cubic foot of antimony weighs 6·7 times 1000 ounces, that is, 6700 ounces.

Knowing the specific gravity of a substance, we can find the weight of any volume of it, by multiplying the given volume, in cubic inches, by its specific gravity, and by 62·4 the weight in lbs. of a cubic foot (1728 cubic inches) of water, and dividing by 1728. Thus the weight of 48 cubic inches of cast copper, the specific gravity of which is 8·6, is

$$\frac{48 \times 8\cdot6 \times 62\cdot4}{1728}, \text{ is, that is to say, } 14\cdot9 \text{ lbs.}$$

As the relative weights of equal volumes of metals have often to be taken into consideration in using metals for constructive purposes, for example, in the covering of roofs, where weight is sometimes a matter of importance, a table of specific gravities follows.

TABLE OF SPECIFIC GRAVITIES AND MELTING-POINTS.

Metals.	Specific Gravities.	Melting-points. (Centigrade.)	Authority for Melting-points.
Antimony	6·7	432	Buillet.
Bismuth	9·8	268·3	Reimsdyk.
Copper (cast)	8·6	} 1054	Violle.
" (wrought)	8·8		
Steel	7·8	} 1300 to 1400	Buillet.
Cadmium	8·6		
Iron (wrought)	7·8	} 1500 to 1600	Person.
Lead	11·36		
Tin	7·29	326·2	Person.
Zinc	7·19	232·7	Person.
Aluminium	2·67	433·3	Person.
Nickel	8·30	1045	Violla.
Platinum	21·25	1450	Pictel.
		1775	Violla.

The melting-points of the metals named are added to the table, as it is often useful to be able to refer to these. The degrees of heat are according to the Centigrade thermometer. This portion of the table is taken from 'Melting and Boiling Point Tables,' by Thos. Carnelley, 1885.

We now proceed to notice more particularly the metals iron (including cast iron and steel), tin, zinc, and copper.

IRON AND STEEL.

(105.) Iron in a state of purity is comparatively little known; the ores of it are various and abundant. In its commercial forms, as plate or sheet, bar, and cast iron, it is well known. As sheet it can be cut into patterns and bent into desired forms; as bar it can be made hot and 'wrought,' that is, shaped by means of the hammer; and when molten it can be run or cast into all sorts of shapes. Cast iron is brittle, crystalline in fracture (§ 95), and not workable by the hammer. In sheet and bar form, iron is malleable, mostly fibrous in fracture, and capable of being welded (§ 102). The presence of impurities in bar iron, that is, the presence of substances not wanted in it at the time being, seriously affects its malleability. Thus the presence of phosphorus, or tin, renders it brittle when cold ('cold-short'), and the presence of sulphur makes it unworkable when hot ('hot-short'). Iron quickly *rusts* (oxidises, § 94) if exposed to damp air, as in the case of iron exposed to all weathers; or to air and water, as with vessels in which barely sufficient water is left to cover the bottoms, the rusting (oxidation) being then much more rapid than when the vessels are kept full. Heated to redness and above, 'scale' (oxide of iron) rapidly forms and interferes greatly with welding. It is impossible to enter here into any consideration of the processes by which iron is prepared from its ores. To two modern processes, however, we shall presently have particularly to refer.

(106.) The effects of the presence of several foreign sub-

stances in iron as impurities has just been alluded to, but the presence in it of 'carbon' we have not spoken of. This is a substance which in its crystalline form is known as the diamond, and in its uncrystalline form as charcoal. The presence of carbon in iron destroys its malleability, but at the same time gives to it properties various, so remarkable and useful to mankind, that to say, as a defect, of a piece of iron with carbon in it, that it is not malleable, is simply equivalent to saying when we have a piece of brass, that it is not a piece of copper. Quite the reverse of being 'matter in the wrong place,' carbon in iron furnishes a compound so valuable on its own account, so entirely of its own kinds (in the plural because its kinds are several), that, if there were other substances not metals, the compounding of which with a metal gave products at all resembling those of iron and carbon, then all such compounds would form a class of their own. The iron and carbon compound, however, stands inconveniently alone. There we shall not leave it, but as aiding the full comprehension of it, notwithstanding that we define *alloys* (§ 115) as compounds of *metals*, shall consider it not as outside but as within this class of substances, as well as also shall speak of iron as being *alloyed* with it. We shall deal with it, however, under the present heading, treating compounds of actual metals later on.

(107.) Iron is alloyed with carbon in proportions varying from say $\frac{1}{2}$ to 5 per cent. When in the proportion of from 2 per cent. upwards, the compound is cast iron, that is, iron suitable for casting purposes; in other proportions it is known as steel. In cast iron the metallic appearance is somewhat modified; in steel it is maintained. Originally steel was made by the addition of carbon to manufactured iron, and the word had then a fairly definite signification; meaning a material of a high tensile strength; that by being heated dull red and suddenly cooled could be made so hard that a file would not 'touch' it, that is, would slide over it without marking it; and that could have that hardness modified or '*tempered*' by further application of heat. But

with the introduction of the Bessemer process of steel making, and of the Siemens' process of making steel direct from the ores, processes by which any desired percentage of carbon can be given, the signification of the word has become enlarged, and now includes all alloys of iron and carbon between malleable iron and cast iron; except that the term 'mild' steel is sometimes applied to those alloys that approach in qualities to malleable iron. Steel plates are now produced equal in toughness, and it is said even excelling the best 'charcoal' plates, and as they are much cheaper, the old charcoal-plate-making process is very generally giving way to the direct process. In practice, however, these plates are found to be more *springy* than good charcoal plates, and not so soft and easy to work.

(108.) As iron is very liable to rust, surface protection is given to it by a coating of tin, or of an alloy of lead and tin, (lead predominating), or of zinc. Plates coated with tin are termed '*tin*' plates; with lead and tin have the name of '*terne*' plates, and if coated with zinc are said to be '*galvanised*.' Terne plates are used for lining packing-cases, also for work to be japanned. Usual sizes of tin and terne plates are 14" x 10", 20" x 14", 20" x 28", and they are made up to 40" x 28".

(109.) Large iron sheets of various gauges coated with tin and having the same appearance as a 'tin' plate are called *Manchester plates*, and sometimes *tinned iron*. But the latter term is more generally applied to sheets of iron which are coated with lead and tin, and are dull like terne plates.

(110.) Iron coated with zinc is not so easily worked as when ungalvanised. In galvanising, the zinc 'alloys' with the surface of the iron, and this has a tendency to make the iron brittle. Galvanised iron is useful for water tanks and for roofing purposes, as the zinc coating prevents rust better than a tin coating. For roofing, however, 'terne' plates are largely used in America, and, kept well painted, are found to be very durable. Owing to the ease with which zinc is attacked by acids, galvanised iron is not suitable for vessels exposed to acids or acid vapours.

COPPER.

(111.) This, the only red metal (§ 93), is malleable, tenacious, soft, ductile, sonorous, and an excellent conductor of heat. For this reason, and because of its durability, it is largely made use of for cooking utensils. It is found in numerous states of combination with other constituents, as well as 'native' (uncombined). Its most important ore is *copper pyrites*. Copper melts at a dull white heat and becomes then covered with a black crust (oxide). It burns when at a bright white heat with a greenish flame. No attempt at explanation of its manufacture will here be made, as any description not lengthy would be simply a bewilderment. For the production of sheet copper it is first cast in the forms of slabs, which are rolled, and then annealed and re-rolled, this annealing and re-rolling being repeated until the copper sheet is brought down to the desired thickness. In working ordinary sheet copper, it is hammered to stiffen it, and 'close the grain.' *Hard-rolled* copper is, however, nowadays produced that does not require hammering.

(112.) In the course of the manufacture of copper it undergoes a process termed '*poling*' to get rid of impurities. We mention this because we shall find (§ 129) a similar process gone through in preparing solders. The poling of copper consists in plunging the end of a pole of green wood, preferably birch, beneath the surface of the molten metal, and stirring the mass with it. Violent ebullition takes place, large quantities of gases are liberated, and the copper is thoroughly agitated. It is doubtful if this poling process is fully understood, for, though it is quite obvious that there may be insufficient poling ('*underpoling*'), it is not easy to explain '*overpoling*.' But overpoling, as a fact, is fully recognised in the manufacture of copper, and the metal is brittle both if the poling is too long continued or not long enough. If duly poled, the cast slab when set displays a comparatively level surface; if underpoled a longitudinal furrow forms on the surface of the slab as it cools; if over-

poled, instead of a furrow, the surface exhibits a longitudinal ridge. Copper, duly poled, is known as 'best selected,' and as 'tough cake' copper.

ZINC.

(113.) Of this metal, known also very commonly as 'spelter,' *calamine* is a very abundant ore; another abundant ore is *blende*. The metal is extracted from its ores by a process of distillation, the metal volatilising (§ 94) at a bright red heat, and the vapour, passing into tubes, condenses, and is collected from the tubes in powder and in solid condition. If required pure, further process is necessary. This metal does not appear to have been known till the sixteenth century. Henkel, in 1741, was the first, at least in Europe, who succeeded in obtaining zinc from calamine. Zinc is hardened by rolling, and requires to be annealed at a low temperature to restore its malleability. Until the discovery of the malleability of zinc when a little hotter than boiling water, it was only used to alloy copper with, and sheet zinc was unknown. Zinc expands $\frac{1}{340}$ th by heating from the freezing to the boiling point of water. The zinc of commerce dissolves readily in hydrochloric and in sulphuric acid; pure zinc only slowly. If zinc is exposed to the air, a film of dull grey oxide forms on the surface; it suffers afterwards little further change. Zinc alloys with copper and tin, but not with lead; it also alloys with iron, for which it is largely used as a coating; iron so coated being known as 'galvanised' iron (§ 108).

LEAD.

(114.) Another metal that is prepared in sheet is lead. This metal was known in the earliest ages of the world; it is soft, flexible, and has but little tenacity. One of its principal ores is *galena*. Being a soft metal, it is worked ('bossed') by the plumber into various shapes by means of special tools, which often saves the making of joints. As it is compara-

tively indestructible under ordinary conditions, it is largely used for roofing purposes and for water cisterns. It is also used for the lining of cisterns for strong acids, in which case the joints are not soldered in the ordinary way with plumber's solder, but made by a process termed '*autogenous soldering*' or '*lead burning*.' Lead prepared in sheet by casting is known as *cast lead*, but when prepared by the more modern method of casting a small slab of the metal and then rolling it to any desired thickness is called *milled lead*.

ALLOYS.

(115.) An alloy is a compound of two or more metals. Alloys retain the metallic appearance, and whilst closely approximating in properties to the metals compounded, often possess in addition valuable properties which do not exist in either of the constituent metals forming the alloy. An alloy of copper and zinc has a metallic appearance and working properties somewhat similar to those of the individual metals it is made up of, and so with an alloy of gold, or silver, and a small percentage of copper. But the latter alloys have the further property of hardness, making them suitable for coinage, for which gold, or silver, unalloyed, is too soft. Like to this addition of copper to gold or silver is the addition of antimony to lead and to tin, by which alloys are obtained harder though more brittle than either lead or tin by itself. The alloy of lead and antimony is used for printer's type, for which lead alone is too soft.

(116.) Alloys are often more fusible than the individual metals of which they are composed. Thus while lead melts at 326°C ., tin at 233°C ., bismuth at 268°C ., and cadmium at 321°C ., an alloy of 8 parts bismuth, 4 tin, and 4 lead, forms what is technically known as a '*fusible*' alloy, meaning an alloy very readily fusible, the particular alloy stated melting indeed at 95°C ., that is, below the boiling-point of water. The addition of a little cadmium to the above forms a still more '*fusible*' alloy, called Wood's alloy, which melts at about 65°C

(117.) We give here the names of some of the more important alloys, with those of the metals of which they are made up.

Bronze	}	Copper and tin.
Bell-metal		
Gun-metal		
Speculum-metal		
Brass	}	Copper and zinc.
Dutch-metal		
Muntz' metal		
Hard solders (see p. 287) ..		
German silver	}	Copper, zinc, and nickel.
Britannia metal		
Pewter	}	Tin, antimony, copper, bismuth, and zinc.
Soft solders (see p. 287) ..		
Type-metal	}	Lead and tin.
	}	Lead and antimony.

ALLOYS OF COPPER AND TIN.

(118.) Alloys of copper and tin are known as bronze, gun-metal, speculum-metal, &c. Some of these alloys possess the property of becoming soft and malleable when cooled suddenly while red-hot, by dipping into cold water, but of being hard and brittle when cooled slowly.

Name of Alloy.	Composition per cent.	
	Copper.	Tin.
Bell-metal	78·0	22·0
Gun-metal	90·0	10·0
Speculum-metal	66·6	33·0
Bronze coinage	95·0	4·0 and 1 zinc.

(119.) Speculum-metal is a very hard, brittle, steel-grey alloy, capable of receiving a very smooth and highly-polished surface.

ALLOYS OF COPPER AND ZINC.

(120.) *Brass*.—Brass is the general name given to alloys of copper and zinc (ordinary brass consists of two of copper to one of zinc); by some writers also to alloys of copper and tin, now better known as *bronze*. Brass was known to the ancients, who prepared it from copper and calamine (§ 113), as they were unacquainted with the metal contained in calamine. We are said to have learned the fusing of copper with calamine from the Germans, and until a comparatively recent date brass, called calamine brass, was thus prepared in this country. Between 1780 and 1800, various patents were taken out for improving the manufacture of brass, by fusing copper and zinc direct instead of employing calamine. The calamine method, however, did not at once die out, as it was thought by some that calamine brass was better than that made direct.

In the manufacture of brass, the copper is first melted, because of its high melting-point, and the zinc, warmed, is then let down by tongs into the crucible containing the molten copper, plunged under its surface, and held there till melted. The mass is then stirred with a hot brass or iron rod, so as to mix the metals, great care being taken not to introduce any cold or damp matter. A little sulphate of sodium 'salt-cake,' or carbonate of sodium 'soda-ash,' thrown into the crucible at the moment of pouring, assists in raising any impurities to the surface, which can then be skimmed off as the mass is poured. With proper management, the loss of zinc is not so great as might be expected, considering the comparatively low temperature at which it volatilises, and the relatively high temperature necessary to melt the copper.

(121.) Brass is harder than copper, and therefore stands wear better; it is very malleable and ductile, may be rolled into thin sheets, shaped into vessels by 'spinning' (see § 124), stamping, or by the hammer, and may be drawn into fine wire. It is well adapted for casting, as it melts easily at a

lower temperature than copper and is capable of receiving very delicate impressions from the mould. It is said to resist atmospheric influences better than copper, but when its surface is unprotected by lacquer, it rapidly tarnishes and becomes black. It has a pleasing colour, takes a high polish, and is cheaper than copper.

(122.) The malleability of brass varies with its composition, and the heat at which it is worked. The malleability is also affected in a very decided degree by the presence of various foreign metals in its composition, even though these are present in but minute quantity. Brass intended for door-plate engraving is improved by the presence of a little tin; or by the presence of a little lead if to be used in the lathe or for casting. Brass for wire-drawing, however, must not contain lead; nor must brass intended for rolling contain antimony, which renders it brittle. Some kinds of brass are only malleable while cold, others only while hot, others are not malleable at all. A good example of the remarkable malleability of certain kinds is furnished by *Dutch Metal*, which contains a large proportion of copper, and which can be hammered into leaves of less than $\frac{1}{37500}$ th of an inch in thickness.

Though extremely tenacious, brass loses its tenacity in course of time by molecular change, especially if subject to vibration or continued tension. It is therefore unfitted for chains or for the suspension of weights. Chandelier chains have been known to lose their tenacity, become brittle, and break; and fine brass wire, which is of course brass in a state of tension, will, in time, become quite brittle, merely hanging in a coil.

(123.) *Muntz' Metal* is a variety of brass consisting of above three parts of copper to two of zinc, with about one per cent. of lead. The alloy is yellow, and admits of being rolled at a red-heat. It is extensively applied for the sheathing of ships, as it is said to keep a cleaner surface than copper sheathing.

Hard Solders.—These are treated further on.

The number of alloys of copper and zinc is considerable, and there is great confusion in respect of their names and composition. The table on opposite page, showing the proportions of some alloys of copper and zinc, is an extract from a table by Dr. Percy.

(124.) *Britannia Metal*.—We follow on with this alloy although not an alloy of copper and zinc alone. Britannia metal is highly malleable and one of the best of the substitutes for silver. It is composed of tin, antimony, copper, bismuth, and zinc, in various proportions, according to the purpose for which it is required. If the alloy is to be 'spun,' that is, worked into shape by specially formed tools whilst revolving in a lathe, a greater proportion of tin is used than when the alloy is only to be rolled. If it is to be cast, the proportion of tin is much less.

ALLOYS OF TIN AND LEAD.

(125.) Alloys of tin and lead furnish us with our soft solders, and are therefore of great practical value. Under the heading 'solders' further particulars of these alloys will be found. The melting points of the soft solders in the Table of Solders (p. 287) can be compared with the melting points (p. 274) of the individual metals of which the solders are composed, as illustrating what has already been stated about the fusing points of alloys often being below those of the metals forming them.

(126.) *Pewter*.—The composition of pewter varies considerably. Common pewter consists of tin and lead alone; the best contains also small percentages of antimony, copper, and bismuth; varying indeed but little from Britannia metal.

SOLDERS.

(127.) A solder is a metallic composition, by the fusion of which metals are united. The requirements of a good solder are twofold. (1) Its melting point must be below that of

No.	Composition per cent.		Specific Gravity.	Colour.	Qualities, &c.
	Copper.	Zinc.			
1	86.3	12.7	8.705	Reddish yellow ..	{ Not so good for rolling, wire-drawing, or hammering as common brass. Behaves irreproachably under hammer, rolls, and in wire-drawing. Working qualities the same as those of common brass. It is said this alloy was puffed up in England, under the name of Mosaic Gold.
2	85.4	14.6	8.591	Yellowish red.	
3	83.02	16.98	8.415	Yellowish red ..	
4	74.53	25.42	8.397	Pale yellow ..	{ Works excellently under the hammer, rolls, and in wire-drawing. Much to be recommended on account of its golden colour. Well adapted for cast ware; after casting very flexible while still hot; hard to cut when cold. Is unsuitable for wire-drawing, hammering, or rolling.
5	66.18	33.82	8.299	Full yellow ..	
6	52.0	48.0	8.229	Nearly gold yellow ..	{ A tough alloy. Too hard to file or turn. Lustre nearly equal to speculum metal (see § 119).
7	32.85	67.15	8.283	Deep yellow ..	
8	31.52	68.48	..	Silver white ..	
9	30.3	69.7	..	Silver white ..	
10	29.17	70.83	..	{ Silver white to silver grey.	

whatever metal is to be joined; (2) it must run easily when melted. It is comparatively easy (§ 116) to fulfil the first condition of a good solder. To fulfil the second requirement, one of the constituents of the solder must be either the same metal as that to be soldered, or a metal which will readily alloy with it, or which will readily coat its surface.

(128.) Solders are 'soft' or 'hard,' according to the temperature at which they melt. Hard solders fuse at a red heat. Soft solders are those which can be applied with a 'soldering iron,' that is to say, a 'copper bit,' or 'plumber's iron,' or with a mouth blowpipe; these solders melt below say 300° C. Hard solders have a much higher fusing point, and require either a forge or a blast blowpipe to apply them. Soldering with hard solders is termed 'brazing.'

(129.) An important particular in the preparation of solders is that they should be well stirred before pouring, preferably with a piece of green wood (§ 112), and the surface of the molten metal exposed as little as possible to the air, so that 'dross' (oxide) shall not form on the surface. A few knobs of charcoal on the molten metal will to a very great extent prevent the formation of dross.

(130.) Examining the soft solders, we see that plumber's solder melts at 227° C., that is to say, at a lower melting point than the metal (lead pipe), for soldering which it is used. Further, it is largely composed of lead. It thus fulfils both the requirements of a good solder. Tinman's solder melts at 160° C. It is used for soldering tin-plate, which, remember, is iron coated with tin. Tin melts at 233° C., a higher temperature than that of its solder, and tin is a constituent of the solder. Again the conditions of a good solder are fulfilled. Tinman's solder is also used for soft-soldering copper, because an alloy of lead and tin will readily coat copper, as also readily alloy with it.

TABLE OF SOLDERS.

No.	Name.	Metals composing it, and their proportion.		Melting-point. (Centigrade.)	Uses.
SOFT SOLDERS.					
Lead. Tin.					
1	Plumber's solder	2	1	227°	For joining lead-pipe.
2	Coarse "	3	1	250°	{ Blowpipe or gas-fitter's solder; some- times used by tinmen. { First class tinman's-solder; used also for soldering zinc, and the 'metal' wheels, floats, &c., of wet gas-meters.
3	Fine "	1	2	171°	
4	" "	1	1½	168°	
5	Very fine "	1	3	180°	Ordinary tinman's solder. Pewterer's solder.
6	Very fusible "	1	1	160°	
7	" "	1	1 and 2 bismuth	95°	
HARD SOLDERS.					
Copper. Zinc. Silver.					
1	Spelter, hardest	2	1	0	For ironwork, gun-metal, &c.
2	" hard	1½	1	0	For copper and iron.
3	" soft	1	1	0	For ordinary brasswork.
4	" finer	2	2	½	For finer kinds of brasswork.
5	Silver-solder, hardest	1	0	4	Hardest, but makes very neat joints.
6	" " hard	1	0	1	Makes a sound joint, and will not burn.
7	" " soft	1	0	2	For general use.

SOLDERING FLUXES.

(131.) Substances that 'flux' or aid the flow of metals when melting or melted are termed '*fluxes*.' The general subject of fluxes is outside our province; we are, however, specially interested in what we have designated '*soldering fluxes*,' namely, those fluxes that facilitate the flow of the solders and of the metals of which they are composed.

(132.) Essentially this 'fluxing' consists in the prevention of the formation of oxide (§ 94) to which metals are very prone when highly heated or molten. The black scale (§ 111) that forms on the surface of copper, for instance on copper 'bits,' when highly heated, is an oxide; also the scale that falls off red-hot iron when hammered (§ 105); and also the 'dross' that forms on the surface of molten lead or molten solder (§ 129).

(133.) The employment of charcoal (carbon) for the purpose of preventing the formation of dross we have already alluded to in speaking of the preparation of solders. Sometimes a layer of it is spread over the surface of the molten metal to keep it from contact with the air; sometimes a layer of grease.

In their character of aiding the flow of metals, fluxes are further applied to the surface of the metals to be soldered, which they clean, as well as aiding the flow of the molten solder when that is applied.

(134.) '*Spirits of salts*' (hydrochloric or muriatic acid) when 'killed' is a most useful flux for soft solders. The 'killing' is done by dissolving zinc in the acid till gas is no longer given off. As the gas is most offensive, the dissolution of the zinc should be effected in the open air. This flux is not one to be used where rust would be serious; though there is very little danger of this, if, after soldering, the joint is wiped with a clean damp rag, and further cleaned with whiting.

(135.) *Resin, or resin and oil* is a good flux for almost any



kind of soft soldering. The surface to be soldered must, however, be well cleaned before applying the flux.

(136.) '*Killed spirits*' (chemically, *chloride of zinc*) is specially useful for tin-plate soldering, because it assists in cleaning the edges to be joined; whereas if resin, or resin and oil, is used, the edge must, as stated, be cleaned previously.

(137.) Spirits of salts not killed is used for soldering zinc because it cleans the surface of the zinc; it acts indeed as chloride of zinc, for this is what it becomes on the application to the zinc, in fact the cleaning is the result of this action. The killed spirits, however, answers equally well as the strong acid if the zinc is bright and clean, so far as the experience of the writer has gone. The 'raw' (unkilled) spirits of salts is improved, as a flux for soldering zinc, by adding a small piece of soda to it.

(138.) Powdered resin, or resin and oil, as a flux, possesses the great advantage over chloride of zinc, that there is no risk of rust afterwards. For this reason resin, or resin and oil, is much used in the manufacture of gas-meters. It is also used, or should be, for the bottoms and seams of oil bottles. The resin and oil flux can easily be wiped off joints immediately after soldering; it is for this reason better than dry resin which has to be scraped off. Even this trouble, however, can be got over if the hot copper bit is dipped in oil before application to the joint to be soldered.

(139.) In 'tinning' a copper bit, that is, coating its point with solder before using it in soldering (a piece of manipulation of much importance as regards the easy working of the bit), the best thing to use is a lump of *sal-ammoniac*. In a small hollow made in the *sal-ammoniac*, the point of the bit, after having been filed smooth and bright, should be well rubbed, while hot, along with some solder; the point of the bit will then become coated with solder ('tinned'). For 'tinning' copper utensils, that is, coating them with tin, *sal-ammoniac* both in powder and lump is largely used. *Sal-ammoniac* water is also used for cleaning copper bits;

the hot bits being dipped into it prior to being used for soldering. Killed spirits, however, acts better. Sal-ammoniac and resin, mixed, is used as a flux for soldering 'sights' on gun-barrels.

(140.) As a flux for lead soldering, plumbers use tallow ('touch'). For pewter, Gallipoli oil is the ordinary flux.

(141.) For hard soldering, the flux is *borax*. This flux is also made use of in steel welding.

TINNING AND RE-TINNING.*

The remarks that follow have mainly in view the needs of the ordinary workshop. The object of covering the surface of a piece of sheet metal with tin is to protect the sheet from chemical action. In the case of iron, tinning prevents oxidation (rusting); in the case of copper cooking vessels, it protects the copper from the action of acids, which might prove injurious to health.

To ensure the proper union or *alloying* of the tin with the iron or other surface to be covered, this surface must be made perfectly clean, and to assist the flow of the molten tin over it a suitable flux must be employed. According to the purpose for which the sheet metal under treatment is to be used, it has to be tinned on one or on both sides. If the sheet is to be tinned on one side only, the tin is placed, either in a molten or a solid state, on the cleaned surface, and heat, from a forge fire or a gas blowpipe, is applied to the under surface, so as to heat the sheet of metal to a temperature sufficient to keep the tin melted if already molten or to melt the tin if this is applied solid. Greater care is needed with the flux when the tinning is on one side only than when on both sides, because there is a tendency for fluxes such as chloride of zinc (killed spirits) to dry and form a skin on the surface of the sheet before the tin has time to cover and unite

* The writer originally prepared this article for the *Ironmonger Diary*, in which it appeared.

with it. Hence fluxes such as sal-ammoniac, resin and tallow are the most suitable fluxes. Where a sheet has to be tinned on both sides the like care is not required, as it can be dipped bodily into molten tin. Care is required, however, that the sheet metal to be dipped is not wet or damp or even very cold, for if it is the tin will fly out, perhaps even to the serious injury of the operator. The sheet should be dipped gradually so that any dampness is steamed off and the sheet warmed before it is finally plunged and submerged in the molten tin.

RE-TINNING COPPER.

In the re-tinning of copper saucepans and other vessels that have been used for cooking, they should first be warmed and all grease, especially near rivets, carefully wiped off. They can then be cleaned by scouring thoroughly until bright with either wet forge scale, or better still, silver sand which has been moistened with lemon juice, and which may be applied either with a cork to secure friction, or with a piece of rough moleskin, or with tow or similar material. It is sometimes necessary, when articles are very dirty and where there are rivets, to first clean them with warm killed spirits before scouring thoroughly bright. In the case of copper moulds, it is often difficult and inconvenient to thoroughly scour the inside crevices. It will be found very helpful in such cases to half fill the moulds with a solution of equal parts lemon juice and spirits of salts, and after a few minutes to rub with a strip of cork. This greatly assists in securing a perfectly bright surface. Better results are obtained with the mixture of lemon juice and raw spirits of salts than with either of these liquids separately. When the vessel to be re-tinned has been thoroughly cleansed, it should be well washed for about a minute with chloride of zinc, and then if it is to be tinned on both sides, it should be gradually and carefully dipped into a bath of molten tin. For re-tinning a vessel on one side only it is advisable to prepare the surface that is to go next the fire by coating it

with a mixture of wet whiting and salt, as the untinned part can then be cleaned more readily afterwards. The vessel being placed on the fire, the tin is applied molten or solid. If molten or after melting, a little fresh mutton fat or a little powdered resin or powdered sal-ammoniac may be dusted over the surface to prevent oxidation. Then the vessel should be moved about gently so that the molten tin passes over the whole of the surface that is to be covered. If there are any spots to which the tin does not readily adhere, it may be wiped over them by means of a piece of tow or by a piece of rod wire that has been previously prepared by being coiled up at one end and tinned. It is useful to have at hand a lump of sal-ammoniac to make similar use of, wiping as it were with it the tin over the spots that do not readily cover. A large cork is also handy to rub the tin over the surface that does not willingly take it. The tin invariably clings about the edges of the vessel, and forms a list which must be shaken off. If it will not shake off, it must be wiped off gently and quickly with a piece of moleskin cloth, this is better than tow. When tinned either on one or both sides, the vessels should at once be washed with hot water, dried with bran, and polished bright, and it is important that this should be done immediately after the tinning if a bright surface is to be ensured. Scouring and pickling liquids may be drained off and saved, and used over and over again, as well as the sand, which will settle at the bottom.

RE-TINNING WROUGHT IRON.

Wrought iron is more difficult to tin than copper, and the surface requires longer time to prepare it to take a coating of tin, and the preparation is more troublesome. The article to be tinned should be heated to redness and afterwards placed in dilute sulphuric acid for about twelve hours and then immersed in chloride of zinc (killed spirits) for about six hours, or, if the article cannot be annealed then it should

be "pickled" in raw spirits of salts for twenty-four hours (it is better to do this in a closed vessel) and afterwards immersed in chloride of zinc in which sal-ammoniac has been dissolved. In either case, if there are patches on the surface which are difficult to clean or tin, it will be advantageous to well rub them with a piece of wet pumice stone. A wrought-iron stewpan which has been previously tinned and is in fair condition, may only require warming to clean off the grease, and judgment must be used as to the length of time required for the pickling, the article being examined at intervals to see whether the surface is clean and bright. The wrought iron vessel thoroughly cleaned by either method, should now be gradually dipped in a bath of molten tin, and allowed to remain there for two or three minutes. If required to be tinned on one side only, the tinning may be done in the same way as already explained for copper.

RE-TINNING CAST IRON.

Cast iron is most difficult to tin, and for this reason one would never recommend an attempt being made to re-tin cast-iron cooking vessels as it pays better to substitute new ones. If cast iron is to have anything like a good appearance when tinned, it must not be spongy, and its surface should be rendered smooth either by grinding, filing or machining, whichever it is most convenient to do. If possible the casting should be made red hot, and then pickled for twenty-four hours in slightly diluted spirits of salts (hydrochloric acid), and immediately on being removed should again be completely immersed for about two hours in chloride of zinc. If the casting cannot be made red hot, it should be immersed for about ten minutes in dilute sulphuric acid and warmed to about 90° Fahrenheit, then pickled in raw spirits of salts for two days, and afterwards allowed to soak for two hours in a mixture of chloride of zinc and sal-ammoniac, about two ounces of the latter to a gallon of spirits.

The casting, by whichever method cleaned, is now ready to be dipped gradually into the tinning pot. Should it not be sufficiently tinned the first time, the surface should be well rubbed with a piece of cork directly the casting is withdrawn from the melted tin, and be again dipped in chloride of zinc, and then once more into the tinning pot. Sometimes it is required to tin a part of a casting and to use the copper bit. When such is the case it is essential that the surface be cleaned and smoothed by scraping or otherwise, then soaked with raw spirits of salts, and then with chloride of zinc in which sal-ammoniac has been dissolved. Then the tin should be applied by means of the copper bit, and a piece of cork used to rub in the tin where there is a difficulty to get it to adhere. Tinning in this way, though not so strong as tinning in the way just described, is sufficiently effective for most purposes.

CHAPTER II.

SEAMS OR JOINTS.

WE notice here and illustrate the more important seams or joints used in metal-plate work. The drawings are intended to aid in the intelligent comprehension of the formation of joints, and not as exact representations of them.

Lap Seam.—In No. 1 is shown how metal plates are arranged for a lap seam which is to be soldered.

Circular Lap Seam.—No. 11 shows how the edge of the bottom of a cylindrical article is bent up previous to soldering. It is evident that this seam is essentially No. 1 seam adapted to the fitting a bottom to a cylinder. Such bottom is called a 'snuffed on' or 'slipped on' bottom.

Countersunk Lap Seam.—This is represented in No. 2. It will be seen that the edge of one of the plates is bent down, so that the edge of the plate to be joined to it may lie in the part bent down, and that the two plates when joined may present an unbroken surface.

Rivetted Lap Seam.—This is shown in No. 8. The amount of lap should not be less than three times the diameter of the rivet.

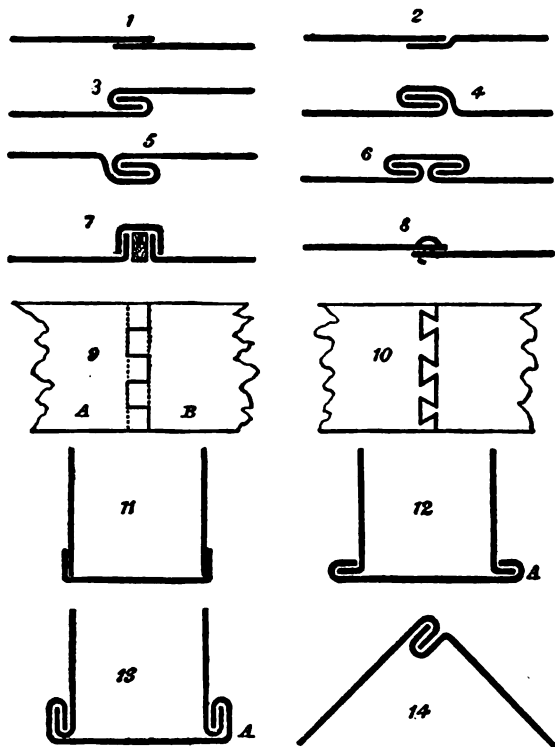
Folded Seam.—No. 3 shows how the edges of plates are prepared for folded seam.

Circular Folded Seams.—With a circular article the folded seam is sometimes in the form of No. 12, which shows a 'paned down' bottom to a cylinder. This seam is essentially No. 13.

Another form of circular folded seam is shown in No. 13. It is really No. 12 seam turned up, so as to lie close against the cylinder (see reference letter A in Nos. 12 and 13).

A bottom thus fitted is called a 'knocked up' bottom. Here again comparison with No. 3 should be made.

Double Folded Seam.—This is shown in No. 6, and needs no explanation. It is used with thick plates, where these when



joined have to present to the eye an unbroken surface; as in the hot-plates of large steam-closets.

Grooved Seam.—This is represented in No. 4. It will be seen that the seam is the same as No. 3, but one plate is countersunk. In fact No. 3 shows the seam as prepared for

countersinking ('grooving') with a 'groover.' Seam No. 6 is used where plates are too thick for grooving.

Countersunk Grooved Seam.—This seam (No. 5) is used when an unbroken surface is required on the outside of an article, for example, in toilet-cans, railway-carriage warmers. It is prepared as No. 3 and then countersunk the reverse way to No. 4.

Box Grooved Seam.—This seam, shown in No. 14, is used for joining plates in 'square work,' as for example where the ends and sides of a deed-box are joined together. It is essentially No. 3 seam.

Zinc-roofing Joint.—The arrangement for this joint is seen in No. 7. This joint admits of the expansion and contraction of the zinc sheets. The edges of two sheets, the wood 'roll,' and the 'roll cap' are shown. The zinc 'clip,' by which usually the sheets of zinc are held down, is not represented.

Brazing Joints.—A brazing joint for thin metal is shown in Fig. 9. The edge of plate A is cut to form laps as represented, and these laps are arranged alternately over and under the edge of plate B.

For thick metal the brazing joint is shown in Fig. 10. It is essentially the same thing as the 'dovetail' joint of the carpenter.

CHAPTER III.

USEFUL RULES IN MENSURATION.

To find the circumference of a circle, the diameter being given.

Multiply the diameter by $3\frac{1}{2}$. In other words, multiply the diameter by 22 and divide by 7.

Or, should closer accuracy be required, multiply the diameter by 3.1416.

Example I.—The diameter of a circle is 8 inches; to find the circumference.

$8 \times 22 = 176$, which divided by 7 gives $25\frac{1}{2}$ inches, the circumference required.

Or, $8 \times 3.1416 = 25.13$ inches.

To find the area of a circle, the diameter being given.

Multiply one-quarter of the diameter or, which is the same thing, half the radius, by the circumference.

Example II.—The diameter of a circle is 8 inches; to find its area.

One-quarter of the diameter is 2 inches; the circumference is $25\frac{1}{2}$ inches.

$2 \times 25\frac{1}{2} = 50\frac{1}{2}$ square inches, the area required.

To find the area of an ellipse, the axes being given.

Multiply the axes together, and multiply the result by .7854.

Example III.—The major axis of an ellipse is 6 inches and the minor 4 inches; to find its area.

$6 \times 4 = 24$, which multiplied by .7854 gives 18.85 square inches nearly.

To find the area of a rectangle.

Multiply the length by the breadth.

Example IV.—The length of a rectangle is 16 inches and the breadth 9 inches; to find its area.

$16 \times 9 = 144$ square inches, that is, one square foot, the area required.

To find the volume of a circular, elliptical, rectangular, or other tank, or vessel, of which the sides are perpendicular to the base.

Multiply the area of the base by the height.

If the answer is required in cubic inches, all the dimensions must be multiplied in inches. If in cubic feet, the dimensions must be in feet. (See Examples.)

Example Va.—The height of a circular tank is 6 feet, and the diameter of the base 8 feet; to find its volume.

By Example II. the area of the base is $50\frac{1}{2}$ square feet, which multiplied by 6 feet gives $301\frac{1}{2}$ cubic feet, the volume required.

Example Vb.—The height of an elliptical tank is 1 foot 6 inches, the base is 6 inches by 4 inches; to find its volume.

By Example III. the area of the base is 18.85 square inches, which multiplied by 18 inches (that is to say, by the height in inches, as the answer is to be in cubic inches) gives 339.3 cubic inches, the volume required.

Example Vc.—The height of a rectangular vessel is 2 feet 3 inches, the length 1 foot 4 inches, and the breadth 9 inches; to find its volume.

We will suppose the answer is required in cubic feet. This being so, it is in feet that the dimensions must be multiplied. Stated in feet, the height is $2\frac{1}{4}$ feet, the length $1\frac{1}{2}$ feet, and the breadth $\frac{3}{4}$ foot.

By Example IV. the area of the base is $1\frac{1}{2}$ feet multiplied by $\frac{3}{4}$ foot, that is, is $\frac{3}{2} \times \frac{3}{4} = 1\frac{1}{4} = 1$ square foot, which multiplied by $2\frac{1}{4}$ feet gives $2\frac{1}{4}$ cubic feet, the volume required.

To find the volume of a right cone.

Multiply the area of the base by the height and divide by 3.

Example VI.—The height of a cone is 6 inches, and the diameter of the base $3\frac{1}{2}$ inches; to find the volume.

The circumference of the base is $\frac{3\frac{1}{2} \times 22}{7} = 11$ inches.

The area of the base is one-half of $1\frac{1}{2}$ inches (the radius) $\times 11 = \frac{1}{2} \times 11 = \frac{11}{2} = 9\frac{1}{2}$ square inches.

And the area $9\frac{1}{2} \times 6$ inches (the height) = $57\frac{1}{2}$, which divided by 3 gives $19\frac{1}{2}$ cubic inches, the volume required.

To find the volume of a frustum of a right cone.

From the volume of the complete cone of which the frustum is a part subtract the volume of the cone cut off.

For example, the volume of the frustum C A B D (Fig. 8a, p. 33) is equal to the volume of the complete cone O A B less the volume of O C D the cone cut off.

The height of the complete cone and that of the cone cut off from it to form the frustum can be found by Problem V., p. 36.

To find the volume of a sphere, the diameter being given.

Multiply the diameter of the sphere by the area of a circle of same diameter, and take two-thirds of the product.

Example VII.—The diameter of a sphere is 8 inches. The area of a circle of same diameter is $50\frac{1}{2}$ (see Example II.); which multiplied by 8 gives $402\frac{1}{2}$ cubic inches, two-thirds of which ($268\frac{1}{3}$ about) is the volume required.

Or, Multiply the cube of the diameter by $\cdot 5236$.

Given the volume of a vessel, any vessel, to find the number of gallons, quarts, or pints that it will hold.

If the volume is in cubic feet, as in Example V a, then, to bring it to gallons, multiply by $6\frac{1}{4}$, there being in a cubic foot of water $6\frac{1}{4}$ gallons about. If the volume is in cubic

inches, divide by 277. The number of cubic inches in a gallon of water is $277\frac{1}{4}$ nearly; but in ordinary calculations, the quarter may be omitted.

If the volume is required in quarts, multiply it, if in cubic feet, by 25; if in cubic inches, divide it by 69. The number of cubic inches in a quart of water is about $69\frac{1}{4}$; in our examples here we have disregarded the fraction.

If the volume is required in pints, multiply it, if in cubic feet, by 50; if in cubic inches, divide it by 35. The number of cubic inches in a pint of water is rather more than $34\frac{1}{2}$; in our examples we have taken it as 35.

Example VIII.—To find the number of gallons, quarts, or pints, contained in the tank of Example Va.

Gallons.— $301\frac{1}{2} \times 6\frac{1}{4} = 1885\frac{1}{2}$ gallons.

Quarts.— $301\frac{1}{2} \times 25 = 7542\frac{1}{2}$ quarts.

Pints.— $301\frac{1}{2} \times 50 = 15085\frac{1}{2}$ pints.

Example IX.—To find the number of gallons, quarts, or pints, contained in the tank of Example Vb.

Gallons.— $339\cdot3 \div 277 = 1\cdot22$ gallons about.

Quarts.— $339\cdot3 \div 69 = 4\cdot92$ quarts about.

Pints.— $339\cdot3 \div 35 = 9\cdot7$ pints about.

Given the number of gallons, quarts, or pints, that a tank or other vessel contains, any vessel, of which the sides are perpendicular to the base, also the dimensions of the base, to find its height.

Divide the number of gallons by $6\frac{1}{4}$; this will give the volume of the required tank in cubic feet. If the quantity is given in quarts, then to ascertain the required volume, multiply by 69. If the quantity is given in pints, multiply by 35.

To find the required height for the tank, divide the volume found as just shown, by the area of the base.

**COMPARATIVE WEIGHTS AND GAUGES OF SHEET IRON, COPPER, AND
ZINC, AND OF TIN-PLATE.**

Gauge.	Equivalent by Birmingham Wire-Gauge.	Tin-Plates of Equivalent Strength.	Approximate Weight per Square Foot.		Zinc. Approximate Weight.		
			Iron.	Copper.	Per Square Foot.	Per Sheet 7 ft. by 3 ft.	Per Sheet 8 ft. by 3 ft.
			lb. oz.	lb. oz.	lb. oz.	lb. oz.	lb. oz.
4	33	0 4 $\frac{3}{4}$	6 4	
5	31	0 5 $\frac{1}{2}$	7 9	
6	30	1 C	0 8	0 9 $\frac{1}{2}$	0 6 $\frac{1}{2}$	9 0	10 5
7	29	..	0 9	0 10 $\frac{1}{2}$	0 7 $\frac{1}{2}$	10 6	11 10
8	28	{ 1 X and D C }	0 10 $\frac{1}{2}$	0 12	0 9	11 12	13 5
9	27	X X	0 11 $\frac{1}{2}$	0 14	0 10 $\frac{1}{2}$	13 5	15 8
—	26	S D C	0 13	0 15 $\frac{1}{2}$			
10	25	{ S D X D X }	0 15	1 0	0 11 $\frac{1}{2}$	15 1	17 3
11	24	{ 1 X X X X D X X S D X X X }	1 0	1 2 $\frac{1}{2}$	0 13 $\frac{1}{2}$	17 4	20 0
12	23	D X X X	1 2	1 5	0 15	19 12	22 11
13	22	D X X X X	1 4	1 8	1 1	22 4	25 7
14	21	D X X X X X	1 6 $\frac{1}{2}$	1 10	1 2 $\frac{3}{4}$	24 12	28 2
15	20	D X X X X X X	1 9	1 12	1 5 $\frac{1}{2}$	28 11	32 10
16	19	..	1 12	2 0	1 8 $\frac{1}{2}$	32 10	37 2
17	18	..	1 14 $\frac{1}{2}$	2 4	1 11 $\frac{1}{2}$	36 8	41 8
18	—	1 14 $\frac{3}{4}$	40 7	
19	17	..	2 3	2 8	2 3 $\frac{1}{2}$	45 0	
20	16	..	2 8	2 14 $\frac{1}{2}$	2 4 $\frac{1}{2}$	49 8	
21	15	..	2 13	3 4	2 8 $\frac{1}{2}$	55 0	
22	2 12 $\frac{3}{4}$	59 0	
23	3 1	64 8	

STRENGTH, SIZES, AND WEIGHTS OF TIN-PLATES.

Strength.	Sizes of Sheets in Inches.	Thickness in Birmingham Wire-Gauge.	Weights per Box and Number of Sheets.
			ew. qrs. lb.
1 O	14 × 10 14 × 20 28 × 10 28 × 20	30	225 sheets weigh 1 0 0 112 " " 1 0 0 112 " " 2 0 0 225 " " 1 1 0
1 X	14 × 10 14 × 20 28 × 10 28 × 20	28	112 " " 1 1 0 112 " " 2 2 0 100 " " 0 3 14
D O	17 × 12½ 34 × 12½ 25 × 17		50 " " 8 3 14
1 X X	14 × 10 14 × 20 28 × 10 28 × 20	26 easy	225 " " 1 1 21 112 " " 1 1 21 112 " " 2 3 14 200 " " 1 1 27
S D C	22 × 15 14 × 10 14 × 20		100 " " 1 1 27 225 " " 1 2 14
1 X X X	14 × 20 28 × 10 28 × 20		112 " " 1 2 14
D X	17 × 12½ 34 × 12½ 25 × 17	25 easy	112 " " 3 1 0 100 " " 1 0 14 50 " " 1 0 14
S D X	15 × 11 22 × 15 14 × 10 14 × 20		200 " " 1 2 20 100 " " 1 2 20 225 " " 1 3 7
1 X X X X	14 × 20 28 × 10 28 × 20	25 full	112 " " 1 3 7 112 " " 3 2 14 200 " " 1 3 13 100 " " 1 3 13
S D X X	15 × 11 22 × 15 14 × 20		100 " " 1 3 13
1 X X X X X	14 × 20 28 × 10 28 × 20		112 " " 2 0 0 112 " " 4 0 0 100 " " 1 1 7
D X X	17 × 12½ 34 × 12½ 25 × 17	24	100 " " 1 1 7 50 " " 1 1 7
S D X X X	15 × 11 22 × 15		200 " " 2 0 6 100 " " 2 0 6
S D X X X X	15 × 11 22 × 15	24 full	200 " " 2 0 27 100 " " 2 0 27

STRENGTH, SIZES, AND WEIGHTS OF TIN-PLATES—*continued*.

Strength.	Sizes of Sheets in Inches.	Thickness in Birmingham Wire-gauge.	Weights per Box and Number of Sheets.
			cwt. qrs. lb.
1XXXXXX	$\left\{ \begin{array}{l} 14 \times 20 \\ 28 \times 10 \\ 28 \times 20 \end{array} \right\}$	23 easy	112 sheets weigh 2 0 21
	$\left\{ \begin{array}{l} 17 \times 12\frac{1}{2} \\ 34 \times 12\frac{1}{2} \end{array} \right\}$	23	112 " " 4 1 14
DXXX	$\left\{ \begin{array}{l} 25 \times 17 \\ 17 \times 12\frac{1}{2} \end{array} \right\}$	23	100 " " 1 2 0
	$\left\{ \begin{array}{l} 34 \times 12\frac{1}{2} \\ 25 \times 17 \end{array} \right\}$	22	50 " " 1 2 0
DXXXX	$\left\{ \begin{array}{l} 17 \times 12\frac{1}{2} \\ 34 \times 12\frac{1}{2} \end{array} \right\}$	22	100 " " 1 2 21
	$\left\{ \begin{array}{l} 25 \times 17 \\ 34 \times 12\frac{1}{2} \end{array} \right\}$	21	50 " " 1 2 21
DXXXXX	$\left\{ \begin{array}{l} 34 \times 12\frac{1}{2} \\ 25 \times 17 \end{array} \right\}$	21	50 " " 1 3 14
DXXXXXX	$\left\{ \begin{array}{l} 34 \times 12\frac{1}{2} \\ 25 \times 17 \end{array} \right\}$	20	50 " " 2 0 7





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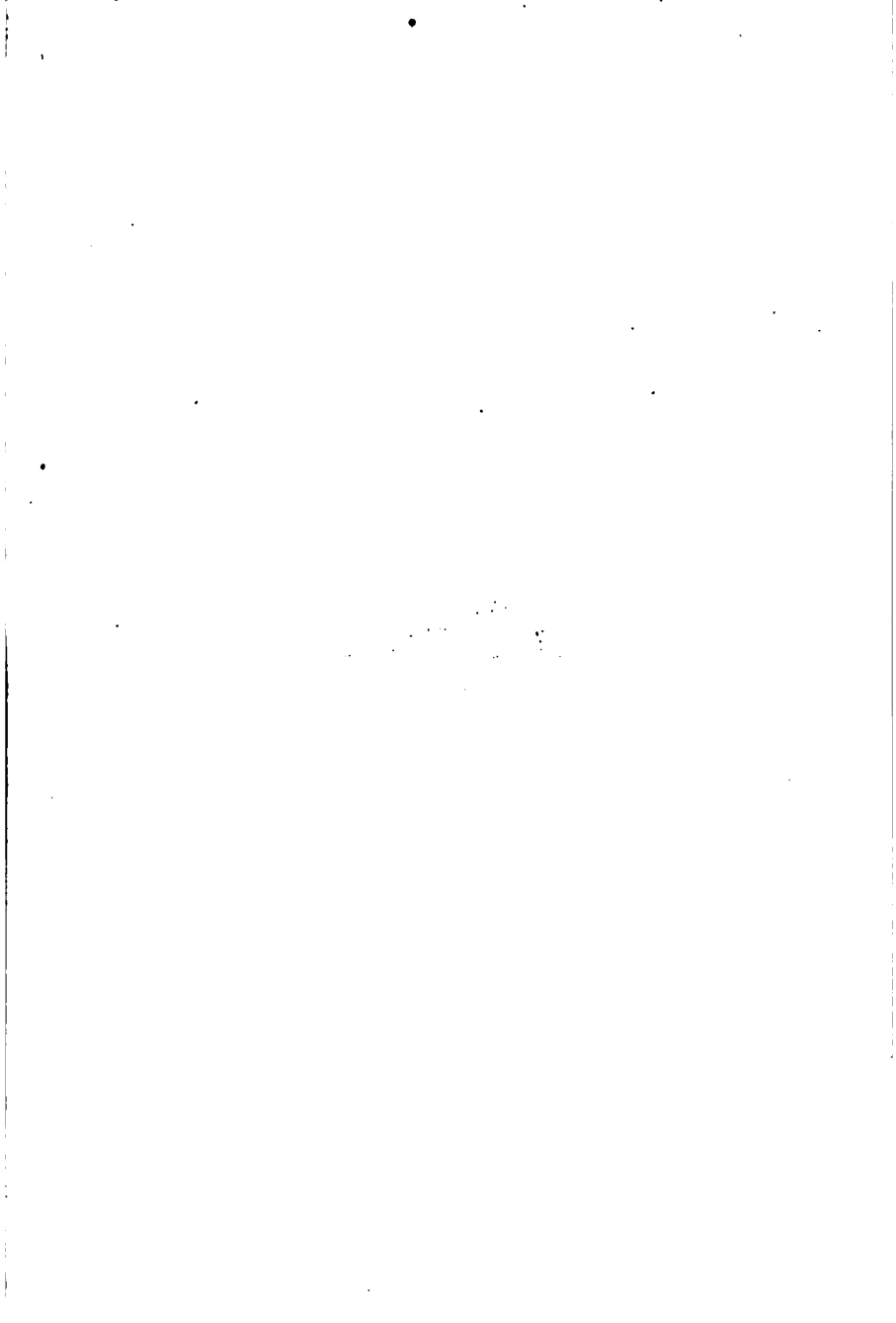
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